Added resistance by means of time-domain models in seakeeping

A. J. Hermans *

1 Introduction

It is well known that to describe the motion of a ship sailing in waves strip-theory gives very good results for many practical hull forms. For this reason not much attention is paid to three dimensional solvers. In recent years computer programs are developed to compute the forces and motions of a ship sailing in waves by means of linear diffraction programs. These frequency-domain codes are in analogy with the programs developed for the zero speed case. This became possible since the one integral expression for the Green's function can be computed rather fast, so the main change in the zero-speed diffraction program is the use of a different subroutine for the Green's function. Also the extra terms in the pressure must be taken care of. In fact the method uses a linearisation around the unperturbed flow around the ship. This may be a good approximation for slender and thin ships. For this class of ships the strip-theory and its modifications give good results, as well. However, in the case of short waves these methods tend to underestimate the added-resistance severely. This becomes a problem if one tries to optimise a hull form if the average weather condition is taken into consideration. If the ship has a blunt hull-form the local steady flow influences the value of the added-resistance greatly. In this paper we present a time-domain method that is capable to solve different kinds of linearised formulations. As an input the program may use the unperturbed flow, double-body flow or the non-linear steady flow. In principle the method can be transformed into a non-linear solver. However, this has not been implemented yet. Experience with the raised panel code RAPID suggests that in the future a similar approach is possible for the unsteady part. The major part of this presentation is based on the PhD theses of Hoyte Raven [1] for the steady part and of Tim Bunnik [2] for the time-domain model.

2 The non-linear formulation

We consider a symmetrical, smoothly-shaped ship sailing with a constant velocity U in incoming waves that propagate in a direction which makes an angle θ with the forward direction of the ship. We choose a coordinate system fixed to the ship and moving with its mean velocity U. The frequency at which the incoming waves are encountered changes due to this forward speed, unless the ship sails in beam waves. The x-axis is along the direction of this current in the symmetry plane of the ship. The z-axis points upwards and the origin lies in the undisturbed free surface $z = 0$. The ship is free to rotate around or translate along any of its axes. The water depth h is supposed to be constant and, therefore, the bottom corresponds to the plane $z = -h$.

We assume that the flow is irrotational and incompressible, a velocity potential Φ exists, which gradient is the velocity of a fluid particle

$$\mathbf{u} = \nabla\Phi$$

Inside the fluid domain this potential satisfies the equation of Laplace, which follows from the conservation of mass

$$\Delta\Phi = 0$$

On the free surface two physical conditions hold. The first is the dynamic free-surface condition, stating that the pressure should equal the atmospheric pressure, which is true when we neglect surface tension. The pressure p inside the fluid follows from the equation of Bernoulli, which relates it to the velocity potential

$$-\frac{p-p_0}{\rho} = \frac{\partial\Phi}{\partial t} + \frac{1}{2}\nabla\Phi\cdot\nabla\Phi + gz - \frac{1}{2}U^2$$

Imposing atmospheric pressure on the unknown free surface $z = \zeta$ gives the dynamic free-surface condition

$$\zeta = \frac{-1}{g}\left(\frac{\partial\Phi}{\partial t} + \frac{1}{2}\nabla\Phi\cdot\nabla\Phi - \frac{1}{2}U^2\right) \quad \text{on } z = \zeta \tag{1}$$

The second is the kinematic condition, stating that a fluid particle cannot leave the free surface, which is mathematically described by

$$\frac{\partial\Phi}{\partial x}\frac{\partial\zeta}{\partial x} + \frac{\partial\Phi}{\partial y}\frac{\partial\zeta}{\partial y} + \frac{\partial\zeta}{\partial t} - \frac{\partial\Phi}{\partial z} = 0 \quad \text{on } z = \zeta$$

If these two conditions are combined, the free-surface elevation ζ can be eliminated, resulting in a condition that only contains the velocity potential

$$\frac{\partial^2\Phi}{\partial t^2} + \nabla\Phi\cdot\nabla\frac{\partial\Phi}{\partial t} + \left(\frac{\partial\Phi}{\partial x}\frac{\partial}{\partial x_\zeta} + \frac{\partial\Phi}{\partial y}\frac{\partial}{\partial y_\zeta}\right) \tag{2}$$

$$\left(\frac{\partial\Phi}{\partial t} + \frac{1}{2}\nabla\Phi\cdot\nabla\Phi\right) + g\frac{\partial\Phi}{\partial z} = 0 \quad \text{on } z = \zeta$$

