# Nonlinear Effect in Diffraction Wave

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## 1. Result of experiment

Diffraction wave observed experimentally has a distinctive difference from the one predicted by linear theory. As seen in Fig.1 the diffraction wave measured down a line parallel to the track of a ship at $y = 0.204L/2$ ($y$ is the breadth-wise distance from the longitudinal center of the ship and $L$ the ship length ) has slightly longer wave length than the theoretically computed wave. The $x$ axis in Fig.1 directs towards forward of the ship. Extension of the wave length is small but the discrepancy is clear after a distance of several wave length. The amount of the extension is almost 10% of the original wave length.

The measured wave in Fig.1 was obtained by a technique proposed in Ohkusu (1996); the theoretical one was computed by a boundary panel method RPM with the free surface condition incorporating the double-model flow as the basic nonuniform steady flow and satisfied on the average calm water surface..

The discrepancy is also seen in full wave patterns. Computed asymptotic crest line ( only the dominant component ) is drawn over the measured diffraction wave contours in Fig.2 . The upper is the $\cos\omega t$ component and the lower $\sin\omega t$ component. Figure 2 is for the same hull form and conditions as Fig.1.

Here we observe the same discrepancy between the experimental and theoretical wave length down the flow: the experimental is longer than the theoretical. Another feature of the actual diffraction wave we recognize in this figure is that the crest line of the transverse wave part looks like an inclined line rather than an arc form theoretically predicted.

Comparison in the radiation wave pattern, which is presented not here but at the presentation, showed clearly that this discrepancy never happens with the radiation wave pattern: the wave length of the measured radiation wave is perfectly in agreement with the theoretical one. What exists for the diffraction, not for the radiation, is "incident wave". This fact leads us to believe the discrepancy would be some nonlinear interaction from the incident wave.

## 2. Wave length effects in tertiary wave interaction

The third order wave-wave interaction ( resonance ) demonstrated, for example, in Longuet-Higgins ( 1962) and Longut-Higgins and Phillips (1962) might explain the discrepancy observed in the diffraction wave ( this is inspired by Molin (2003) ). We reformulate the theory to adapt it to our case of a ship at non-zero cruising speed.

The coordinate system fixed to a ship is defined as shown in Fig. 3: the $x$ axis and the relative current $U$ representing the cruising speed direct backward of the ship. The incident wave going into the positive $x$ is

$$\phi_0 = a_0 e^{k_0 z} \sin(k_0 x - \omega_e t) \tag{1}$$

A component of the diffraction wave propagating into $\theta$ ( We take only a dominant longer-wave component. Hereafter all the arguments are for this component unless specially noticed. )

$$\phi_2(\theta) = a_2 e^{k_2 z} \sin(k_2(\theta) P - \omega_e t) \tag{2}$$

where

$$P = x\cos\theta + y\sin\theta, k_2 = \frac{g}{U^2}\frac{1 + 2\tau\cos\theta - \sqrt{1 + 4\tau\cos\theta}}{2\cos^2\theta}$$

We remake the theory for tertiary interaction of two wave trains $\phi_0$ and $\phi_2$ to be adapted for our case at the finite $U$. The free surface condition for the third order interaction $\phi_{21}$ on $z=0$ is:

$$\begin{aligned} & (\frac{\partial}{\partial t}+U\frac{\partial}{\partial x})^2\phi_{21}+g\frac{\partial\phi_{21}}{\partial z} \\ = \;& a_0^2 a_2 g\sqrt{k_0k_2}[\sigma_-\Gamma_-\cos^2\frac{\theta}{2}(1+\frac{4\sqrt{gk_0}\sqrt{gk_2}\sin^2\frac{\alpha}{2}}{\sigma_-^2-g\Gamma_-}) \\ & +\sigma_+\Gamma_+\sin^2\frac{\theta}{2}(1-\frac{4\sqrt{gk_0}\sqrt{gk_2}\sin^2\frac{\beta}{2}}{\sigma_+^2-g\Gamma_+}) \\ & +\sqrt{gk_0}(k_0-k_2+\frac{k_2}{4}\sin^2\theta)+\sqrt{gk_2}(k_0+k_2)\cos\theta] \end{aligned} \tag{3}$$

