Scattering of Water Waves by Two Thin Symmetric Inclined Plates

by

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Summary : Scattering of surface water waves by two thin symmetric inclined plates present in finite or infinite depth water is investigated here assuming linear theory and hypersingular integral equation formulation. The numerical results for the reflection coefficient are depicted graphically against the wave number for different configurations of the plates. It is seen that if the depth of submergence of the mid points of the plates below the free surface is of the order of one-tenth of the depth of the water bottom, then the corresponding deep water results hold effectively good.

1. Introduction : Study of water wave scattering by various configurations of obstacles present in water of finite and infinite depth is very important in designing certain breakwaters. Parsons and Martin [1,2] applied hypersingular integral equation formulation for investigating scattering problems involving a single thin straight or curved plate which is either submerged or surface-piercing. Midya *et al.* [3] used this method to study the problem of scattering of water waves by a thin inclined plate in finite depth water. Here we have investigated the effect of two thin symmetric inclined plates on the surface waves for two cases viz. when the plates are present in deep water and also in finite depth water.

2. Formulation : The y-axis is taken vertically downwards into the water in which two symmetric plates Γ_1, Γ_2 of length $2b$ are present, inclined to the vertical at an angle α, y-axis being the line of symmetry. Γ_1, Γ_2 are represented parametrically as

$$\Gamma_1, \Gamma_2 : x = \pm(a + bt\sin\alpha),\ y = d - bt\cos\alpha, -1 \le t \le 1 \tag{2.1}$$

with

$$a > b\sin\alpha,\ d > b\cos\alpha \text{ for deep water (DW)},$$

$2a$ being the distance between the mid-points of the plates and d being the depth of the mid-points below the free surface and

$$a > b\sin\alpha,\ d > b\cos\alpha \text{ and } h > d + b\cos\alpha$$

for finite depth water (FDW), h being the bottom depth. Assuming linear theory and irrotational motion, the incident wave potential can be represented by $Re\{\phi_0(x,y)e^{-i\sigma t}\}$ where

$$\phi_0(x,y) = 2g(y)e^{-il_0(x-a)}$$

with

$$g(y) = \begin{cases} e^{-Ky} & \text{for DW} \\ \dfrac{\cosh k_0(h-y)}{\cosh k_0 h} & \text{for FDW} \end{cases}$$

and $l_0 = K(k_0)$ for DW(FDW), k_0 being the unique real positive root of the transcendental equation $k\tanh kh = K$.

If $Re\{\phi(x,y)e^{-i\sigma t}\}$ denotes the scattered wave potential then $\phi(x,y)$ satisfies

$$\nabla^2\phi = 0 \text{ in the fluid region,} \tag{2.2}$$

$$K\phi + \phi_y = 0 \text{ on } y = 0, \tag{2.3}$$

$$\frac{\partial\phi}{\partial n} = 0 \text{ on } \Gamma_i (i = 1, 2),$$

$\dfrac{\partial}{\partial n}$ denoting the normal derivative at a point on Γ_i,
(2.4)

$$r_0^{1/2}\nabla\phi \text{ is bounded as } r_0 \to 0 \tag{2.5}$$

where r_0 is the distance from any submerged edge of the plates,

$$\begin{array}{l} \nabla\phi \to 0 \text{ as } y \to \infty \text{ for DW,} \\ \frac{\partial\phi}{\partial y} = 0 \text{ on } y = h \text{ for FDW} \end{array} \tag{2.6}$$

and

$$\phi(x,y) \sim \begin{cases} \phi_0(x,y) + R\phi_0(-x,y) & \text{as } x \to \infty, \\ T\phi_0(x,y) & x \to -\infty, \end{cases} \tag{2.7}$$

R and T are the unknown reflection and transmission coefficients respectively to be determined.

3. Method of Solution : Due to the geometrical symmetry of the two plates about the y-axis, $\phi(x, y)$ can be split into its symmetric and antisymmetric parts (w.r. to x) as given by

$$\phi(x, y) = \phi^s(x, y) + \phi^a(x, y)$$

where

$$\phi^s(-x, y) = \phi^s(x, y) \text{ and } \phi^a(-x, y) = -\phi^a(x, y)$$

so that the analysis can be restricted to the region $x \geq 0$ only.

