RADICAL DESIGN OPTIONS FOR WAVE-PROFILING WAVE ENERGY CONVERTERS

F.J.M. Farley Maritime Energy Developments Ltd, 8 Ch de St Pierre, 06620 Le Bar sur Loup, France *fjmfarley@wanadoo.fr*
R.C.T. Rainey WS Atkins Oil & Gas, Euston Tower, London NW1 3AT, U.K. *rod.rainey@atkinsglobal.com*

SUMMARY

Wave-profiling wave energy converters, typically in the form of a long articulated raft lying perpendicular to the wave crests, are one of the oldest concepts in this field, yet still one of the most promising. This paper considers two radical design options. Both depart from existing technology, but may offer attractive economics in the long term.

The first is a development of the first author's "buckling raft", tested at model scale 25 years ago (Farley 1982). It addresses the survivability and cost issues in the light of modern developments in offshore engineering. It is a radical design, which makes extensive used use of concrete and rubber. The second is even more radical, being an all-rubber design like a dracone. The articulations are eliminated, and power is extracted instead from internal pressure fluctuations, which are amplified by the propagation of "bulge waves" along the device. Energy is stored as elastic energy in the dracone fabric, rather than the potential energy of a conventional raft.

Attention is also drawn to the fundamental mathematical question of the existence of linear solutions in cylinders in head seas (Ursell, 1968), and the possible existence of trapped modes (Porter and Evans, 1998).

1. INTRODUCTION

Extracting power from the angular motion at the joints of an articulated raft is one of the oldest concepts in wave energy devices. It was certainly proposed (among other concepts) in the 1950s by the Japanese wave energy pioneer Yoshio Masuda, and the earliest related patents are probably 200 years old. In relatively recent times it has been appreciated that for good energy capture the device needs to be resonant, like a radio aerial, which means that a long device, with many articulations, needs to propagate vertical wave motion along it with the same speed and wavelength as the water waves to be absorbed (Farley, 1982). The fundamental problem this poses is that the period of such a vertical wave motion will be close to the natural heave period of an individual raft segment. If the segment has waterplane area W and average draft d, then its mass is ρWd, or $2\rho Wd$, say, including its heave added mass. Thus its natural heave period is:

$$2\pi\surd(2\rho Wd/\rho gW) \;=\; 2\pi\surd(2d/g) \qquad (1)$$

This is typically too short – e.g. if the raft has a mean draft of 2m, then it is 4 seconds, compared with period of the waves to be captured, which is typically 10 seconds.

The well-known PELAMIS wave energy device (currently the world's most commercially advanced, being now past the full-scale prototype stage and in series production for a wave farm in Portugal, see www.oceanpd.com), overcomes this problem by having the raft oscillate in an inclined plane in which the resolved force of gravity is much smaller. See the paper at the 2001 Workshop (Rainey, 2001), and the discussion which points out the practical advantages of this approach over the original concept in Farley (1982), which was to reduce the effective stiffness by compressing the raft axially with cables, to encourage buckling.

Fig 1. The prototype PELAMIS (photo courtesy OPD Ltd., Edinburgh, UK)

Compressing the raft axially does keep the stresses in the segments compressive, however, allowing concrete to be used extensively as the structural material, with important economic benefits. The original buckling concept is therefore developed further in Section 2, to address the practical difficulties just mentioned, by radical means. The design would require considerable development, however, and is a complete departure from the PELAMIS philosophy of using existing technology. It is effectively a possible long-term development of the present PELAMIS concept.

Section 3 describes an even more radical option. This is to eliminate the hydrostatic stiffness ρgW in (1) that is responsible for the troublesome short natural period, by setting $W = 0$. This gives a fully-immersed device, just beneath the surface. The necessary wave-making changes in immersed volume are produced by making the device flexible, so that its cross-sectional area *A* (perpendicular to its longitudinal axis) can change. Long flexible fluid-filled tubes, in the form of dracones, have been used for many years for transporting liquid cargoes, or for defensive booms, see Figure 2.

