A MODAL SYSTEM FOR CLASSIFICATION AND SIMULATION OF NONLINEAR SLOSHING IN A NEAR-SQUARE BASE TANK WITH FINITE DEPTH

ODD M.FALTINSEN[1], OLAV F. ROGNEBAKKE[2] & ALEXANDER N.TIMOKHA[3]

The adaptive third order analytical modal approach by Faltinsen & Timokha [3] is modified to handle three-dimensional sloshing in a rectangular base basin. (New results are presented and details will be given in a future full-scale article.) The assumptions are incompressible fluid, irrotational flow, no overturning waves, no roof impacts and right contact angle at the wall. In addition, the lateral and angular external forcing is sufficiently small and characterised by the dimensionless parameter $\epsilon \ll 1$. This gives an infinite-dimensional system of ordinary differential equations accounting for possible amplification of any natural modes. It couples nonlinearly time-dependent (modal) functions $\beta_{i,j}$ describing time-evolution of natural surface modes associated with wave patterns (in dimensionless form)

$$f_i^{(1)}(x)f_j^{(2)}(y); \quad f_i^{(1)}(x) = \cos(\pi i(x+1/2)), \quad f_j^{(2)}(y) = \cos(\pi j r(y+1/2r))$$

(r is the breadth/width ratio). The coordinate system is fixed relative to the tank with the origin in middle of the mean free surface; x and y-axes are in the mean free surface and parallel to tank walls.

The adaptive system is simplified for sloshing in a rectangular base basin with similar breadth and width and finite fluid depth by assuming only two primary dominating modes of $O(\epsilon^{1/3})$ caused by excitation frequency close to the lowest natural frequency of linear sloshing. This ordering reduces the adaptive system to a finite-dimensional nonlinear system for the nine lowest modal functions. The nonlinear system is completed by an infinite-dimensional linear system for the remaining modal functions. If cross-waves are not excited, it coincides with the modal system by Faltinsen *et al.* (2000) [2] derived for two-dimensional sloshing. The two-dimensional results by Faltinsen & Timokha (2001) [2] can be used as an indicator of the limitation of our three-dimensional model to capture longitudinally excited primary mode in terms of depth/breadth ratio h and forcing amplitude. If the forcing amplitude is sufficiently small, h should be larger than 0.24.

The simplified modal system is validated by new experimental data on resonant sloshing in a square base tank due to horizontal forcing. The experiments consider longitudinal (parallel to the walls) and diagonal forcing with the depth/breadth ratios $h = 0.508$ and 0.34 and the forcing amplitude/breadth ratio 0.0078. Arai, Cheng & Inoue (1993) [1] conducted also experiments for a square base tank with $h = 0.5$ and established 'swirling' phenomena at the main resonance. Since transients did not die out even after approximately 120 excitation periods (this highlights minor dissipation effect), some additional efforts were made to identify the measured data in terms of expected steady-state motions. The classification of experimental results was made by both visual observations and post-experimental analysis of recorded wave elevations near the tank walls. A Bubnov-Galerkin scheme combined with asymptotic technique

[1]Department of Marine Technology, NTNU, N-7491 Trondheim, Norway (oddfal@marin.ntnu.no)
[2]MARINTEK, P.O.Box 4125, N-7450 Trondheim, Norway;
[3]Institute of Mathematics of NASU, Tereschenkivska, 3 str., Kiev, 01601, Ukraine.

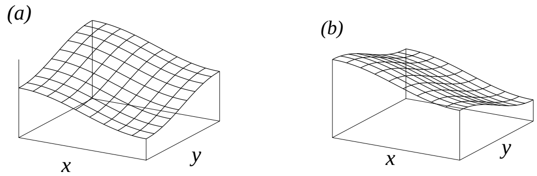

FIGURE 1. Square (diagonal) wave patterns.

is used to find analytically the steady-state waves correct to $O(\epsilon)$ by means of a secular system of nonlinear algebraic equations coupling the dominant amplitudes of the primary modes. The solution depends on three coefficients m_i, which are functions of h and excitation frequency. The coefficients are approximately constant for $h > 1$ and vary very slowly for $h_1 < h < 1$ (here $h_1 = 0.337\ldots$ is the critical depth corresponding to change from hard to soft spring response of two-dimensional sloshing).

The analytical scheme establishes three types of possible dominant steady-state wave responses for longitudinal excitation, namely, 'planar' (two-dimensional), 'swirling' (rotary motions) and so-called 'squares'-like three-dimensional steady-state waves formed by a combination of the two 'squares' (diagonal) wave patterns shown in figures 1 (a,b). By adopting a stability analysis scheme that neglects perturbations in non-leading modes, we were able to calculate effective frequency domains for the different wave behaviour and find critical depths where either the frequency domains of stable regimes or their wave response may change dramatically. Summarised theoretical and experimental results are presented in figure 2 (a,b) for longitudinal and diagonal forcing respectively. Theoretical and experimental effective frequency domains of different steady-state wave motions agreed well for both longitudinal and diagonal forcing. For longitudinal excitation and $h > h_1$ they have the same qualitative structure as for resonant sloshing in a circular basin, namely, the small neighbourhood of the primary resonance consists of two zones, where stable 'swirling' and irregular (no stable steady-state solutions) motions are realised. Stable 'planar' sloshing occurs for excitation frequencies slightly away from the main resonance. This zone falls into two non-connected regions, i.e. for lower and larger excitation frequencies than the primary one. Left region (for lower frequencies) contacts with the frequency domain of irregular waves ('chaos'), while the right region partly overlaps the effective domain for 'swirling'.