Then $\phi^{s,a}(x, y)$ satisfy equations (2.2) to (2.6) together with

$$\phi^s_x(0, y) = 0,\ \phi^a(0, y) = 0,$$

for $y \geq 0$ for DW and $0 \leq y \leq h$ for FDW.

Let the behaviours of $\phi^{s,a}(x, y)$ for large x be represented by

$$\phi^{s,a}(x, y) \to g(y)\left[e^{-il_0(x-a)} + R^{s,a}e^{il_0(x-a)}\right]$$

where $R^{s,a}$ are unknown constants, then $R^{s,a}$ are related to R and T by

$$R, T = \frac{1}{2}(R^s \pm R^a)e^{-2il_0a}. \tag{2.8}$$

The Green's integral theorem is applied to the functions

$$\psi^{s,a}(x, y) = \phi^{s,a}(x, y) - g(y)e^{-il_0(x-a)}$$

and $\mathcal{G}^{s,a}(x, y; \xi, \eta) = G(x, y; \xi, \eta) \pm G(-x, y; \xi, \eta)$ where for finite depth water $G(x, y; \xi, \eta)$ is given by

$$G(x, y; \xi, \eta) = ln\frac{r}{r'} - 2\int_{C_1}\frac{e^{-k(y+\eta)}}{k-K}\cos k(x-\xi)dk$$

$$-2\int_{C_2}\frac{e^{-kh}L(k, y)L(k, \eta)}{k(k-K)\Delta(k)}\cos k(x-\xi)dk, \tag{2.9}$$

with

$$r, r' = \{(x-\xi)^2 + (y \mp \eta)^2\}^{1/2},$$

$$L(k, y) = k\cosh ky - K\sinh ky,$$

$$\Delta(k) = k\sinh kh - K\cosh kh,$$

and C_1, C_2 are along the positive real axis in the complex k-plane indented below the pole at $k = K$ for C_1 and below the poles at $k = K, k_0$ for C_2.

For deep water $G(x, y; \xi, \eta)$ comprises of only the first two terms in (2.9).

This produces

$$\begin{aligned}\phi^{s,a}(\xi, \eta) &= 2g(\eta)e^{il_0a}(\cos l_0\xi, -i\sin l_0\xi)\\ &\quad -\frac{1}{2\pi}\int_{\Gamma_1}F^{s,a}(p)\frac{\partial\mathcal{G}^{s,a}}{\partial n_p}(x, y; \xi, \eta)ds_p\end{aligned}$$

where $p = (x, y) \in \Gamma_1, q = (\xi, \eta)$ and $F^{s,a}(p)$ denote the discontinuities of $\phi^{s,a}$ across Γ_1 at the point p so that $F^{s,a}(p)$ vanish at the end points of Γ_1.

Application of the boundary condition (2.4) leads to the hypersingular integral equations

$$\frac{1}{2\pi}⨎_{\Gamma_1}F^{s,a}(p)\frac{\partial^2\mathcal{G}^{s,a}}{\partial n_p\partial n_q}(x, y; \xi, \eta)ds_p = h^{s,a}(q),\ q \in \Gamma_1$$

which after parametrisation produces

$$⨎_{-1}^{1}\left[\frac{-1}{(\tau-t)^2} + K^{s,a}(\tau, t)\right]f^{s,a}(t)dt = h_1^{s,a}(\tau),$$

$$-1 < \tau < 1 \tag{2.10}$$

where $f^{s,a}(t)$ are related to $F^{s,a}(p)$ and must vanish at $t = \pm 1$ and $K^{s,a}$ and $h_1^{s,a}$ are known bounded functions.