Fig 2. Dracone used as protective barrier (photo courtesy Dunlop Fabrications, Manchester, UK)

What is required is a much more elastic type of dracone, which can propagate a large "bulge wave" along it (see Lighthill 1978, section 2.2, for an account of bulge waves, and their existing applications e.g. to the propagation of pressure pulses along blood vessels), at the same speed as the wave. The fluid inside the dracone will then experience a resonant amplification of the exterior pressure, which is used to drive a piston at the end of the dracone.

Section 4 highlights a purely mathematical difficulty inherent in the analyses presented, which is that the problem of wave propagation along an infinite semi-immersed cylinder actually has no solution (Ursell, 1968). This is an interesting, and possibly important, mathematical point.

2. IMPROVED "BUCKLING RAFT"

The improved "buckling raft", sized for the Benbecula wave climate, is shown in Figure 3, taken from Farley, (2006).

It is much larger than the prototype PELAMIS, displacing about 5,000 tonnes rather than about 600. An axial compression of 10,000 tonnes is provided by a single large internal cable, like a suspension bridge cable. As the device bends vertically at the joints, the cable is free to slide up and down in a vertical slot in the beam structure. But in the horizontal plane the cable is constrained by the slot, so there is no tendency for the device to buckle horizontally. Similarly in large waves the cable contacts the end of the slot, which puts a soft limit on the buckling action in the vertical plane.

The vertical bending stiffness required at the joints, for the optimum performance described in Farley (1982), is 1,250 MN-m/rad. This appears to be feasible with rubber springs as shown in Figure 3, or, for lower losses, with pneumatic rubber springs which operate like car tyres, but at much higher pressure.