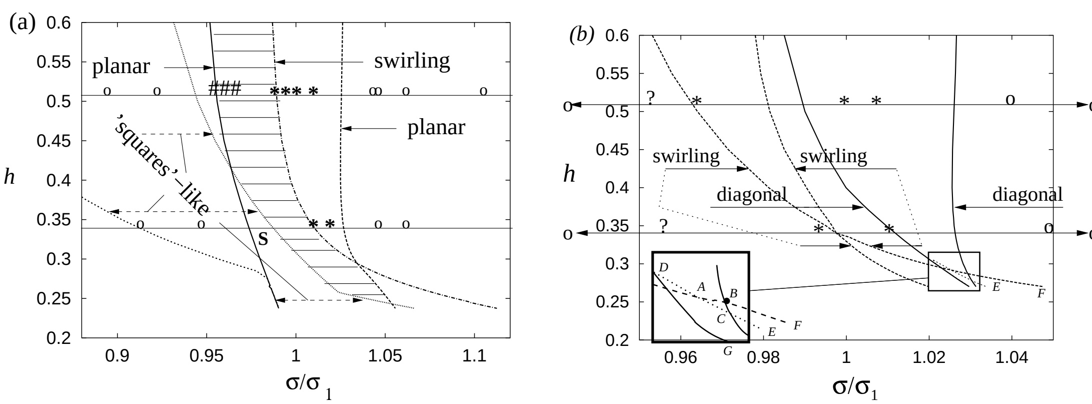

FIGURE 2. Theoretical ranges of stable steady-state motions for **(a)** longitudinal and **(b)** diagonal excitations; depth/breadth ratio h versus excitation/lowest natural frequency σ/σ_1 with fixed forcing amplitude/breadth ratio $H = 0.0078$. Shaded area in (a) = 'chaos' (vanishes for $h < 0.25$). Experiments ($h = 0.508$ and 0.34): 'o' – 'planar'(a)/'diagonal'(b) waves, '*' – 'swirling', '#' – 'chaos', 'S' – 'squares'-like and '?' – regular beating.

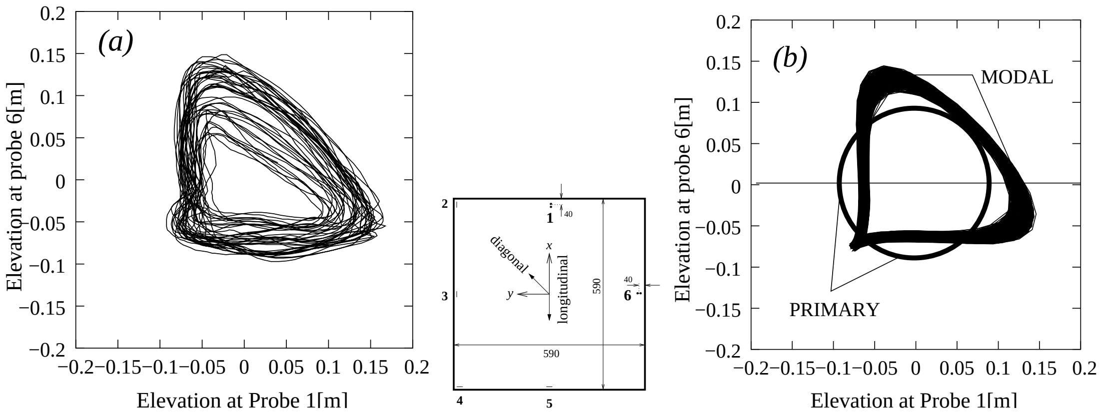

FIGURE 3. Comparisons between experimental and theoretical results for steady-state 'swirling' modes. Diagonal excitation $H = 0.0078, h = 0.508, \sigma/\sigma_1 = 1.008$. (a) presents the parametric graph from the experiments. (b) gives theoretical predictions in the framework of primary modes $O(\epsilon^{1/3})$ ('PRIMARY') and by the modal system (up to $O(\epsilon)$, 'MODAL').

The effective domain for 'squares'-like modes is in general overlapped with left region of 'planar' waves. Moreover, its steady-state amplitude is significantly larger than the amplitude of the 'planar' waves. Therefore, experiments will most probably not capture steady-state 'squares'-like waves with static initial fluid state. The situation changes with decreasing h. The frequency domain of stable 'squares'-like waves then occupies the small vicinity of the primary resonance instead of the regions of irregular waves, while the 'swirling' frequency domain moves away from the primary resonant zone. The frequency domain where three-dimensional phenomena occur, increases with increasing forcing amplitude .