To solve the equations (2.10) $f^{s,a}(t)$ are approximated as

$$f^{s,a}(t) = (1-t^2)^{1/2}\sum_{n=0}^{N}a_n^{s,a}U_n(t) \tag{2.11}$$

where $U_n(t)$'s are Chebyshev polynomials of the second kind and $a_n^{s,a}$ are unknown constants. Substitution of (2.11) into (2.10) and collocating at $N+1$ points $\tau = \tau_j$ produces

$$\sum_{n=0}^{N}a_n^{s,a}A_n^{s,a}(\tau_j) = h_1^{s,a}(\tau_j),\ j = 0, 1, \ldots, N \tag{2.12}$$

where $\tau_j = \cos\left(\frac{2j+1}{2N+2}\pi\right), j = 0, 1, \ldots, N$ and

$$A_n^{s,a}(\tau) = \pi(n+1)U_n(\tau) + \int_{-1}^{1}(1-t^2)^{1/2}K^{s,a}(\tau, t)U_n(t)dt.$$

The systems of linear equations (2.12) are solved to obtain the unknown constants $a_n^{s,a}$ and then $R^{s,a}$ are calculated from the following formulæ:

$$R^{s,a} = \pm e^{2il_0a} + 2e^{il_0a}$$

$$\int_{\Gamma_1} F^{s,a}(p) \frac{\partial}{\partial n_p} \left[p(y)(i \cos l_0 x, \sin l_0 x) \right] ds_p$$

where

$$p(y) = \begin{cases} e^{-Ky} & \text{for DW} \\ \dfrac{2\cosh k_0 h \cosh k_0(h-y)}{2k_0 h + \sinh 2k_0 h} & \text{for FDW} . \end{cases}$$

Ultimately R and T are found by using the relations (2.8).

4. Numerical Results :

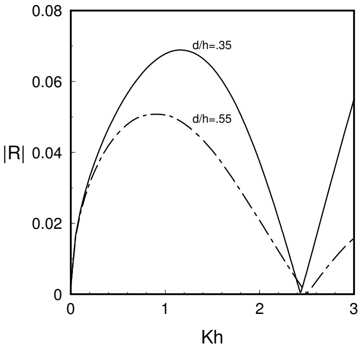

Fig.1: Reflection coefficient for two vertical plates for different depth
a/h=.3, b/h=.15, $\alpha=0^0$

In Figure 1, $|R|$ is depicted for two thin vertical plates ($\alpha = 0$) submerged in finite depth water. It is seen that the graphs of $|R|$ are exactly similar to those obtained by Das *et al.* [4] who investigated oblique wave scattering by two vertical plates in finite depth water by Galerkin method.

The figure 2 displays $|R|$ for different depths of submergence of the plates in water of uniform finite depth h for $\alpha = 45^o$ and $\frac{a}{b} = 1$. It is noticed that when $\frac{d}{h} = .1$, almost all the data points for $|R|$ for finite depth water coincide with those for deep water, obtained by taking $\frac{a}{b} = 1, \frac{d}{b} = 1,\ \alpha = 45^o$. However when $\frac{d}{h}$ increases $|R|$ decreases rapidly which is quite plausible as we are concerned with surface waves.

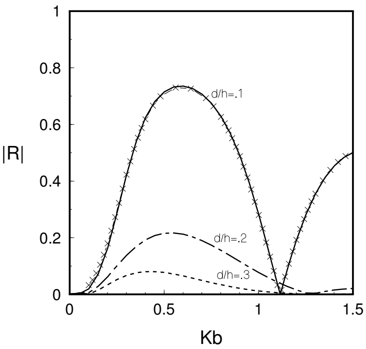

Fig.2 : Reflection coefficient for different depths.
a/b=1, b/h=.1, $\alpha=45^0$

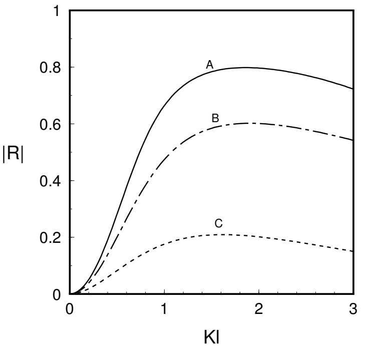

Fig.3 : Reflection coefficient for closed vertical plates.
α=0, a/b=.1, l=d+b, μ=(d-b)/(d+b)=.01(A),.05(B),.25(C).

