A FULLY-NONLINEAR HIGH-ORDER SPECTRAL 3D MODEL for GRAVITY WAVES GENERATION and PROPAGATION

David LE TOUZÉ, Félicien BONNEFOY & Pierre FERRANT

Équipe Hydrodynamique et Génie Océanique/Laboratoire de Mécanique des Fluides
École Centrale de Nantes, B.P. 92101, F-44321 NANTES Cedex 3, France
e-mails: David.Letouze@ec-nantes.fr ; Felicien.Bonnefoy@ec-nantes.fr ; Pierre.Ferrant@ec-nantes.fr

ABSTRACT

A new model for simulating the fully-nonlinear generation and propagation of gravity waves is proposed. This three-dimensional model is based on a fast spectral method known as 'High-Order Spectral' (HOS). This method originally proposed by West *et al.* [1] and Dommermuth & Yue [2] was limited up to now to simulations of initially imposed wave fields, freely evolving in a domain where a double space periodicity condition was supposed. Here additional potential techniques are employed to calculate forcing terms, making it possible to reproduce the full generation and propagation process of three-dimensional wave fields, starting from rest. Two kinds of previously validated additional potentials are proposed: inner [3] and inlet [4]. The possibilities of our fully-nonlinear model are illustrated on the interaction of two orthogonal wave trains, and on the focusing of a directional Pierson-Moskowitz spectrum wave field.

INTRODUCTION

In the design and operation of naval and offshore structures the accurate estimation of hydrodynamic interaction effects represents a major concern. The knowledge of the sea-state and the capability of models to reproduce it precisely are therefore crucial. Up to now, the potential theory approximation remains best suitable to numerically reproduce such sea conditions over long time ranges. To that goal, mainly Boundary Element Models (BEM) have been developed in the last decades. Nonetheless, due to their complexity realistic sea-states require fine grid definitions. This implies for BEM costly calculations in three dimensions, limiting the accuracy obtainable. Other much less employed potential methods have also been developed in past years to that same aim (cf. e.g. [1], [2], [5], [6], [7]), among which in particular spectral techniques that present specific and attractive fast convergence, high accuracy and fast resolution properties. However, those models were not able up to now to simulate the generation of wave fields, only their evolution. Here we present a fully-nonlinear model, able to reproduce the whole wave generation and propagation process, and benefiting from the spectral attractive specificities. It is also suitable to be used as fast and accurate incident wave simulator in the frame of combined potential/viscous calculations.

FORMULATION

There is not enough room here to expose the details of the formulation we employ, only its main lines will thus be recalled. Please refer to [3] and [4] for more information concerning the additional potential generation solutions we use, and to [8] for a full description of the models.

High-Order Spectral core

The HOS core we have developed is formulated in a parallelepipedic bounded domain with a no-flux condition imposed on the vertical boundaries, contrary to the other HOS models (cf e.g. [1], [2] & [5]). Those latter models are indeed expressed in an open domain with a double space periodicity condition imposed, of which our model is free. We include also in those 'regular' HOS formulations the Dirichlet-Neumann Operators (DNO) (cf. e.g. Bateman *et al.* [6]) very similar to HOS in their practical implementation (same number of FFT evaluations, same quantities evaluated, etc. (cf. [8] for a discussion on the comparison of the two techniques)).

In our bounded domain where the potential satisfies the Laplace equation, the free surface is considered to be single-valued. The fully-nonlinear free surface boundary conditions (FNFSBCs) can be written in a classical semi-Lagrangian way. The first possibility is then to plot a spectral expansion such as

$$\phi(x,y,z,t) = \sum_{(n_x,n_y)=(0,0)}^{(N_x,N_y)} A_{n_x n_y}(t) \frac{\cosh[k_{n_x n_y}(z+1)]}{\cosh[k_{n_x n_y}]} e^{i(k_{n_x}x + k_{n_y}y)} \qquad \text{in } D \tag{1}$$

directly in those FNFSBCs. This leads to the assembling of a system of equations that is solved at each time step by an iterative scheme (GMRES), to get both the unknown amplitudes $A_{n_x n_y}(t)$ of the natural modes (n_x, n_y) of the domain, and the elevations at the $N_x N_y$ collocation nodes spread on the free surface at its instantaneous position. It was this 'direct method' that we used first (cf. [3]).

