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Abstract

Prediction of planing forces on prismatic hulls far exceeding expectations by inconsistent theory

by

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"An essential characteristic of the application of mathematics to systems of great complexity (like fluids in motion) is that progress can be made only through an efficient cooperation between theory and experiment" Sir James Lighthill (1986)

Introduction

It is my hope to acquaint this astute group of specialists with the experimental facts of planing which are necessary to accommodate in any realistic theory and to show that by so doing to achieve remarkable agreement with data beyond expected ranges. A wealth of data exists at Davidson Laboratory, Stevens Institute of Technology obtained from measurements made on a family of models summarized by Savitsky (1964). Figure (1) on the last page displays the geometry of the tested prismatic hulls.

Definitive observations of flow patterns

Each of 4 models of deadrises of 0,10,20 and 30 degrees was towed free to heave at fixed successive trims at several loadings and at four speeds. Lift, drag and moment were measured and lengths of the wetted keel and chine were obtained from underwater photography.

These photographs of the wetted surface of prismatic models at rest and at planing speeds have revealed :

The wetted keel length at speed remains equal to the static length;

The forebody(FB) waterplane at beam Froude number (Fn) > 1 is nearly triangular with greatly increased apex angle by rotation of the at-rest waterline about the fixed apex

This is attended by commensurate increase in wetted chine as can be seen in Fig. (1).

Above water it is seen that the flow breaks away from chines and stern beyond a certain Froude number and that the fine spray and main spray leave the hull at angle nearly twice the dynamic semi-apex angle. This is indicative of a strong transverse velocity along the waterline.

One must follow the above advice of Sir James to respond to these observations in formulating a theory. Long before this advice Marshall. Tulin (1957) did so, being informed by his observations when water skiing! In the following his ingenious procedure is applied to hulls of Vee-sections.

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Outline of inconsistent theory

Application of slender-body theory (SBT) to a FB without an afterbody (AB) the velocity potential is taken to be that of a hydrofoil at its infinite Fn limit.

$$1)\qquad \varphi = \frac{1}{\pi}\int_{-a}^{a} dy'\gamma \arctan\frac{z}{y-y}$$

Folding the integration and dividing the range into a very large part, 0 to b(x), and from b to a(x) with a/b only slightly greater than 1.0,

The equation of the surface of prismatic hulls and components of the normal are:

$$(2)\quad z = y\tan\beta - x\tan\tau\,;\ \ (\partial F_x = \tan\tau\,;\ \partial_y F = -\tan\beta\,;\partial_z F = 1)/\sqrt{\tan^2\beta + \tan^2\tau + 1}$$

The integral equation arising from satisfaction of the kinematical condition is taken as:

$$(3)\quad v\tan\beta\ \ \frac{1}{\pi}\int_0^1 dt\frac{v(t)}{t-s} = U\tan\tau - v_b\int_1^{(a/b)^2}\frac{dt}{t-s}\,;\ 0<s<1$$

where v is the transverse velocity on the underside of the distribution of vorticity; v(b(x))= $v_b\,;t=(y'/b)^2\ ;\ s=(y/b)^2$

The inconsistencies are, of course, the inclusion of the transverse velocity as the first term in (2), the introduction of the singularity in the fluid along the waterline presented by the last term and the retention of the trigonometry functions rather than their arguments, However the experiments require a strong transverse velocity which is found to be:

$$(4)\quad v_b = 2Udb/dx = 2U\tan\alpha \text{ for triangular waterplanes;}$$

α is the semi-apex angle. ***This gives agreement with measured spray angle!***

Without inclusion of transverse velocity one ignores the expeimental evidence that this component dominates along the waterlines and the necessary attenuation of lift with increasing deadrise would be missed. With it agreement with measured lift is secured as seen below. As trim is not to exceed 8 degrees there is no need to use the argument. This restriction allows the use of large angles of deadrise.