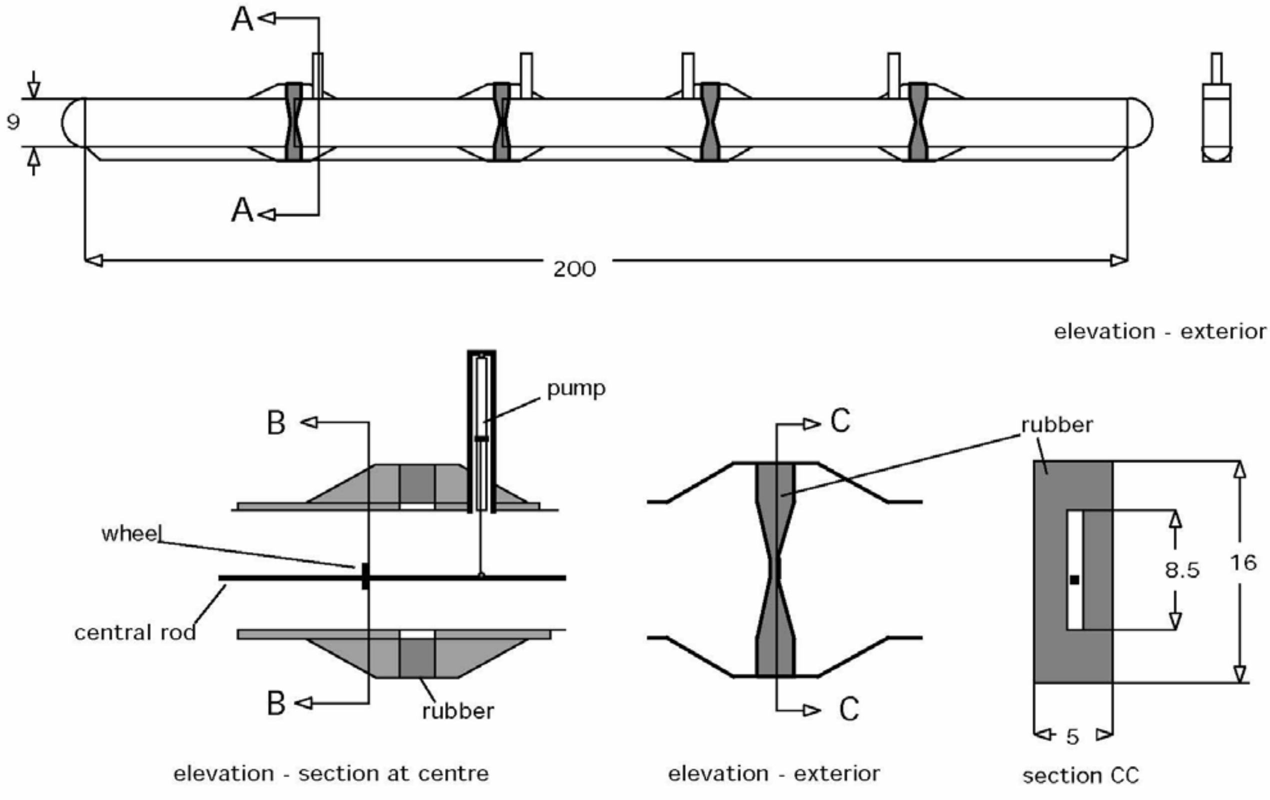

Fig 3. Buckling raft sized for Benbecula wave climate. Dimensions in metres.

The main structure is in compression almost everywhere, so it can be concrete, which is an order of magnitude cheaper than steel, provided the stresses are compressive, as here. The power take off can either be pneumatic, if pneumatic rubber springs are used, or hydraulic, from the movement of the tensioning cable, as shown in Figure 3.

The simplest case for calculating performance is a device which is not tuneable (although this device could be tuned, by varying the cable tension), but has the same power RAO in all conditions. The power RAO then needs to peak at the peak of the annual-average wave spectrum, which is at a period of 12.5 seconds according to WERATLAS (1997). This is the approach which has been followed in the design of Figure 3. A calculation using a numerical simulation allows for the power limit produced by cable contact at the ends of the slot, gives an annual average power at Benbecula of 2,400 kW. This is before conversion losses, which might reduce it to 2,000 kW, given the use of hydraulic or high-pressure pneumatic conversion. Overall, therefore, the power-to-weight ratio is similar to PELAMIS.

This suggests that PELAMIS may already be near optimum, as far as power-to-weight ratio goes. The design of Figure 3 seeks instead to find improvements in cost-effectiveness, by departing from the original PELAMIS philosophy of using only existing technology. The larger scale, and the extensive use of cheap materials (both to buy and maintain), may offer significant economic advantages in the longer term, albeit at the cost of more development work, especially on the rubber components.

3. "BULGE-WAVE" DEVICE

An even more radical design, relying even more on elastic material, is shown schematically in Figure 4. The articulated floating raft is replaced by a flexible dracone, which can propagate bulge waves along its length (with coordinate x) as shown. Consider the device in regular incident water waves of amplitude B:

$$\eta = B\sin(k(x - Ct)) \qquad (2)$$

which will induce a bulge wave of the same wavenumber k and speed C. This is in general not the same as the "natural" bulge wave speed C^*, that a bulge wave would have in still water.

The system is most easily analysed on the simplifying assumption that the device is infinitely long, so that the bulge wave amplitude has reached an equilibrium. Consider this equilibrium as a problem in steady flow, by adopting a frame moving with the wave (in the classical manner, see e.g. Lamb, 1932, Art 250).

By conservation of fluid, the axial fluid velocity (other components being negligible) inside the dracone where its cross-sectional area is A is $C(A_0/A)$ where A_0 is its mean cross-sectional area. If we write:

$$A = A_0 + dA \qquad (3)$$

then the internal velocity is $C(1 - dA/A_0)$.