Alternatively, the FNFSBCs can be written by means of the Zakharov surface potential [9]:

$$\frac{\partial \phi^S}{\partial t} = -\eta - \frac{1}{2}\left|\vec{\nabla}\phi^S\right|^2 + \frac{1}{2}\left(1+\left|\vec{\nabla}\eta\right|^2\right)\left(\frac{\partial\phi}{\partial z}\right)^2 \qquad \text{on } z=\eta(x,y,t) \tag{2}$$

$$\frac{\partial \eta}{\partial t} = \left(1 + \left|\vec{\nabla}\eta\right|^2\right) \frac{\partial \phi}{\partial z} - \frac{\partial \phi^S}{\partial x}\frac{\partial \eta}{\partial x} - \frac{\partial \phi^S}{\partial y}\frac{\partial \eta}{\partial y} \qquad \text{on} \quad z = \eta(x,y,t) \tag{3}$$

This way the only remaining non-surfacic quantity is the potential vertical derivative $\partial\phi/\partial z$. The HOS technique consists in expanding this last quantity located at the *exact* free surface position, in Taylor series about the undisturbed free surface position $z = 0$. An iterative process can then be settled to obtain $\partial\phi/\partial z$ through that development, starting from known surface quantities (η and the surface potential ϕ^S). This iterative process is also solved in a pseudo-spectral manner (through expansions like (1)), but this time by means of advantageous FFTs, benefiting from evaluations made at the fixed surface $z = 0$. However, by plotting in the FNFSBCs (3) the solution obtained for $\partial\phi/\partial z$ out of this iterative process, one keeps the full expression of these FNFSBCs *still at their exact position*. The surface quantities, again at this instantaneous position, can next be updated through the time-marching scheme. This is to say that *the HOS technique retains the fully-nonlinear feature of the solution*, despite the Taylor expansion and the following iterative resolution, which is *inner* and *not correlated* to the main resolution. This model is therefore not linked to classical perturbation series expansions.

Additional spectral generation techniques

As we are willing to generate wave fields, and since our HOS core is formulated in a bounded domain, we have developed specific additional potential solutions, that add forcing terms to the core problem acting as wave generators. Two techniques have been proposed: the first one is *inner* and consists in adding submerged singularities into the domain, whose potential engenders waves. The specifically designed singularities to that aim are optimized unsteady spinning dipoles (cf. [3]). The alternative technique is to impose an *inlet* flux condition on a wall of the tank, to simulate the presence of a wavemaker. The additional potential employed in that case is then calculated by means of another adapted spectral expansion (cf. [4]), allowing to retain the attractive features of the spectral method.

ILLUSTRATIVE RESULTS

Validation of the model

The model presented in this paper associates to main components: a non-periodic HOS resolution core on the one hand, and additional potential solutions employed to generate waves on the other hand. The validation of the inner generation by use of optimized unsteady spinning dipoles has been previously realized on target wave packets reproducing [3], with a spectral core relying on the direct resolution at the time. The validity of the inlet generation additional potential has also been established by extensive tests made on our model SWEET (second-order 3D wave tank) relying on that same technique (cf. e.g. [4]).

Hence, we will here restrict ourselves to showing an example of test validating the HOS core, with an inner generation in present case. It consists in comparing our model results to the highly accurate spectral steady solution of Rienecker & Fenton [10], which provides fully-nonlinear wave profiles of regular waves up to the Stokes' limit, based on a Newton's iterative procedure. In figure 1, one can notice the very good agreement between the steady profile and our unsteady calculation, and this already with moderated slope order ($m = 3$). The steepness shown is $2A_x/\lambda_x = 8\%$.

