SLAMMING EFFECTS ON ELASTIC CONE

Y.-M. Scolan, ESIM, 13451 Marseille cedex 20, France, scolan@esim.fr

Introduction

Combined hydroelasticity and slamming effects have been studying for many years. This is of great interest for analyzing the high speed vessel behavior in waves. These effects also revealed somewhat crucial for FPSO or cruisers depending on their encountered sea states and their local stern or bow configurations. Faltinsen (2000) presented a large review of these effects for plate with small deadrise angle. In particular it is concluded that maximum pressure cannot be used to estimate maximum stresses. Kvålsvold (1995) also investigated the fully coupled hydroelastic problem with applications to the wet-deck configurations. The Euler or Timoshenko beam models were combined with a Wagner model.

This technique is applied here to the elastic cone penetrating a flat free surface of liquid during a free drop test. The wedge can be treated as well in the same way. The method of solution is inspired by the method developed by Korobkin and Khabakhpasheva (1999). The formulation is mainly analytical and hence it does not require large computer resources. This is particularly suitable at the preliminary design stage in order to evaluate the trends and some features of the phenomenon. However this approach has limitations and drawbacks due to the basic assumptions of the model.

The obtained results concern a free fall of a cone with a finite outer radius. The duration of the drop is very short, less than 10 milliseconds. The flow separation is not modelled. The so-called structural inertia phase is reproduced but not the free vibration phase as identified by Faltinsen (2000). No attention is paid concerning the stresses even if these quantities are the most important for designers. However the stresses are easily calculated from the present approach. This abstract rather focuses on the modelling of the successive steps during the penetration into the liquid. In particular the influence of elasticity is commented through a parametric study in term of the thickness of the shell. Results have been obtained for the elastic wedge as well. They are not described here but the calculations show precisely when air cushion or entrapped air pocket may appear.

Structural problem

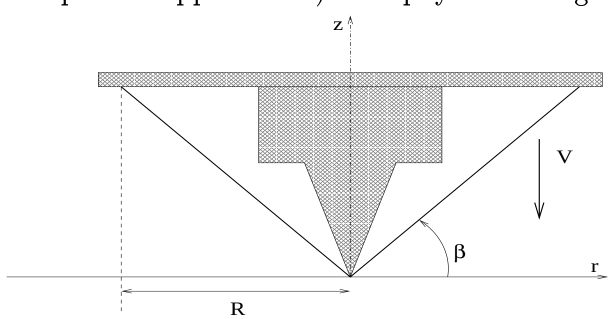
The structural problem is formulated within the linear elasticity theory. The partial differential equation for the elastic deformation $w(r,t)$ is

$$D\Delta\Delta w + \rho_d H(\ddot{w} - \dot{V}) = p(r,t), \quad D = \frac{EH^3}{12(1-\nu^2)}, \tag{1}$$

where ρ_d is the density of the shell, H its thickness, E the Young modulus and ν its Poisson coefficient. The dot stands for the time derivative. The unknown velocity follows from Newton law

$$M_c\dot{V} = M_c\gamma + F(t) \tag{2}$$

where M_c is the total mass of the moving body, $F(t)$ is the hydrodynamic force and γ is the acceleration of gravity (usually negligible for the present applications). The physical configuration is illustrated below



The deformation is expressed through a modal decomposition: $w(r,t) = \sum_n^\infty A_n(t)w_n(r)$. The mode shapes $w_n(r)$ depend on the boundary conditions: here the cone is clamped both at its center and along its outer boundary. Only axisymmetric solutions are retained. hence $w_n(r)$ appears as linear combination of the zeroth order Bessel and modified Bessel functions

$$w_m(r,t) = C_m\Big[J_0(\frac{k_m r}{R}) - I_0(\frac{k_m r}{R})\Big] - \Big[Y_0(\frac{k_m r}{R}) + \frac{2}{\pi}K_0(\frac{k_m r}{R})\Big], \tag{3}$$

where C_m are given constants. The wave number k_m are zeroes of the following equation

$$\Big[J_0(k) - I_0(k)\Big]\Big[Y_1(k) + \frac{2}{\pi}K_1(k)\Big] - \Big[J_1(k) + I_1(k)\Big]\Big[Y_1(k) + \frac{2}{\pi}K_1(k)\Big] = 0 \tag{4}$$

This "dry" decomposition is sufficient at that stage since the considered simulation of the penetration into the liquid is supposed to take place during a time interval which is of the same order than the first "dry" natural period. As a example it is considered a cone with radius $R = 0.128m$, density $\rho_d = 2700Kg/m^3$, thickness $H = 1.5mm$, Young modulus $E = 1.2 10^{11} N/m^2$ and Poisson coefficient $\nu = 0.3$. The first natural period is $T_{n1} \approx 0.0015s$ and its immersion lasts about $T_{imm} = 0.0015s$ with a initial velocity $V_{ini} \approx 8.3m/s$ and a deadrise angle $\beta = 6^o$.