We can now write down the internal pressure head p from the Bernoulli steady-flow formula, not forgetting the hydrostatic component, which is $-\eta$ on the surface, assuming the device exactly follows the wave profile (it is in fact unimportant if the device fails to follow the profile very closely, because the differential pressure across the dracone wall will be unaffected):

$$\begin{aligned} p &= -\eta + (0.5/g)[C^2 - (C(1 - dA/A_0))^2] \\ &= -\eta + (C^2/g)dA/A_0 \qquad (4) \end{aligned}$$

But by definition of the distensibility D of the dracone (Lighthill, 1978, Ch 2, eqn (17)):

$$dA/A_0 = D\rho g p \qquad (5)$$

and the "natural" bulge-wave speed C^* of the dracone satisfies (Lighthill 1978, Ch 2, eqn (31)) $\rho C^{*2} D = 1$ so that $dA/A_0 = gp/C^{*2}$ so that finally:

$$p = -\eta + p(C/C^*)^2$$

i.e. $$p = -\eta/(1 - (C/C^*)^2) \qquad (6)$$

This is a very simple result. If the dracone is very flexible, then C^* is small, and from (6) the pressure in the dracone is small too, as expected. The pressure inside the dracone is also in phase with η (i.e. positive maximum in a crest), so the dracone bulges out in the

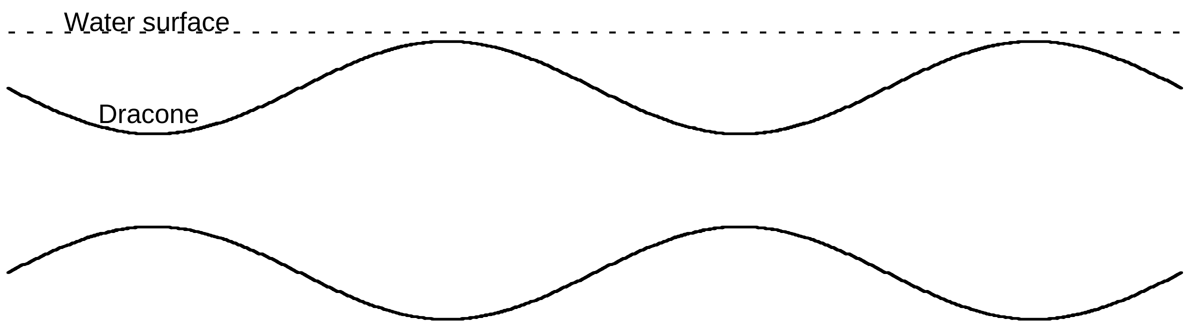

Fig 4. Bulge wave device

crests, as expected, following the vertical "extensional motion" in the wave, which will stretch A from a circle into an ellipse. As the flexibility of the dracone is reduced, C^* increases, until at resonace $C^* = C$. Thereafter the pressure inside changes sign, to become in anti-phase to η. So the dracone contracts in the crests.

To get the power, we need the fluid velocity in the dracone, which we already have above as $C(1 - dA/A_0) = C(1 - gp/C^{*2})$. This is the fluid velocity seen in the moving frame of reference, which is in the opposite direction to that of the wave propagation. In the usual earth-fixed frame of reference, and in the direction of wave propagation, it is:

$$\begin{aligned} Cgp/C^{*2} &= Cg[-\eta/(1 - (C/C^*)^2)]/C^{*2} \\ &= Cg\eta/(C^2 - C^{*2}) \end{aligned} \qquad (7)$$

If we write the horizontal surface particle velocity in the wave (in the direction of wave propagation) in this earth-fixed frame as V, then $g\eta = g(V/\omega) = \omega V/k = CV$ so the horizontal velocity in the dracone, in the earth-fixed frame, comes to:

$$V/(1 - (C^*/C)^2) \qquad (8)$$

This is another very simple result. For a very flexible dracone, C^* is small, so the horizontal velocity inside the dracone approaches that outside, as expected. As the flexibility of the dracone is reduced, C^* increases, and the velocity inside starts to exceed that outside but still be in phase with it. Then after resonance at $C^* = C$ the velocity inside changes sign, to become in anti-phase to the velocity outside.