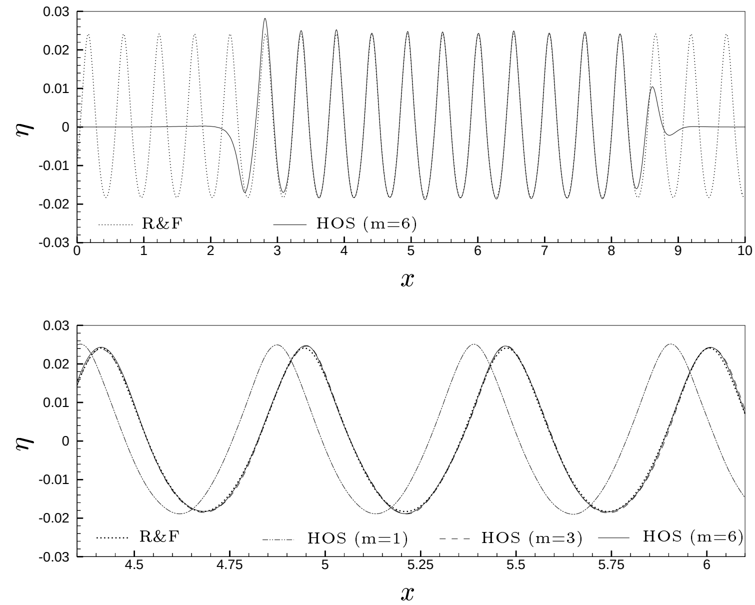

Figure 1: Comparison of the model to the steady solution; $2A_x/\lambda_x = 8\%$.

Nonlinear interaction of two orthogonal wave trains

In order to further illustrate the possibilities of the model, a nonlinear interaction case of two orthogonal two-dimensional wave trains progressing respectively along the x- and y-axis has been selected. Each of them is generated as a regular wave train modulated by the smooth time window function $f(t)$ defined as

$$f(t) = \left(1 - \exp^{\log 0.1 \frac{(t-T_i)}{2T}}\right)\left(1 - \exp^{\log 0.1 \frac{(T_f - t)}{2T}}\right) \quad \text{if} \;\; T_i \le t \le T_f \qquad \text{and} \qquad f(t) = 0 \;\; \text{else} \tag{4}$$

where T is the wave period. At the time they are generated, these windowed regular wave trains have for parameters: wavelength $\lambda_x = 0.5$, steepness $2A_x/\lambda_x = 7.5\%$ and window bounds $[T_{i_x} = 0;\ T_{f_x} = 7T_x]$ along x-axis ; and respectively $\lambda_y = 0.35$, $2A_y/\lambda_y = 6.5\%$, $[T_{i_y} = 6T_y;\ T_{f_y} = 13T_y]$ along y-axis. The generation is achieved by means of two orthogonal optimized unsteady spinning dipoles located at $(x_d^1 = 1.5;\ z_d^1 = -0.25)$, and $(x_d^2 = 0.75;\ z_d^2 = -0.25)$ in the $(L_x = 10) \times (L_y = 6) \times 1$ domain. The figure 2 shows two snapshots of the free surface, at an early stage of the interaction, and later when a sharp peak occurs. To cast more light on the interaction itself, the free surface elevation is recorded at three locations in the domain: first at the place where the high peak lies in the bottom view (probe 2), and also at the same coordinates on the domain borders (probes 1 and 3) where only one of each of the two wave trains is present all along its propagation. It must be noticed that the signals measured on those probes 1 and 3 already result from the fully-nonlinear behavior of the propagation. However, the difference clearly visible in figure 3, between the superposition of these two nonlinear signals and the actually measured peak at probe 2, demonstrates the additional nonlinear *interaction* of those two wave trains, as expected.