where $\sigma_\pm=\sqrt{gk_0}\pm\sqrt{gk_2}$, $\Gamma_\pm=\left|\vec{k}_0-\vec{k}_2\right|$.The vectors are defined as

$$\vec{k}_0\equiv(k_0,0),\ \vec{k}_2\equiv(k_2\cos\theta,k_2\sin\theta)$$

$\alpha$ and $\beta$ are the angles between $\vec{k}_2-\vec{k}_0$ and $\vec{k}_0$, and between $\vec{k}_2+\vec{k}_0$ and $-\vec{k}_0$.

The reduction in the wave number ( equivalently the increase in the phase velocity ) of $\phi_2$ resulting from its interaction from $\phi_0$ is given by

$$\frac{\Delta k_2(\theta)}{k_2(\theta)}=(k_0a_0)^2\frac{\sqrt{gk_2(\theta)}}{2\omega_e}F(\theta,\vec{k}_2^*) \tag{4}$$

Here we omit the details of $F(\theta,\vec{k}_2^*)$ but it is a function explicitly given by $\theta$ and $\vec{k}_2$ normalized by $k_0$. It is noticed that $\Delta k_2/k_2$ is independent of the amplitude of $\phi_2$.

Here we introduce an approximation. Diffraction wave is composed of wave components progressing into different directions emanating from the ship. For each direction different wave number $k_2(\theta)$ is assigned. There happens tertiary interaction between each other of those component, and between one of them and the incident waves. The interaction given by (3) is proportional to the square of the steepness of the interacting wave (3) and independent of the amplitude of the interacted wave. Since the steepness of every component of diffraction wave is much less than that of the incident wave, we may assume that the interaction from the incident wave to components of the diffraction wave will be predominant and it may be ignored the other way round. The interaction between two components of the diffraction wave may ignored with the same reason. The interaction from the incident wave to the diffraction wave is independent of the amplitude of the diffraction wave and consequently under our assumption the diffraction wave is expressed in a pseudo linear way as

$$\zeta=\int_0^{\pi-\alpha_0}H_2(\theta)e^{i[(k_2-\Delta k_2)(x\cos\theta+y\sin\theta)-\omega_e]}d\theta \tag{5}$$

$\Delta k_2$ is given by (3). The limit angle $\alpha_0(=\cos^{-1}(1/4\tau)$ ) of the wave propagation is also affected by the interaction but here it is kept as it was in the original linear expression for the sake of simplicity.

### 3. Results

One example of the reduction in the wave length $\Delta k_2$ computed by (3) with the $\theta$ component of a diffraction wave is shown in Fig. 4. As expected the wave length is lengthened into $X$ direction ($\theta\approx 0$) and shortened into $y$ direction ($\theta\approx\pi/2$). Magnitude of extension in the wave length in the $x$ direction is only 5% despite the steepness squared is exaggerated in this computation two times larger than that in Figs. 1 and 2. The tertiary interaction does not fully explain the discrepancy between the measured and the theoretical diffraction wave pattern.

Contour lines computed by (4) with the modified wave number and with assuming uniform amplitude function $f(\theta)$ =constant is given in Fig.5. Qualitatively the extension in the wave length along the $x$ axis is reproduced. The "line-like" crest line of the transverse wave part is not presented despite the large reduction in the wave length of the component going forward of the ship.

The reduction in the length of the components propagating forward will be expected to affect added resistance of the ship. Practical implications of the tertiary interaction will be presented at the Workshop.