Care must be taken with the definition of the derivatives in this condition. The gradient, ∇, is defined as the vector with partial derivatives in x, y and z-direction. The partial derivatives $\frac{\partial}{\partial x_\zeta}$ and $\frac{\partial}{\partial y_\zeta}$, however, are here defined as operators working on a function that is defined at the free surface $z = \zeta$, so for $F = F(x, y, \zeta(x, y))$, these partial derivatives relate as follows to the partial derivatives $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$

$$\frac{\partial F(x,y,\zeta(x,y))}{\partial x_\zeta} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial z}\frac{\partial\zeta}{\partial x}$$

*TU Delft, Faculty of Electrical Engineering, Mathematics and ComputerScience, Department of Applied Mathematical Analysis, Mekelweg 4, 2628 CDDelft, The Netherlands, e-mail a.j.hermans@ewi.tudelft.nl

and

$$\frac{\partial F(x,y,\zeta(x,y))}{\partial y_\zeta} = \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}\frac{\partial \zeta}{\partial y}$$

So implicitly, the vertical partial derivative is hidden in these expressions. The partial derivatives $\frac{\partial}{\partial x_\zeta}$ and $\frac{\partial}{\partial y_\zeta}$ can be obtained by calculating the differences between points on the free surface, so we can use very simple difference schemes for a flat plane. We consider finite water depth, hence

$$\frac{\partial \Phi}{\partial n} = 0 \quad \text{at } z = -h$$

The condition on the hull of the ship becomes

$$\frac{\partial \Phi}{\partial n} = \frac{\partial \boldsymbol{\alpha}}{\partial t} \cdot \mathbf{n} \quad \text{on } H(t) \tag{3}$$

where $\boldsymbol{\alpha}$ is the displacement and $H(t)$ the exact position of the hull in the ship-fixed coordinate system.

To obtain a unique solution, we have to impose a radiation condition. This condition states that waves generated by the ship should propagate away from the ship (in the steady case behind the ship).

3 Decomposition of the potential

It is very time consuming to solve the non-linear equations formulated in the previous section, especially when the ship has a forward speed and is sailing in waves. With the increase of computer power, non-linear calculations become more and more promising. With the present state of computer technology, however, it is not yet possible to calculate the non-linear time-varying flow around a sailing ship within acceptable time limits yet. We therefore decided to split up the potential in a steady and an unsteady part. For the time being the unsteady potential will be linearised. The appropriate small parameter is the wave steepness $\varepsilon = A/\lambda$, where A is the amplitude and λ the length of the time-dependent wave.

The velocity potential is now decomposed into a steady, time-independent part Φ_s, and an unsteady, time-dependent part Φ_u. We retain the linear terms for Φ_u.

$$\Phi(\mathbf{x},t) = \Phi_s(\mathbf{x}) + \phi_u(\mathbf{x},t) \tag{4}$$

For the steady potential $\Phi_s(\mathbf{x})$ several descriptions are used. for slender and/or thin ships it is common practice to replace this potential by the unperturbed steady potential Ux. The next step is that for slowly moving blunt bodies one replaces this by the double body potential or at finite forward speed by the solution of the non-linear problem. Before deriving the linearised equations for the unsteady potential we consider the steady potential in more detail.