Hydrodynamic Wagner problem

Difficulties appear in the coupling of the structural problem to the hydrodynamic problem. The linearized Wagner problem is based on the so-called flat disk approximation. This approximation is valid for small deadrise angle β. The boundary conditions are linearized and imposed on the initial water level $z = 0$. The displacement potential ϕ satisfies the equations

$$\begin{cases} \Delta\phi = 0 & \tilde{z} < 0 \\ \phi = 0 & \text{on } \rho > 1 \text{ and } \tilde{z} = 0 \\ \phi_{,\tilde{z}} = a(t)\Big[-h(t) + a(t)\rho\tan\beta + w(a(t)\rho,t)\Big] = \kappa(a(t)\rho,t) & \text{on } \rho < 1 \text{ and } \tilde{z} = 0 \\ \phi \to 0 & (\rho^2 + \tilde{z}^2) \to \infty \end{cases} \tag{5}$$

where $\tilde{z} = z/a(t)$ and $r = a(t)\rho$. $h(t)$ is the height of penetration into the liquid. The variable $a(t)$ appears as the radius of the contact line which separates the wetted surface ($\rho < 1$) from the free surface ($\rho > 1$); it is part of the unknowns. The usual way to solve this problem is to turn the equations through a Hankel transform of ϕ (see Sneddon, 1966). In the domain ($0 < \rho < 1, z = 0$) one obtains

$$\phi(\rho,0,t) = \Phi(\rho,t) = \int_1^{1/\rho} \frac{\chi(\rho\nu,t)\mathrm{d}\nu}{\sqrt{\nu^2 - 1}}, \quad \chi(\mu,t) = \frac{2}{\pi}\int_0^{\mu} \frac{\sigma\kappa(a(t)\sigma,t)\mathrm{d}\sigma}{\sqrt{\mu^2 - \sigma^2}}. \tag{6}$$

By imposing finite displacements at the edge $r = a(t)$, the classical Wagner condition yields the additional equation for $a(t)$. This condition is turned into

$$\sum_{n=1}^{\infty} A_n(t) \int_0^1 \frac{x w_n(ax)\mathrm{d}x}{\sqrt{1 - x^2}} = \sum_{n=1}^{\infty} A_n(t) Q_n(a) = h(t) - a\frac{\pi}{4}\tan\beta \tag{7}$$

In the following developments the quantity $\underline{A}$ will denote a vector with coefficients $A_n(t)$. In order to avoid a double time derivative of the displacement potential (which appears in the pressure), it is convenient to evaluate directly the velocity potential φ after replacing the deformation $w(r,t)$ with its modal decomposition. On the plane $z = 0$ and the interval $0 < r < a(t)$, the potential reads

$$\varphi(r,a) = -\frac{2V(t)}{\pi}\sqrt{a^2 - r^2} + \sum_{n=1}^{\infty} \dot{A}_n(t)\Phi_n(r,a), \quad \Phi_n(r,a) = \frac{2r}{\pi}\int_1^{a/r} \frac{\nu \mathrm{d}\nu}{\sqrt{\nu^2 - 1}} \int_0^1 \frac{x w_n(rx\nu)\mathrm{d}x}{\sqrt{1 - x^2}} \tag{8}$$

Fully coupled hydroelastic problem

Korobkin and Khabakhpasheva (1999) proposed a method of solution to solve the time differential system. It is first imposed that the velocity V only depends on time and V_{ini} is the velocity at initial contact instant. Then a new variable q is introduced

$$Dq(r,t) = -\rho_d H(\dot{w} - V + V_{ini}) - \rho_f\varphi \qquad \dot{q} = \Delta\Delta w \tag{9}$$

These relations are projected onto the complete family of eigenfunctions $w_m(r)$ with the associated inner product.

