NON-LINEAR WAVE INTERACTION WITH A SQUARE CYLINDER

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1 Introduction

A companion paper offered at the workshop (Jamois *et al.*, 2005a) describes the application of a high-order Boussinesq model to oblique wave interaction with a vertical plate. This model, under development at EGIM, is further described in Jamois *et al.* (2004, 2005b). It has proved to properly reproduce the run-up effect, attributed to third-order interactions between the incoming and reflected wave-fields (Molin *et al.*, 2005).

Here we move one step backward and focus on second-order quantities, with the same aim of validating the Boussinesq model against reference results. There have been numerous studies dealing with second-order wave interaction with vertical circular cylinders (e.g. see Molin & Marion, 1986; Eatock Taylor & Hung, 1987; Newman, 1996; Ferrant, Malenica & Molin, 1999). However, in its present stage, the Boussinesq model can only handle (wall-sided) rectangular geometries, as it is based on regular cartesian discretizations in the horizontal plane. So we consider the parent case of a vertical cylinder with a square cross section, standing on the sea-floor. The first and second-order diffraction problems are solved numerically with WAMIT, convergence being assessed through successively finer discretizations. The Boussinesq model is run with incoming regular waves of such low steepnesses that no (third-order) run-up effects are observed. Fourier analysis of the time series yields fundamental and double frequency components that are compared with the results from WAMIT.

2 Test cases

In dimensional form, the waterdepth h is taken equal to 1 m, while the square cylinder side d is 2 m. WAMIT calculations have been made for two headings (0 and 45 degrees) and three wave periods (2.30, 1.45 and 1.16 s), leading to $kh = 1, 2$ and 3. The low-order version of WAMIT has been used.

The Boussinesq model was run only at zero degree heading. Advantage was taken of the symmetry to model only one half of the square cylinder, protruding from one of the side-walls. The numerical domain has a width of 12 m and a length of 10 wavelengths, with the (half) square starting 4 wavelengths from the wave generation zone. Of these the first two are used to damp out reflected waves, meaning that they propagate freely only over two wavelengths. This short distance was chosen in order to minimize nonlinear interactions between the incoming and reflected wave systems. It might have the drawback that the second-order forcing at the free surface is confined to a small domain. Another damping zone, also two wavelengths long, is located at the lee end of the domain.

3 First-order results

We consider the case $kh = 3$ with normal incidence. The Boussinesq model was run with a wave steepness H/L equal to 0.002, leading to incoming waves of wavelengths $L = 2.094$ m. At such a low wave steepness non-linear effects do not appear. The wavemaker region is relaxed over a single wavelength in the direction of propagation. Input wave conditions are obtained using the theoretical stream function solution given by Fenton (1988). The discretization used for this linear case, is $\Delta y = L/20 = 0.1047$ m, $\Delta x = 0.1043$ m (the end of the structure should lie half way between grid points) and $\Delta t = T/20 = 0.058$ s. Consequently, the half square cylinder dimensions actually are 2.09 x 1 m. The simulation was run up to a stationary state. Figure 1 show the free surface elevations around the cylinder and the vertical pressure profile at midpoint on the weather side computed by the Boussinesq model and by WAMIT. A very good agreement is obtained between the two numerical models. Some weak discrepancies appear in the vicinity of corner points. They might be linked to some numerical inaccuracies due to the steep gradients present around exterior corners.