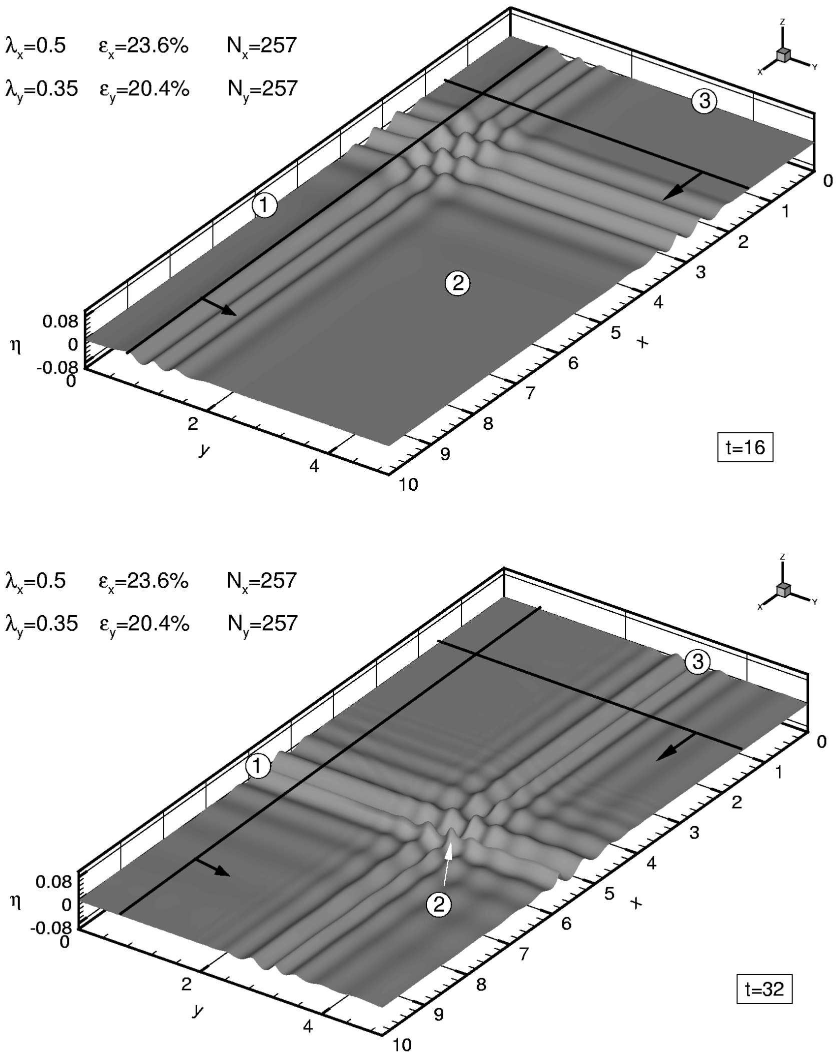

Figure 2: Fully-nonlinear interaction of two orthogonal wave trains.

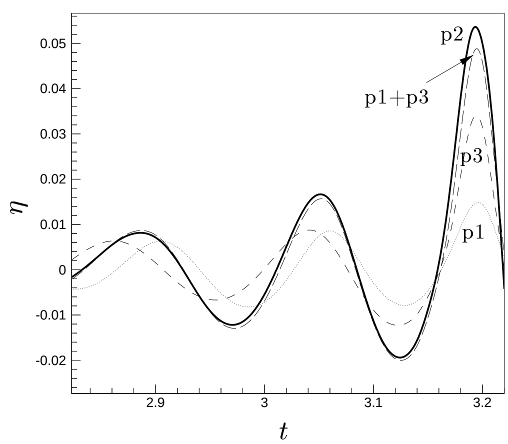

Figure 3: Nonlinear interaction effects: elevations at the probes.

Focusing of a directional spectrum wave field in a 3D wave tank

Another illustration of the capabilities of our model is shown in figure 4 where the focusing of a directional wave field is simulated. The chosen spectrum is a Piersen-Moskowitz one, of significant height $H_s = 0.053m$ and peak frequency $f = 0.4Hz$. The directional spreading is governed by a classical distribution: $cos^{2s}\left(\frac{\theta-\theta_0}{2}\right)$ where $\theta_0 (= 0)$ is the main angle and $s = 10$. The focusing itself is obtained by imposing equal phases of all directional components, at $(x = 20m;\ y = 15m;\ t = 35s)$ in the ECN wave tank $(50m \times 30m \times 5m)$. Here the generation is realized by means of an inlet flux condition corresponding to a serpent-type wavemaker.