The effect of two closely situated vertical plates in deep water on $|R|$ is manifested in figure 3. New parameters $l = d + b$ and $\mu = \frac{d-b}{d+b}$ are introduced in order to compare the results with those obtained by Evans [5] for a single vertical plate in deep water. The agreement between the two results is seen to be highly satisfactory if $\frac{a}{b}$ is chosen to be 0.1.

The figure 4 depicts $|R|$ for a wedge-shaped barrier for two cases - when the vertex of the wedge is upwards ($\alpha = -60^o$) and when it is downwards ($\alpha = 60^o$). It is seen that the reflection coefficient is much higher for the wedge with vertex downwards than that for the wedge with vertex upwards for the same depth ($\frac{d}{b} = 1$) and separation ($\frac{a}{b} = |\sin\alpha|$) of the mid-points of the plates.

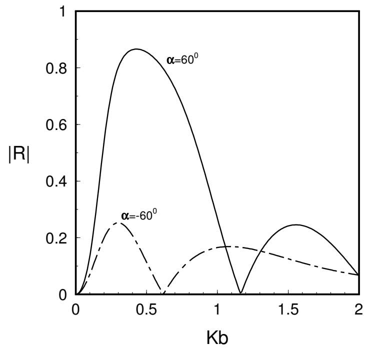

Fig.4 : Reflection coefficient for wedge shaped obstacle.
d/b=1, a/b=sin60^0

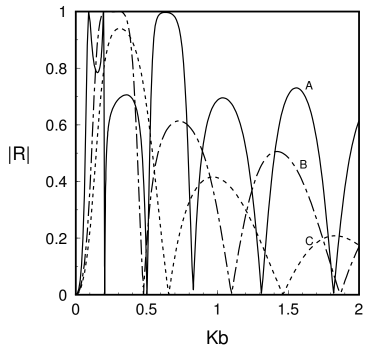

Fig.5 : Reflection coeficient for a horizontal plate at different depths.
$\alpha=\pi/2$, a/b=1, d/b=.1(A), .3(B), .5(C)

The figure 5 shows $|R|$ for a single horizontal plate in deep water ($\alpha = 90^o, \frac{a}{b} = 1$) kept at different depths below the free surface. As the depth increases multiple reflection occurs and $|R|$ attains the value unity for particular wave numbers.

The behaviour of $|R|$ is shown in figure 6 for a horizontal plate with slit for different slit lengths by taking $\alpha = 90^o$, $\frac{d}{b} = .3$, $\frac{a}{b} = 1.5, 1.3, 1.1$. It is observed that as the slit length increases the overall reflection coefficient increases as a result of the increase in the effective length of the horizontal plate.

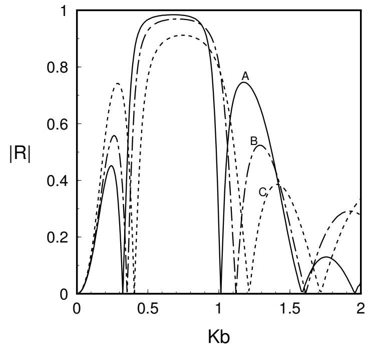

Fig.6 : Reflection coefficient for a horizontal plate with slit.
$\alpha=\pi/2$, d/b=.3, a/b=1.5(A), 1.3(B), 1.1(C)

5. Conclusion : The hypersingular integral equation formulation is employed to study the scattering problems involving two thin symmetric inclined plates in finite and infinite depth water. It is observed that if the plates are submerged to a depth of order of one tenth of the depth of water they can be considered to be submerged in deep water. Known results for a single vertical plate in deep water and for two vertical plates in finite-depth water are produced. New results for wedge-shaped barriers are also established.

Acknowledgement : This work is supported by National Board for Higher Mathematics, INDIA.

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Question by : J.N. Newman
The feature in Figure 6 can be explained using the simple argument in my paper in J. Fluid Mechanics on long symmetric obstacles (circa 1964).

Author's reply:
I agree with your argument. That may be the case. However I have not thought of it properly; I will certainly look at that point more deeply.

Question by : M. Meylan
Are the zeros in modR($|R|$) robust under change in symmetry of plates, etc…?

Author's reply:
Yes, for all configurations of the plates we have got zeros for the reflection coefficient for some wavenumber.