The steady potential

We first consider the still water case, hence in front of the ship the free surface is unperturbed. The non-linear free surface conditions for the steady potential are

$$\frac{\partial \Phi_s}{\partial x}\frac{\partial \zeta_s}{\partial x} + \frac{\partial \Phi_s}{\partial y}\frac{\partial \zeta_s}{\partial y} - \frac{\partial \Phi_s}{\partial z} = 0 \quad \text{on } z = \zeta_s, \tag{5}$$

ζ_s is the steady free-surface elevation that satisfies

$$\zeta_s = -\frac{1}{2g}\left(\nabla\Phi_s \cdot \nabla\Phi_s - U^2\right). \tag{6}$$

On the hull, the steady flow satisfies the no-flux condition

$$\frac{\partial \Phi_s}{\partial n} = 0 \quad \text{on } H \tag{7}$$

For a long time one has linearised these equations and solved the remaining linearised free surface condition. To compute this potential several methods are used. To compute the wave elevation along the hull the Dawson method is well known. It performs reasonably well for a variety of ship hulls. It is common practice to decompose the steady potential as follows

$$\Phi_s(\mathbf{x}) = \Phi_r(\mathbf{x}) + \phi(\mathbf{x}), \tag{8}$$

where $\Phi_r(\mathbf{x})$ equals the double body potential. This seems to make sense, however one must be a little bit more precise. The question is in what sense is this an asymptotic expansion. No mention is made about small parameters in this context. If one takes the slenderness parameter B/L or D/L where B is the beam and D is the draft of the ship and applies a straight forward perturbation technique it is consistent to replace the double body flow by the unperturbed flow Ux. It is well known that this leads to a non-uniform expansion near the bow and the stern of the ship, where a stagnation point is situated. Because of this phenomenon it is more convenient to look at the slow-ship linearisation first. This is done by several authors in the seventies and eighties. Well known is the work of Baba et al [3, 4], Newman [5], Eggers [6] and Brandsma [7] after the pioneering report of Ogilvie [8] in 1968. Brandsma shows that a strickt expansion with respect to the Froude number, with the assumption that the potential function and its derivatives are ofthe same order of magnitude, the free surface condition as derived byEggers is asymptotically consistent if applied at the double body freesurface $z = \zeta_r$. If one introduced the new z coordinate $z' = z - \zeta_r$ and drops the primes in the coordinates, the free surface condition becomes

$$\begin{aligned} &\phi_z + \frac{1}{g}\left[\Phi_{rx}^2\phi_{xx} + 2\Phi_{rx}\Phi_{ry}\phi_{xy} + \Phi_{ry}^2\phi_{yy} + \right. \\ &(3\Phi_{rx}\Phi_{rxx} + 2\Phi_{ry}\Phi_{rxy} + \Phi_{rx}\Phi_{rzz})\,\phi_x + \\ &\left.(3\Phi_{ry}\Phi_{ryy} + 2\Phi_{rx}\Phi_{rxy} + \Phi_{ry}\Phi_{rxx})\,\phi_y\right] = D(x,y) \ \text{ at } \ z = 0, \end{aligned} \tag{9}$$

where $D(x,y)$ is determined by the double body potential. We have

$$D(x,y) = \frac{\partial}{\partial x}\left[\zeta_r(x,y)\Phi_{rx}(x,y,0)\right] + \frac{\partial}{\partial y}\left[\zeta_r(x,y)\Phi_{ry}(x,y,0)\right] \tag{10}$$

and

$$\zeta_r = \frac{1}{2g}\left[U^2 - \Phi_{rx}^2(x,y,0) - \Phi_{ry}^2(x,y,0)\right] \tag{11}$$

In the sequel we either use as steady potential the double body potential or the solution of the complete non-linear steady problem obtained by RAPID ([1])

The unsteady potential

We first decompose the free surface elevation in a steady and an unsteady component as well. The total free surface elevation is written as

$$\zeta_t(x,y,t) = \zeta_s(x,y) + \zeta_u(x,y,t) \tag{12}$$

where the steady level ζ_s is given in (6). If we retain linear terms with respect to the unsteady potential the unsteady contribution becomes

$$\zeta_u = -\frac{1}{g}\left(\frac{\partial \phi_u}{\partial t} + \nabla\Phi_s \cdot \nabla\phi_u\right) \Big/ \left(1 + \frac{1}{2g}\frac{\partial}{\partial z}\left(\nabla\Phi_s \cdot \nabla\Phi_s\right)\right) \tag{13}$$

on $z = \zeta_s$. If we now retain the linear terms with respect to $\phi_u(\vec{x},t)$ in the expression for the dynamic and kinematic free surface condition and eliminate the free surface elevation we obtain the result derived by Newman in 1978 and used by Bertram [9] in 1996. The final expression is transferred to a condition along the steady free surface $z = \zeta_s$, we obtain on $z = \zeta_s$