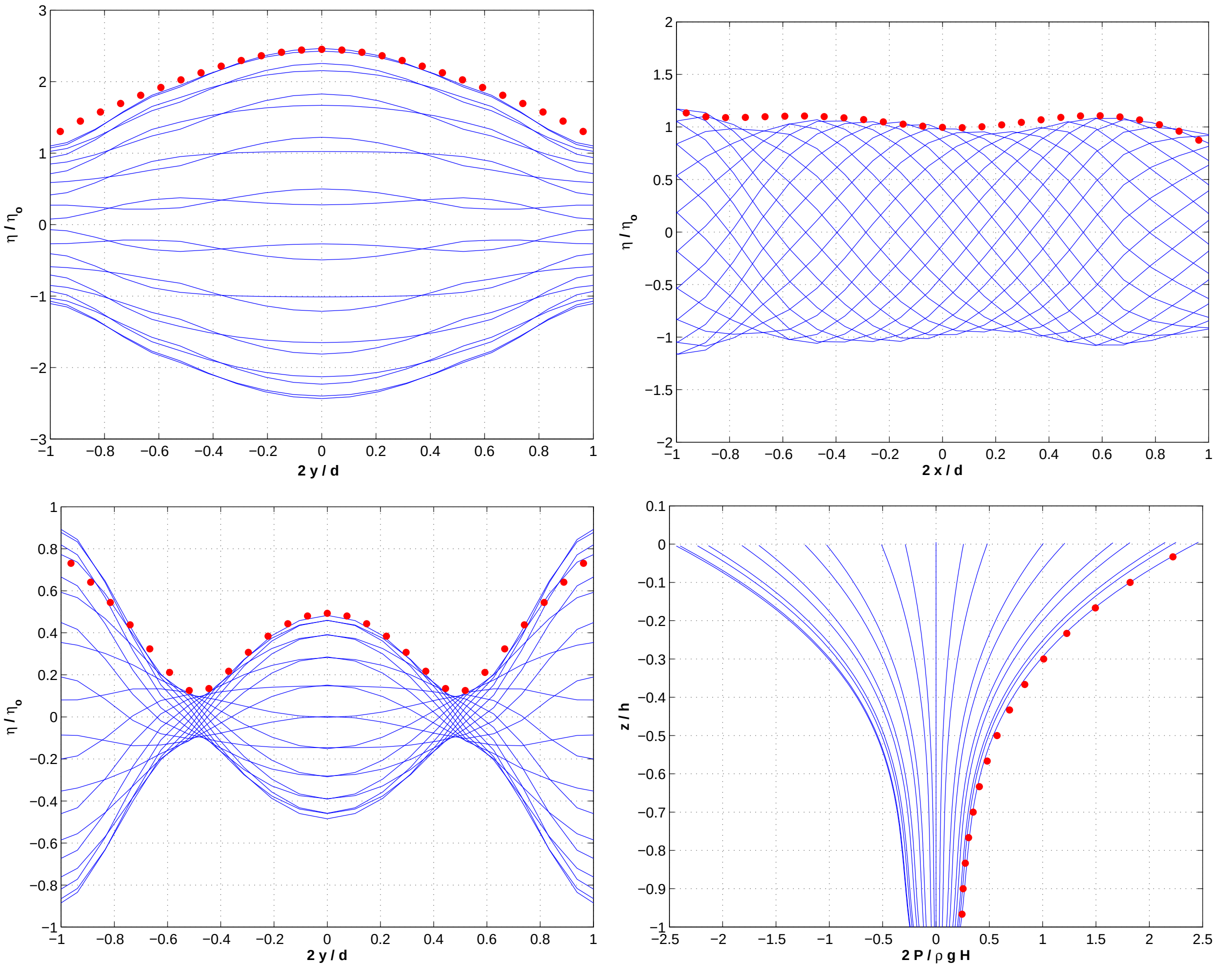

Figure 1: First-order free surface elevations around the structure, on the weather side (up left), the side (up right) and the lee side (down left), and first-order vertical pressure profile at midpoint on the weather side of the cylinder computed with the Boussinesq approach (solid lines) and WAMIT (dots).

4 Second-order results

We focus on the second-order diffraction potential at the double frequency 2ω. Figure 2 shows WAMIT results at zero incidence and for $kh = 1, 2, 3$. It shows, in non-dimensional form $(2\,\omega\,|\varphi_D^{(2)}(x, y, 0)|\,d/(g\,A^2))$, the modulus of the second-order scattered potential along the waterline. Two discretizations were used, leading to quasi identical values. In the coarse one 2730 panels are used on one quadrant of the square cylinder and 2700 panels on one quadrant of the free surface. In the refined one the numbers are 4840 and 4720 respectively.