$$\int_0^R w_n(r)w_m(r)rdr = \delta_{nm}U_m = \begin{cases} U_m & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases} \tag{10}$$

In order to use Newton law (2), the hydrodynamic force must be turned into

$$F(t) = -\frac{d}{dt}\Big(M_a V\Big) - \frac{d}{dt}\Big(\underline{\dot{A}R}(a)\Big), \quad R_n(a) = 2\pi\rho_f\int_0^a \Phi_n(r,a)rdr = 4\rho_f\int_0^a x^2 Q_n(x)dx \tag{11}$$

where $M_a = \frac{4}{3}\rho_f a^3$ is the added mass of a flat disk with radius $a(t)$. Then an explicit expression of the velocity follows from the time integration of Newton law

$$V(t) = \frac{M_c(\gamma t + V_{ini}) - \underline{\dot{A}R}(a)}{M_c + M_a} \tag{12}$$

A change of variable ($a(t)$ instead of t) is performed. In that purpose Wagner condition is time differentiated to provide the Jacobian of this transformation

$$\frac{dt}{da} = \frac{\frac{\pi \tan\beta}{4} + \underline{Q}'(a)\underline{A}}{V(t) - \underline{Q}(a)\underline{\dot{A}}} \tag{13}$$

The final differential system to be solved for the unknowns vectors $\underline{q}$ and $\underline{A}$ reads

$$\left\{ \begin{array}{lcl} \underline{\dot{A}} & = & \Big[\lambda - W(a)\Big]^{-1}\Big[Z\underline{q} + \underline{g}(a) - \underline{O}(V - V_{ini})\Big] \\ \underline{\dot{q}} & = & \underline{A} \end{array} \right. \tag{14}$$

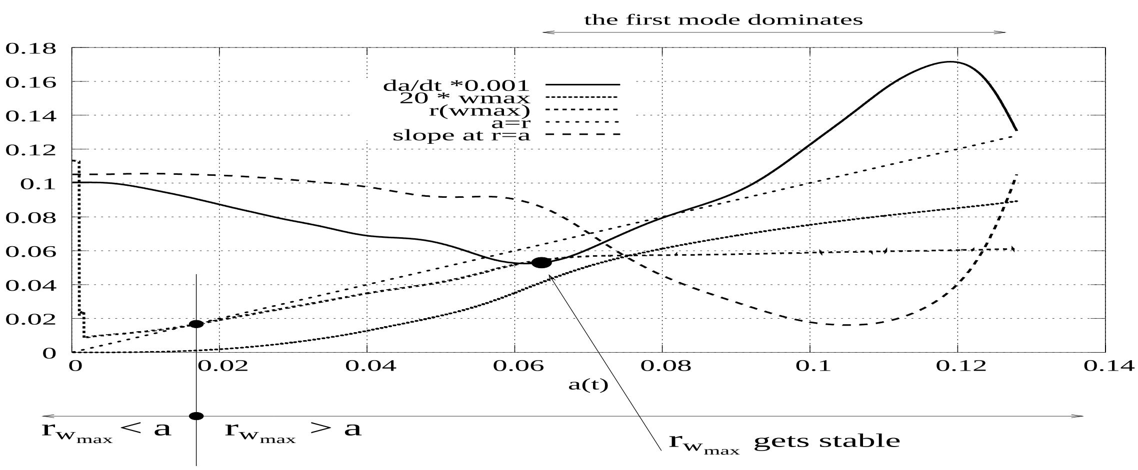
where the matrices λ, $W(a)$, Z and vectors $\underline{g}(a)$ and $\underline{O}$ have coefficients

$$W_{mn}(a) = \frac{2a^3}{\pi}\int_0^1 z^2 Q_m(az)Q_n(az)dz, \quad g_m(a) = \int_0^{a(t)} \varphi_0(r,a,t)w_m(r)rdr, \tag{15}$$