$$\begin{aligned}
&\frac{\partial^2 \phi_u}{\partial t^2} + 2\nabla\Phi_s \cdot \nabla \frac{\partial \phi_u}{\partial t} + \nabla\Phi_s \cdot \nabla\left(\nabla\Phi_s \cdot \nabla\phi_u\right) + \\
&\frac{1}{2}\left(\frac{\partial \phi_u}{\partial x}\frac{\partial}{\partial x_{\zeta_s}} + \frac{\partial \phi_u}{\partial y}\frac{\partial}{\partial y_{\zeta_s}}\right)\|\nabla\Phi_s\|^2 + g\frac{\partial \phi_u}{\partial z} + \qquad (14)\\
\zeta_u \frac{\partial}{\partial z}&\left(\frac{1}{2}\left(\frac{\partial \Phi_s}{\partial x}\frac{\partial}{\partial x_{\zeta_s}} + \frac{\partial \Phi_s}{\partial y}\frac{\partial}{\partial y_{\zeta_s}}\right)\|\nabla\Phi_s\|^2 + g\frac{\partial \Phi_s}{\partial z}\right) = 0.
\end{aligned}$$

Far away from the ship, where the steady flow is uniform, so $\Phi_s = Ux$, this condition reduces to the Kelvin condition

$$\frac{\partial^2 \phi_u}{\partial t^2} + 2U\frac{\partial^2 \phi_u}{\partial x \partial t} + U^2\frac{\partial^2 \phi_u}{\partial x^2} + g\frac{\partial \phi_u}{\partial z} = 0 \quad \text{on } z = 0 \tag{15}$$

The linearised unsteady potential is solved in the time-domain with the nonlinear steady potential obtained by RAPID and with the double body potential in the short wave case by means of a *ray* approach.

Asymptotic formulation

It can be shown that the free surface condition reduces to

$$\frac{1}{g}\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]^2 \phi + \frac{\partial}{\partial z}\phi = 0 \quad \text{on } z = 0, \tag{16}$$

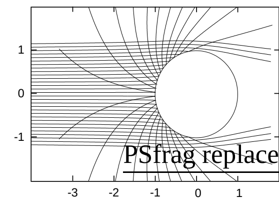
where z is taken with respect to the double body free surface. With the ray expansion for the potential

$$\phi(\mathbf{x},t;k) = \sum_{j=0}^{N} \frac{a_j(\mathbf{x})}{(\mathrm{i}k)^j}\, \mathrm{e}^{\mathrm{i}kS(\mathbf{x}) - \mathrm{i}\omega t}, \tag{17}$$

we derive the eikonal equation for the phase function $S(\mathbf{x})$

$$(1 - \mathbf{u}\cdot\nabla S)^4 - \nabla S \cdot \nabla S = 0, \tag{18}$$

PSfrag replacements



PSfrag replacements

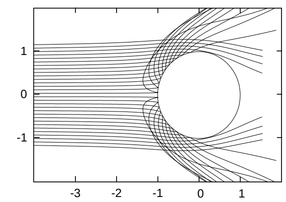


Figure 1: Ray pattern for a cylinder with $\tau = 0.25$

Figure 2: Ray pattern for a cylinder with $\tau = 0.5$

PSfrag replacements

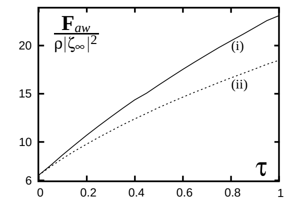


Figure 3: Added resistance for (i) a circular cylinder and (ii) a sphere

and the transport equation for the amplitude function $a_0(\mathbf{x})$

$$\{2\nabla S + 4(1 - \mathbf{u}\cdot\nabla S)^3\mathbf{u}\} \cdot \nabla a_0 + a_0 MS = 0, \tag{19}$$

where $MS = \Delta_3 S - 2\mathbf{u}\cdot\nabla(\mathbf{u}\cdot\nabla S)(1 - \mathbf{u}\nabla S)^2$.