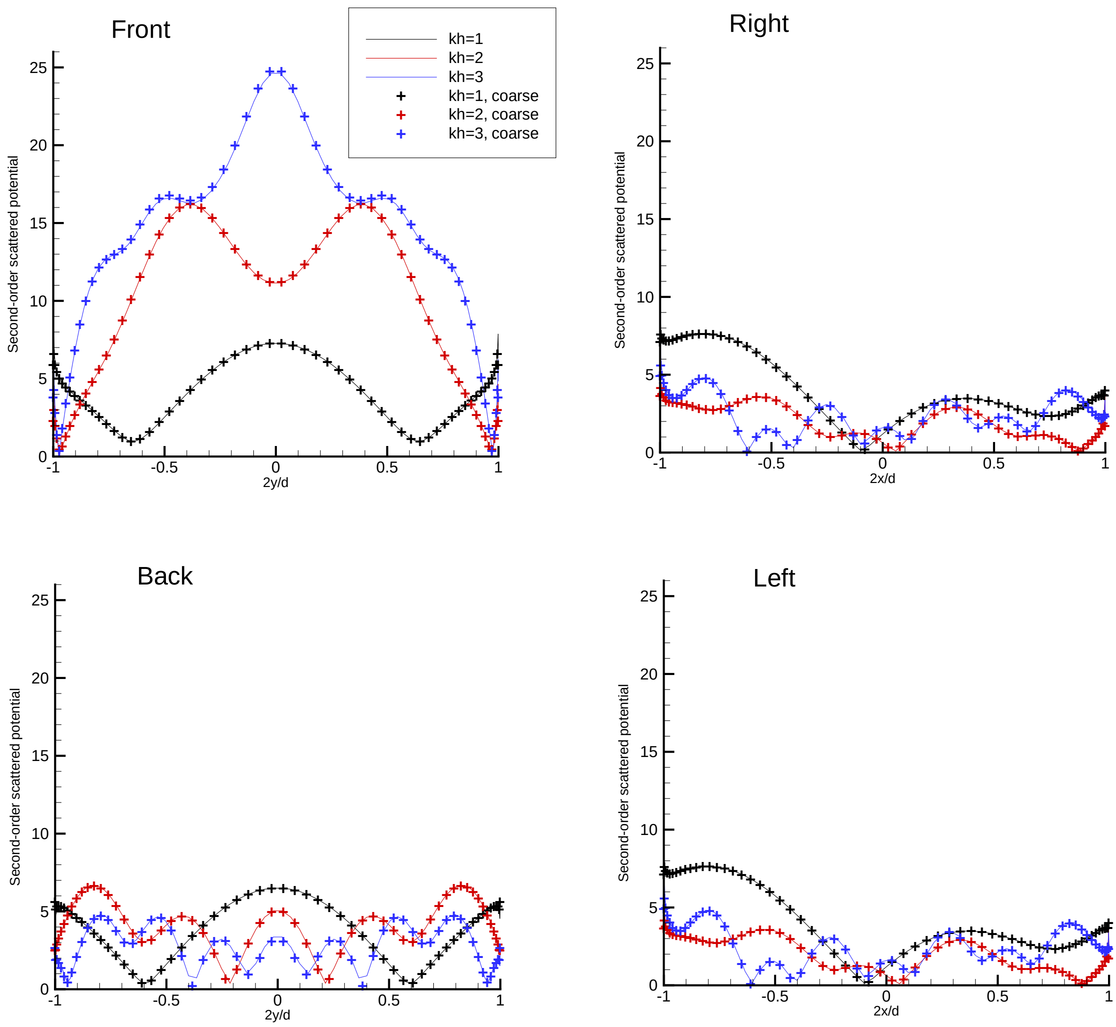

Figure 2: Second-order scattered potential along the waterline, as obtained by WAMIT, for $kh = 1$ (black), 2 (red) and 3 (blue).

The Boussinesq model was run at kh values of 2 and 3, for incoming wave steepnesses H/L of 1, 2 and 3 %. The second-order potential at $z = 0$ was derived from the potential at the free surface $\widetilde{\Phi}(x, y, \eta, t)$ by dividing it with $1 + \omega^2\,\eta/g$ and extracting the double frequency component through Fourier analysis. (At these kh values the incident component of the second order potential is negligible.)