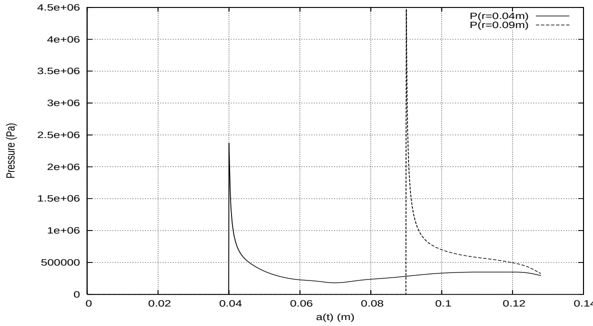
$$\lambda_{mm} = -H\frac{\rho_d}{\rho_f}U_m, \quad Z_{mm} = \frac{D}{\rho_f}\left(\frac{k_m}{R}\right)^4 U_m, \quad O_m = -\frac{\rho_d H}{\rho_f}\int_0^R r w_m(r)dr \tag{16}$$

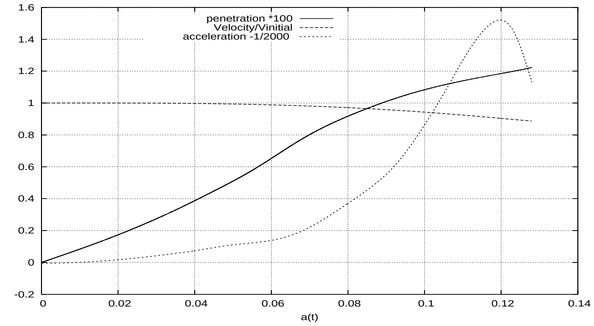
Results and discussion

The most important quantity in the present problem, is $\dot{a}$. It measures the velocity of expansion of the wetted surface. A parametric study is achieved in term of this variable. One of the reasons is that, consistently with the assumptions of Wagner model, the maximum of pressure is calculated from $P_{max} = \frac{1}{2}\rho_f\dot{a}^2$ and the peak of pressure occurs at $r = a(t)$. This follows from a matching of two solutions: one is calculated asymptotically at the contact point $r = a(t)$ from the previous developments, the other is calculated by Wagner (1932) at the spray root. A first application is considered for the already given data and the total mass of the falling body is $M_c = 30Kg$. The following figure shows the variation of $\dot{a}$ with $a(t)$ and its correlation to other variables as the position of the point where the maximum of deformation occurs $r_{w_{max}}$, the slope at point $r = a(t)$ (this is the sum of $\tan\beta$ and $\partial w/\partial r(a,t)$) and the maximum of deformation w_{max}.

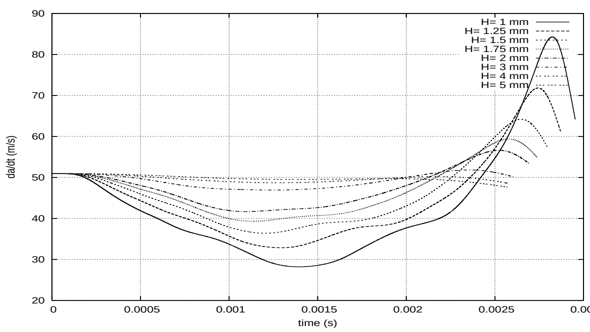


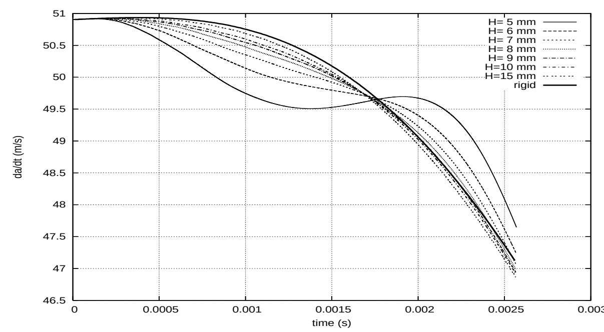
The velocity at which the wetted surface expands is clearly not regular. Up to the point where the first mode gets dominant, $\dot{a}$ decreases monotonically. This is due to the fact that the deformation increases and its maximum occurs close to the contact point; in spite of the fact that the local deadrise angle slightly decreases. The straight line $r = a$ indicates when $r_{w_{max}}$ is below or above the contact point. It is shown that the change occurs at the early stage of penetration. At about $a = 0.06m$, there is a stabilization of this point, meaning that the first mode dominates. Then the immersion accelerates as the local deadrise angle decreases. A maximum is reached before the complete immersion, while the local deadrise angle must come back to β. The reason is that the cone is clamped along its outer boundary. One should note that the velocity $\dot{a}$ may reach about $180m/s$. For thinner shells it can be much higher. The simulation ends up with a maximum deformation of $w_{max} \approx 4.5mm$. Before that it clearly appears that the local slope gets smaller than 4^o when $a(t) > 0.07m$. This limit is known to be the lower bound of the linearized Wagner model. This suggests that other phenomena may appear as ventilation. Beyond this point, the standard pressure calculation within potential theory is not allowed. The pressure variation is plotted on the figure below (left). Two points of measurement (P_1,P_2) are identified $r = 0.04m$ and $r = 0.09m$ on both sides of the limit $a(t) > 0.07m$ where the slope gets very small. One notes that P_2^{max} is about twice P_1^{max}. On the other hand P_2 is divided with a factor 4 within a time interval of $3\ 10^{-5}s$. One can expect that this is too rapid to be caught by any existing sensors! For the same simulation, the figure on the right shows the corresponding variations of the penetration, velocity and acceleration with $a(t)$. The velocity undergoes a small decreasing and ends with a default of 10 %. The acceleration reaches a very high value more than 300 times the gravity. After a maximum is reached at about $a = 0.12m$, the rapid change of the local slope makes the hydrodynamic force decrease substantially.