In Figure 1 and 2 we show the ray pattern for a sphere for two values of $\tau = \omega U/g$ and in Figure 3 the added resistance

Numerical Formulation

We now continue with the numerical formulation as proposed by Prins [12], Sierevogel [10] and applied by Bunnik [2] for the finite speed case. We write

$$\phi_u(\mathbf{x},t) = \phi_{\text{inc}}(\mathbf{x},t) + \phi(\mathbf{x},t)$$

and we write the total unsteady perturbed potential function as a source distribution over the boundaries of the computational domain an integral expression for the velocity. The same can be done for the velocities. If $\mathbf{x}$ is inside the fluid domain, on the hull, or on the free surface, this results in

$$\phi(\mathbf{x},t) = \iint_{\partial\Omega\setminus B} \sigma(\boldsymbol{\xi},t)\, G(\mathbf{x},\boldsymbol{\xi})\; \mathrm{d}S_\xi \tag{20}$$

$$\nabla\phi(\mathbf{x},t) = (1-T)\sigma(\mathbf{x},t)\,\mathbf{n} + \iint_{\partial\Omega\setminus B} \sigma(\boldsymbol{\xi},t)\, \nabla_{\mathbf{x}} G(\mathbf{x},\boldsymbol{\xi})\; \mathrm{d}S_\xi \tag{21}$$

We use the following time independent source function

$$G(\mathbf{x},\boldsymbol{\xi}) = -\frac{1}{4\pi r} - \frac{1}{4\pi r'} \qquad r = |\mathbf{x} - \boldsymbol{\xi}| \quad r' = \left|\mathbf{x} - \boldsymbol{\xi}'\right|$$

where $\boldsymbol{\xi}'$ is the mirror of the source point with respect to the bottom and

$$T = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega\setminus\partial\Omega \text{ or } \mathbf{x} \in B \\ \frac{1}{2} & \text{if } \mathbf{x} \in \partial\Omega\setminus B \\ 0 & \text{if } \mathbf{x} \notin \Omega \end{cases}$$

x10
-3

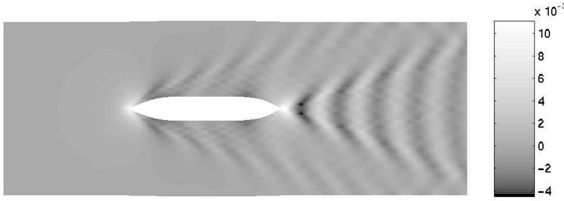


Figure 4: Steady wave pattern, scaled with the length of the ship, $Fn = 0.2$.

For the time derivatives we introduce second order difference schemes

$$\frac{\partial^2 \phi^i}{\partial t^2} = \frac{1}{(\Delta t)^2}\left(2\phi^i - 5\phi^{i-1} + 4\phi^{i-2} - \phi^{i-3}\right) + O\left((\Delta t)^2\right) \quad (22)$$

$$\frac{\partial \phi^i}{\partial t} = \frac{1}{\Delta t}\left(\frac{3}{2}\phi^i - 2\phi^{i-1} + \frac{1}{2}\phi^{i-2}\right) + O\left((\Delta t)^2\right) \quad (23)$$

The unsteady potential's space derivatives are also discretized.

4 Results for an LNG carrier

We apply our model to a 125,000 m^3 LNG carrier sailing at Froude numbers $Fn = 0.14$, $Fn = 0.17$ and $Fn = 0.2$ in water with a depth $h = 175$ metres. We compare our predictions for the motions of the ship and the added resistance at these Froude numbers with measurements from MARIN. Calculations will be done for wave angles $\theta = 0$ (head waves), $\theta = \frac{\pi}{4}$ (bow-quartering waves) and $\theta = \frac{\pi}{2}$ (beam waves). We also vary the length of the incoming waves. The total carrier was divided into 2380 panels.

Figure 4 shows the steady wave pattern of the LNG carrier when it sails at Froude number $Fn = 0.2$. To calculate it, RAPID used 60 panels per wavelength, and 14 panels in transverse direction.