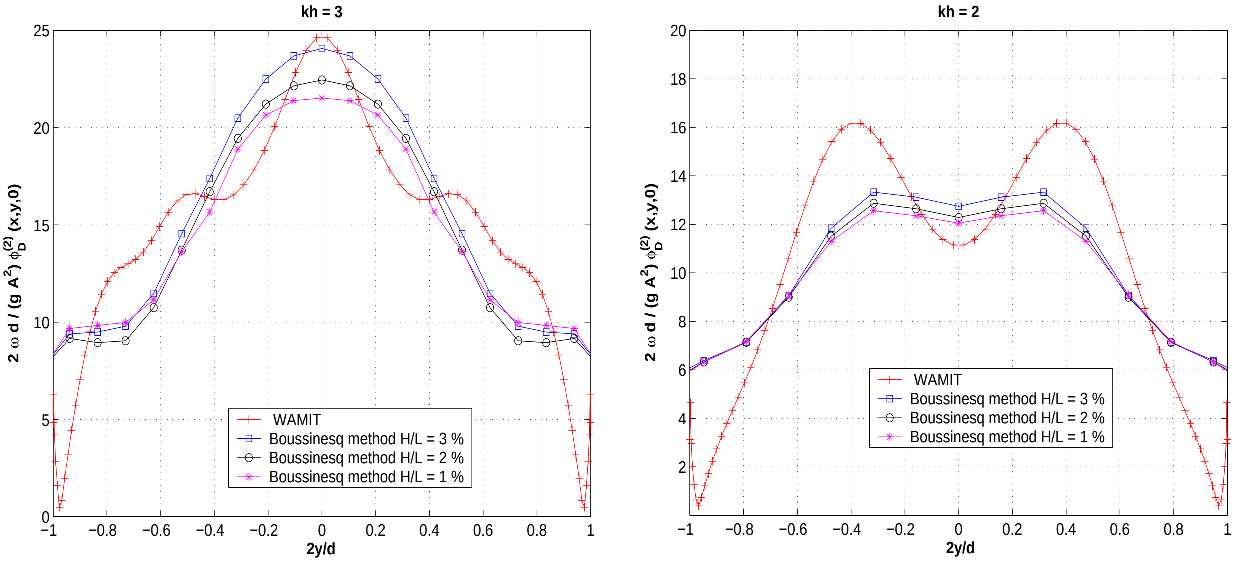

Figure 3: Second-order scattered potential along the waterline. Comparison between WAMIT and results derived from non-linear simulations with the Boussinesq model. $kh = 3$ (left) and $kh = 2$ (right).

Figure 3 shows the obtained moduli of $\Phi_D^{(2)}$ on the weather side of the cylinder, compared with the results from WAMIT. The Boussinesq model provides quite similar results when the steepness varies, suggesting that the differences are actually of higher (fourth?) order. The agreement between WAMIT and the Boussinesq model can only be qualified of "fair".

These results are very preliminary. Further investigations will be presented at the workshop.

References

Eatock Taylor R. & Hung S.M. 1987 Second-order diffraction forces on a vertical cylinder in regular waves, *Applied Ocean Research*, **9**, 19-30.

Fenton J.D. 1988 The numerical solution of steady water waves problems, *Computers and Geosciences*, **14**, 357-368.

Ferrant P., Malenica Š. & Molin B. 1999 Nonlinear wave loads and runup on a vertical cylinder. In *Advances in Fluid Mechanics, Nonlinear Water Wave Interaction, WIT Press.*

Jamois E., Fuhrman D.R., Bingham H.B., Molin B. & Remy F. 2005a Oblique wave interaction with reflective structures by a high-order velocity potential Boussinesq-type model, *Proc. 20th IWWWFB Conf.*, Longyearbyen.

Jamois E., Fuhrman D.R. & Bingham H.B. 2005b Wave-structure interactions and non-linear wave processes on the weather side of reflective structures, in preparation.

Jamois E., Kimmoun O., Molin B. & Stassen Y. 2004 Nonlinear interactions and wave run-up near a gravity base structure, *Proc. ICCE Conf.*, Lisboa.

Molin B. & Marion A. 1986 Second-order loads and motions for floating bodies in regular waves, *Proc. OMAE Conf.*, Tokyo.

Molin B., Remy F., Kimmoun O. & Jamois E. 2005 The role of tertiary wave interactions in wave-body problems, *J. Fluid Mech.*, **528**, 323-354.

Newman J.N. 1996 The second-order wave force on a vertical cylinder, *J. Fluid Mech.*, **320**, 417-443.