The behavior of $\dot{a}$ is very much illustrative of the phenomenon reproduced by Wagner model. The thickness can be chosen as a parameter and the following figures show its influence on the time variation of $\dot{a}$. The initial velocity is now $V_{ini} \approx 4.2m/s$. The thickness varies from $H = 1mm$ up to $H = 15mm$





It is worth noting that the thickness $H = 5mm$ separates two kinds of behavior. Above $H = 5mm$ the elasticity strongly influences the successive steps of the phenomenon (as identified previously), especially the duration of immersion. Below $H = 5mm$ the rigid and elastic cases look like each other. A parametric study should be carried on in term of nondimensional parameters in order to generalize these two different behaviors.

Conclusions

Hydroelastic effects of a cone impacting a flat free surface of liquid is studied. Wagner model is coupled with a linear elastic shell model. The obtained results are now compared with available experimental data (Donghy 2002) and other numerical results.

Acknowledgments

This work is partly supported by Direction Générale pour l'Armement, and CEPM project M6406/01 in partnership with Bureau Veritas, Saipem SA (formerly Bouygues Offshore) and Principia RD.

References

Donghy B, 2002, Etude de l'interaction fluide structure lors de l'impact hydrodynamique. PhD Thesis, University of Nantes.
Faltinsen O.M., 2000, "Hydroelastic slamming", J. Mar. Sci. Technol. Vol. 5, No 2, pp 49–65.
Korobkin A.A. & Khabakhpasheva T.I., 1999, Periodic wave impact onto an elastic plate. 7^{th} Conf. of Numerical Ship Hydrodynamics, Nantes.
Kvålsvold J., 1994, Hydoelastic modelling of wetdeck slamming on multihull vessels. PhD Thesis; University of Trondheim.
Scolan, Y.-M. & Korobkin A.A. 2001a Three-dimensional theory of water impact. Part 1. Inverse Wagner problem. *J. Fluid Mech.* **440**, 293-326.
Sneddon I.N., 1966 Mixed boundary value problems in potential theory. J. Wiley & Sons, Inc.
Wagner H. 1932 Über Stoss- und Gleitvorgänge an der Oberfläche von Flüssigkeiten. *ZAMM* **12**, 193-215.
Zhao, R. & Faltinsen, O., 1992 Water entry of two-dimensional bodies. J. Fluid Mech., 246, pp 593–612.

Question by : K. Takagi
I have two questions on the comparisons with Donguy's experiments:
1) did you average your numerical results in the area of the pressure gauge when you compared with the experiments?
2) is the elasticity important at the condition of Donguy's experiments?

Author's reply:
1) No but it will be done. We can expect that the ramp will be reproduced and a lower pressure peak is also expected.
2) Yes, thicknesses were small: 0.5 mm up to 1.5 mm. Plastic deformations were observed after few drop tests for the lowest thicknesses.

Question by : T. Miloh

In applying Newton's law for closure (statment of momentum conservation) one needs to determine explicitely the time derivative of the added masscoefficient. Here you chose to use the Wagner expanding disc model based on the expression given in Lamb for a disc of zero thickness. One way to improve the theory is maybe to use the added mass of an expanding/penetrating cone used for example by Shiffman and Spencer (analytic result).

Author's reply:
This can be done of course, but this is not consistent with other approximations

----