In the computer code we may choose the steady potential. The RAPID steady potential is used in to compute the added resistance for three values of the Froude number. To do so first the first order quantities such as the added mass(moment) and damping matrices must be computed. Taking into account first order motions the added resistance is computed and compared with experimental values obtained at MARIN. The results for head seas and bow-quartering waves are shown in Figure 5 and 6. The computed and measured results are given for $Fn = 0.2,\ 0.17,\ 0.14$ top-down Figure 7 shows the added resistance computed with the non-linear flow (top), the double-body flow (middle) and the uniform flow (bottom). Although the predicted motions of the ship were not that much different, we see large differences between the predicted added resistances. The use of the double-body flow results in a large underestimation of the added resistance, and the use of the uniform flow in a huge underestimation of the added resistance. These underestimations cannot be caused by the small differences between the predicted motions. Therefore, there must be another explanation. Since the differences between the predicted added resistances do not seem to be caused by the first-order fluid quantities on the hull of the ship (otherwise there would have been larger differences between the motions), nor the motions, they must be caused by the predicted wave elevation on the steady waterline of the ship.

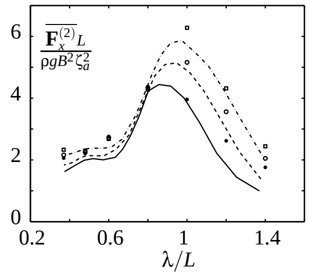


Figure 5: Added resistance in head waves

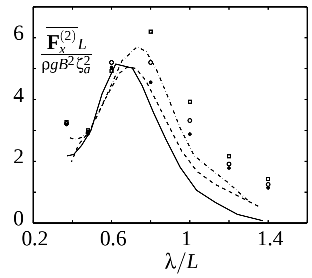


Figure 6: Added resistance in bow-quartering waves

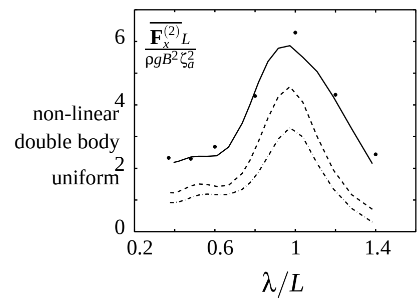


Figure 7: Added resistance in head waves and for $Fn = 0.2$. The asterisks correspond to measurements.

References

[1] H.C. Raven, *A solution method for the non-linear ship wave resistance problem*, PhD thesis, TUDelft, Delft (1996)

[2] T.H.J. Bunnik, *Seakeeing calculations for ships, taking into account the non-linear steady waves*, PhD thesis, TUDelft, Delft (1999).

[3] E. Baba and K. Takekuma, A study on free-surface fbw around bow of slowly moving full forms, *J. Soc. of Naval Arch. Japan* **137** (1975).

[4] E. Baba and M. Hara, Numerical evaluation of a wave-resistance theory for slow ships, *Proc. of the 2nd Int. Conf. on Numerical Ship Hydrodynamics*, Berkeley (1977).

[5] J.N. Newman, Linearized wave resistance theory, *Proceedings of the Int. seminar on wave resistance*, Soc. of Naval Arch. Japan (1976).

[6] K. Eggers, Non-Kelvin dispersive waves around non-slender ships, *Schiffstechnik*, **28** (1981).

[7] F.J. Brandsma, *Low Froude number expansions for the wave pattern and the wave resistance of general ship forms*, PhD thesis, TUDelft, Delft, (1987).

[8] T.F. Ogilvie, *Wave resistance: the low-speed limit*, report no. 002,Univ. of Michigan, Dept of Nav. Arch. Mar. Eng., Ann Arbor (1967).

[9] V. Bertram, *A 3-d Rankine panel method to compute added resistance of ships*, Technical report 566, TU of Hamburg (1996).

[10] L.M. Sierevogel, *Time-domain calculations of ship motions*, PhD thesis, TUDelft, Delft (1998).

[11] A.J. Hermans, Slowly moving hull forms in short waves, *J. of Engineering Mathematics*, **25** (1991), 63-75.

[12] H.J. Prins, *Time-domain calculations of drift forces and moments*, PhD thesis, TUDelft (1995).