The two snapshots of figure 4 illustrate the focusing process, first during its development (on the left), and second when the focused peak actually occurs (on the right). One can notice that this peak has shifted to $(x = 22.07m;\ y = 15m;\ t = 35.48s)$ with respect to the prescribed data, highlighting again the fully-nonlinear nature of the simulation. Another proof of the presence of strong nonlinearities is to be found in the maximum local slope measured on that peak, that exceeds 60%.

Those different three-dimensional results are obtained in a few hours on a 1.7GHz PC. And the numerical cost

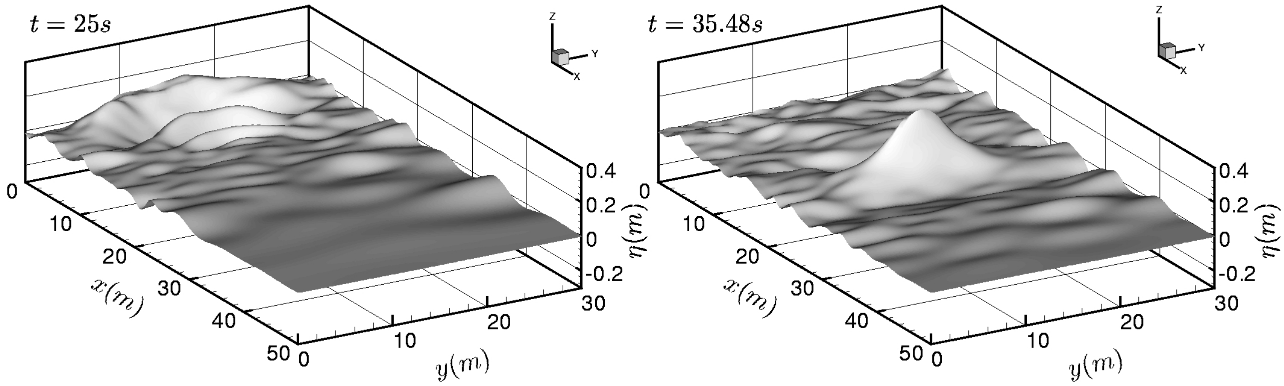

Figure 4: Fully-nonlinear focusing of a generated directional wave field of Pierson-Moskowitz spectrum.

increases almost linearly ($O(N_x N_y \log_2[N_x N_y])$) with the total number of modes/nodes retained: $4s$ calculation per time step in the focusing case where 16700 modes were employed, and $18s$ calculation per time step in the interaction case with 66000 modes.

CONCLUSION

A new three-dimensional model is proposed based on a fast spectral method (HOS). This fully-nonlinear model is able not only to simulate the nonlinear evolution of periodic existing wave patterns as already existing ones, but to reproduce the whole process of generation from rest and propagation of any non-breaking wave field. Validation is made by conclusive comparison with a highly accurate solution for fully-nonlinear regular wave profiles. Illustrative results include the nonlinear interaction of two orthogonal wave packets, and the focusing of a Pierson-Moskowitz spectrum wave field. As future works, further comparisons of these results to other methods, analytical or numerical, will be achieved, as well as to tests to be realized in the new ECN wake tank. Wave train interaction will as well be studied more in details, and results discussed with regards to the description by Longuet-Higgins [11] of such interactions in two dimensions. As for the model itself, two lines of progress will be followed in near future. The first one will consist in attempting to extend our inner optimized unsteady dipole technique to any fully three-dimensional generation. Secondly, higher-order inlet conditions will be implemented, so as to be able to reproduce the actual fully-nonlinear condition found on physical wavemakers.

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