The power take-off is simply a pump at the downwave end of the device, with the same pressure-volume characteristic as the dracone, so that it perfectly terminates the bulge wave. It could be a single large piston, of the same diameter as the dracone, with a diaphragm seal, driving one or more hydraulic rams. The power produced will be the product of volume flow rate in the direction of the wave, times the pressure. This evidently stays positive (as it should because the bulge wave is travelling in the same direction as the wave) - when $C^* < C$, pressure and velocity are both positive in a crest, and when $C^* > C$ they are both negative. The instantaneous power is:

$$\begin{aligned} &A_0[V/(1 - (C^*/C)^2)][-\eta\rho g/(1 - (C/C^*)^2)] \\ = \ &A_0V\eta\rho g/[-(1 - (C^*/C)^2)(1 - (C/C^*)^2)] \end{aligned} \qquad (9)$$

This is the instantaneous wave power flux through A_0 in the undisturbed waves, which is $A_0V\eta\rho g$, times a dynamic amplification factor which is always positive.
The annual-average power follows from the annual-average root-mean-square values of V and η. At Benbecula, WERATLAS (1997) gives them as 0.5 m/s and 0.85 m respectively, so the annual-average wave power flux though $A_0 = 10$ m^2 cross-section (same as PELAMIS, say) is 10(0.5)(0.85)1(9.81) = 45 kW.

For an annual average power of, say, 300 kW before conversion losses, we could choose $C^* = 0.82C$ giving a dynamic amplification of $-1/[(1 - 0.82^2)(1 - 0.82^{-2})] = 6.3$. The peak of the annual-average wave spectrum at Benbecula is given above as 12.5 seconds, which is a wave speed C of $g12.5/2\pi = 19.5$ m/s, so we require $C^* = (\rho D)^{-0.5} = 0.82\times19.5 = 16$ m/s. Hence the required distensibility D of the dracone is $1/16^2 = 0.004$ kPa^{-1}. If the wall thickness is 1% of its diameter, say, then Lighthill (1978) Ch 2 eqn 25 gives the required elastic modulus of the dracone material as 100/0.004 = 25,000 kPa = 25 MPa. This is typical of rubber.

The associated strains in the dracone material follow from dA/A_0, which is given above as $(V/C)/(1 - (C^*/C)^2) = kB(1 - (C^*/C)^2)$. So in a wave of steepness $kB = 0.1$, say, $dA/A_0 = 0.1/(1 - 0.82^2) = 0.3$. The strain in the material is half that, i.e. 15%. This is certainly feasible for rubber, which can easily take 10 times that strain. The issue will be the hysteresis losses, which need to be kept as small as possible. Hysteresis in synthetic rubber is fortunately a very well-researched topic, because of the automotive application. Car tyres likewise need low hysteresis losses, to prevent them overheating.

4. Existence of propagating waves along cylinders

The above analysis assumes that the steady state envisaged, with waves outside the dracone propagating steadily in the direction of the bulge waves, does exist. This is not clear from a mathematical point of view, because waves will not propagate along a semi-immersed cylinder in head seas (Ursell, 1968). In the case of a fully-immersed cylinder in head seas, there is also the possibility of trapped modes (Porter & Evans, 1998). These could both be important effects.

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Farley, F.J.M., Rainey, R.C.T.
‘Radical design options for wave-profiling wave energy converters

Discusser - D.V. Evans:

The idea is similar to the Lancaster flexible bag device developed by Michael French 30 years ago. The advantage of the bulge wave idea is that it is a resonant device which is essential for efficient energy capture.

Reply:

Certainly the Lancaster flexible bag was an influence on us - it was Michael French who first pioneered the use of rubber in WECs, as a fatigue-free, corrosion-free material. But he used it in lieu of a piston between air and water, in an otherwise rigid ship-like structure. We have no rigid structure, and are using it to store energy by stretching. And can achieve resonance, as you say.