When the excitation is in the diagonal plane and $h \geq h_1$, stable steady-state solutions exist for all excitation frequencies close to the main resonance and there is a zone where 'swirling' and 'diagonal' modes of comparable amplitude co-exist. It is only the 'diagonal' and 'swirling' waves that can be stable. However, the theory establishes a very narrow range of stable 'squares'-like waves for lower depths (area GDE in figure 2 (b)) and there is even a zone of 'chaos' (no stable steady-state solutions) about $h \approx 0.286$ (see curvilinear triangle ABC in figure 2 (b)). Since the importance of dissipation increases with decreasing fluid depth (Faltinsen & Timokha [4]), it is possible that this small frequency domain with 'chaos' will not be physically realised. There were some problems in identifying which steady-state solution is realised for diagonal forcing in the frequency domain where 'swirling' and 'diagonal' sloshing of similar amplitude co-exist (labelled '?' in figure 2 (b). Additional experimental and theoretical studies on possible waves in this frequency domain are therefore needed. The experiments should be done for longer time series and the possibility of stable non-harmonic solutions should be theoretically investigated. Additional forthcoming analysis should clarify the situation for arbitrary excitation direction and increasing amplitude. There is probably a set of critical angles where the frequency domain with 'chaos' disappears, while increasing amplitude may lead to larger domain for 'chaotic' motions.

Quantitative comparison of the theoretical and experimental wave elevations was also presented. The validation of steady-state regimes used direct numerical simulations with initial conditions calculated from asymptotic periodical solutions (up to $o(\epsilon)$). An example of experimental and theoretical results are presented in figures 3 (a,b), where

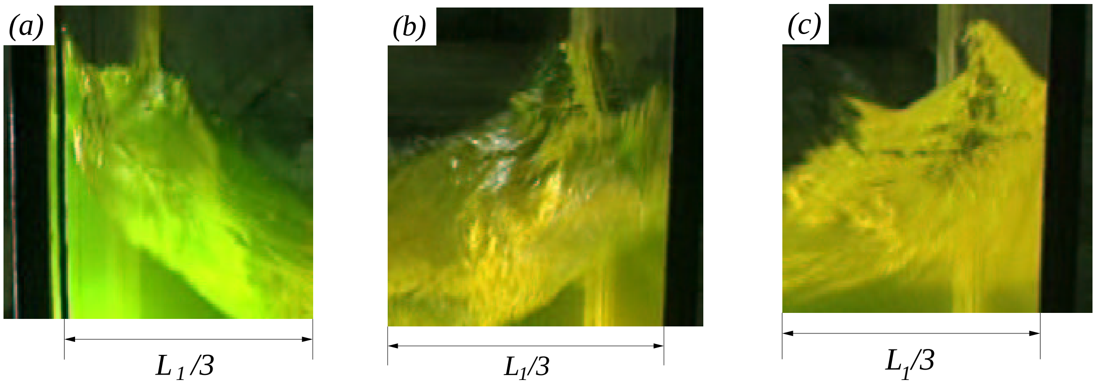

FIGURE 4. Photos from the experimental series demonstrating local phenomena near the wall occurring for three-dimensional waves. Actual width of the photos is approximately 1/3 of the tank length L_1.

the positions of wave elevation probes 2 and 6 are also shown (numbers in [mm]). Theory and experiment agreed very well for ''planar' and 'diagonal' motions, while the calculated wave elevations near the wall for 'swirling' mode gave up to 30% lower maximum wave elevation amplitude than in the experiments. However, the error was significantly larger when using only dominating modes (up to 200%). This confirmed implicitly that modification of the modal system should account for some higher modes having the same order of magnitude as formally dominating ones. Another source of improvements is connected with unsolved local phenomena documented by video for 'swirling' regimes. These local phenomena appeared as very steep waves with possible run-up and overturning (see figure 4). Local phenomena may significantly increase the measured values at the walls.

A forthcoming study should also focus on quantitative description of both transient and steady-state solutions up to $O(\epsilon)$ by direct numerical integration of our modal system. This will require experimental data on initial free surface shapes and velocities. Preliminary calculations showed that actual three-dimensional sloshing depends strongly on small changes in initial conditions. However, initial conditions give negligible changes in time evolution for frequency domains where 'planar' or 'diagonal' steady regimes are expected. Another future study will address resonant sloshing with small deviations between breadth and width. Systematic studies for increasing forcing amplitude and decreasing fluid depth including intermediate depth should be done for a tank with similar lengths and breadth. The intermediate depth case requires Boussinesq-type ordering and strongly multimodal structure in order to describe progressively activated modes. A further problem is to estimate dissipative effects during sloshing, especially for intermediate and shallow depths when they have a dominating character (Faltinsen & Timokha (2002) [4]).

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[3] FALTINSEN, O.M.& TIMOKHA, A.N. 2001 Adaptive multimodal approach to nonlinear sloshing in a rectangular rank. *J. Fluid Mech.* **432**, 167-200.

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