## References


Molin B, Remy F, Kimmoun O and Ferrant P: The third order interaction and wave run-up, 18th IWWWFB (2003)

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Longuet-Higgins M S and Phillips O M: Phase velocity effects in tertiary wave interaction, JFM 12 (1962)

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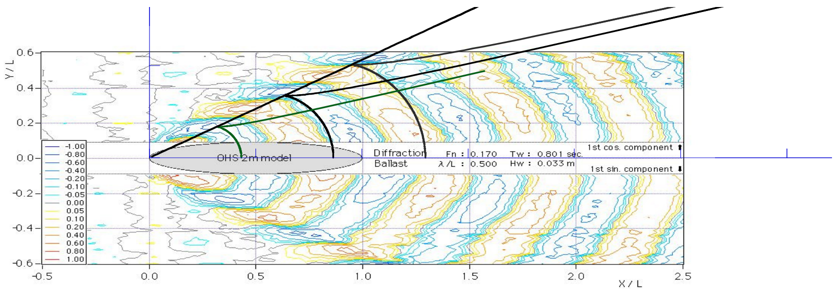

Fig.1 A full diffraction wave pattern

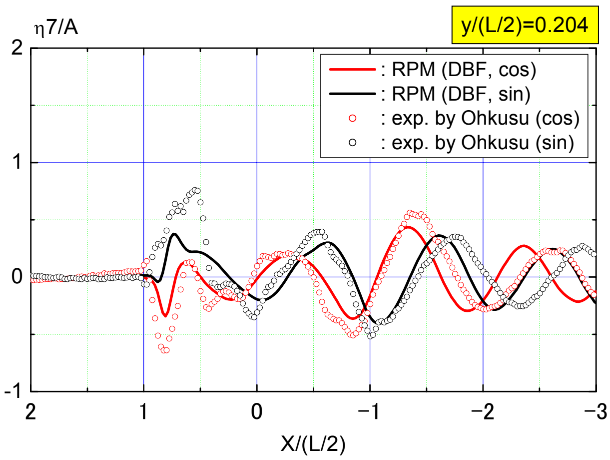


Fig.2 Diffaraction wave on y=0.204L/2

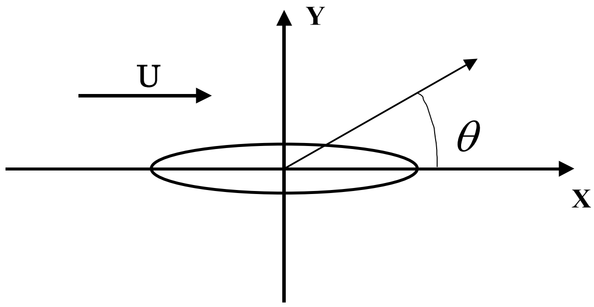


Fig. 3 Coordinate system

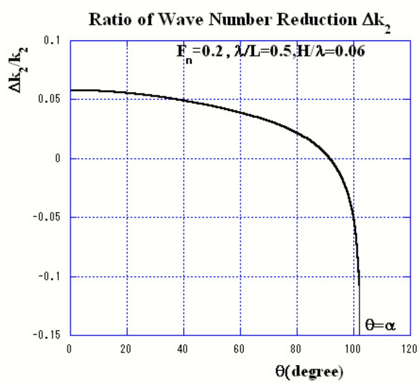


Fig.4 Wave number vs direction

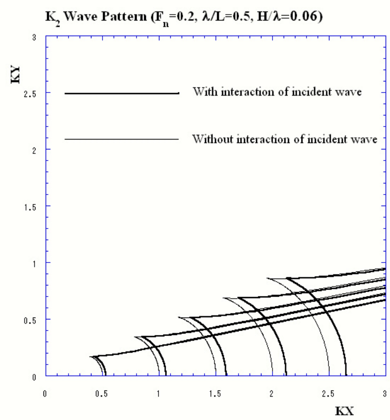


Fig. 5 Wave pattern