The inversion provides two terms, one singular the other regular:

$$(5)\qquad v = \left(\frac{s}{1-s}\right)^{\mu}\left(-\frac{1}{\pi\mu}(2\varepsilon)^{\mu} + U\tan\tau\right) + \frac{2}{\pi}v_b s^{\mu}(1-s)^{\nu}\int_0^{\sqrt{2\varepsilon}} dq\frac{q^{-2\beta/\pi}}{q^2+(1-s)}$$

$\mu = \frac{1}{2}-\frac{\beta}{\pi};\ \nu = \frac{1}{2}+\frac{\beta}{\pi};\ \varepsilon = \frac{a-b}{b} << 1$. To have a finite velocity at the waterline:

$$\varepsilon = \left(\frac{\pi\mu}{v_b}U\tan\tau\right)^{1/2\mu} = \left(\sqrt{\pi}\mu\frac{\Gamma[1+\beta/\pi]}{\Gamma[1/2+\beta/\pi]}\tan\beta\right)^{1/2\mu}\ ;\text{ a function of only deadrise.}$$

Γ is the Gamma function. . The integral in the finite part of (4) is a Hypergeometric function from which the following results are obtained:

Longitudinal and transverse velocities:

$$(6)\qquad u=-\frac{\pi}{2}A_b\left\{\begin{array}{l}\dfrac{s^{\frac{1}{2}+\mu}}{(1-s)^{\mu}}\,{}_2F_1[1,\mu;1+\mu;\dfrac{2\varepsilon}{s-1}]+\\ \dfrac{1}{2}\left(\dfrac{\Gamma[1-\frac{\beta}{\pi}]\Gamma[1+\frac{\beta}{\pi}]}{\Gamma[\frac{3}{2}]}\right)-\dfrac{s^{1+\beta/\pi}}{1-\beta/\pi}\,{}_2F_1[1-\frac{\beta}{\pi},\frac{\beta}{\pi}-\frac{1}{2};2-\frac{\beta}{\pi};s]\end{array}\right\}\cos\beta\sin^2(\tau)$$

$$(7)\qquad v=\left(\frac{s}{1-s}\right)^{\mu}{}_2F_1[1,\mu;1+\mu;\frac{2\varepsilon}{s-1}]\cos\beta\tan\tau$$

$$(8)\qquad \text{Lift: } L=\pi\rho\left(\frac{1}{2}-\frac{\beta}{\pi}\right)b^2(a)U^2\tau\,;\quad 0<\tau<0.14$$

$$(9)\qquad C_L=\frac{\pi}{2}\left(\frac{1}{2}-\frac{\beta}{\pi}\right)A_R\tau\rightarrow\frac{\pi}{4}A_R\tau\ \ for\ \beta\rightarrow 0\,.$$

Expressed as a lift coefficient based on wetted area and aspect ratio:yielding half of the lift of the low-aspect-ratio wing result at zero deadrise as expected.

The total problem: Fore-and aft bodies interacting

The induction of vertical velocity by the FB loading on the AB and vice versa involves two coupled integral equations. One cannot use SBT to find the downwash induced by the triangular FB waterplane vorticity loading upon the rectangular AB-waterplane vorticity because the slope of the flow is by SBT equal to the trim angle. Detailed numerical calculations via vortex lattice by Lai et al (1994) show that the pressures drop nearly vertically along the FB-AB juncture from relatively large values to small values which decay slowly to zero.So there is some dynamic loading which arises from a weak upwash. Using an alternate form of the 3D representation of vorticity distributions the integral equation for the AB pressure distribution can be written as:

$$(10)\qquad w_{AB}=\frac{1}{4\pi}\frac{a}{b}\int_1^{l/a}dx'C(x')\int_{-1}^{1}\frac{y'dy'}{\sqrt{1-(y')^2}}\left[\frac{\sqrt{(x-x')^2+(\frac{b}{a})^2(y-y')^2}}{(x-x')(y-y')}+\frac{1}{y-y'}\right]$$

Where as found by Lai the pressure coefficient is distributed elliptically with C(x) the maximun along the centerline. Calculations of the vertical component induced by FB show flat lateral values dropping rapidly with x, enables the boundary condition to be satisfied along y = 0. With the substitution $y'=\cos\vartheta$ the integral equation for C(x) in $1<x<l/a$ is:

$$(11)\qquad \int_1^{l/a}dx'C(x')\left\{\frac{2E(k)\sqrt{(x-x')^2+s^2}+\pi(x-x')}{x-x'}\right\}=-\frac{4b}{\pi a}(\tan\tau+w_{FA}(x,0.0)))$$

where $k=s/\sqrt{(x-x')^2+s^2}$; s = b/a; E(k) is the complete Elliptic integral of the 2nd kind; and w(x,0,0) is:

$$w_{FA}(x,0,0)=\int_0^1 dx\frac{a}{b}\int_{-1}^{1}dy'v(y')\left(\left[\frac{\sqrt{(x-x')^2+(\frac{b}{a})^2(y')^2}}{(x-x')}-1\right]\frac{1}{y'}-\frac{2}{y'}\right)$$,where the last term

annuls the first term on the RHS of (11). leaving the integral equation with $\subset(x-x',y';s)$ the kernel in (11) as:

(12)

$$\int_{1}^{l/a} dx' C(x') \subset = -\frac{b}{\pi^2 a}\left(\int_{0}^{1} dx \frac{a}{b(x')}\int_{-1}^{1} dy' v(y')\left[\frac{\sqrt{(x-x')^2 + (\frac{b(x')}{a})^2 (y')^2}}{(x-x')} - 1\right]\frac{1}{y'}\right)$$

Since b/a << 1 the coefficient of the RHS is very small and as the role of y′ in the radical is strongly diminished. The RHS vanishes in about one beam **hence the dynamic lift on the AB is indeed small.** The numerator of the kernel is a regular function. Attempts to expand the numerator about x = x′ encounter singularities. The non-singular term in the kernel, π , produces an unknown constant involving the integral of C(x) which appears to be a "show –stopper" for numerical inversion . The two requirements at the FB-AB juncture and Kutta condition are also issues. This is the present state of this theory.

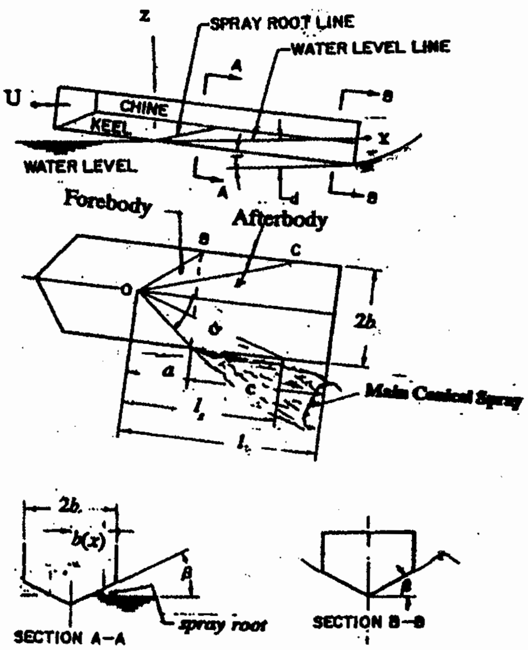

Fig. (1) Geometry of prismatic model and definition of lengths

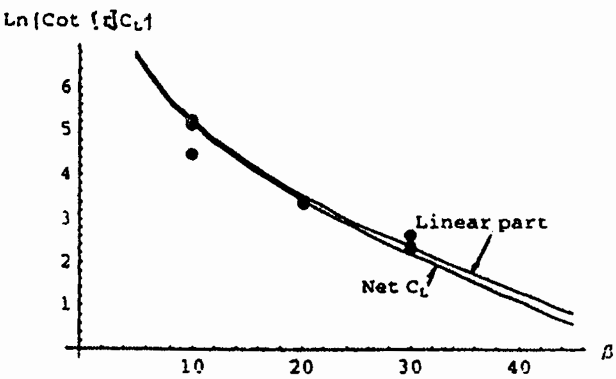

Fig. (2) Comparison of normalized lift coefficient with data vs daedrise from 10 to 30 degrees sans afterbody.

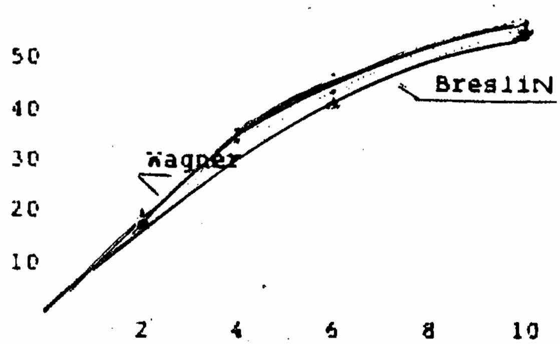

Fig (3) Agreement with data of semi-apex angle from 20 to 50 degs. vs. trim.