Nonlinear irregular wave forces on near-shore structures by a high-order Boussinesq method

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This abstract considers predicting the wave loads on fixed coastal structures of a size that they can be assumed to lie in the diffraction regime, so that the structure's influence on the flow can be ignored. In particular, we are interested in near-shore structures such as windmill foundations, which can be exposed to irregular, highly nonlinear waves in intermediate to shallow depth water. The loads on such structures are typically estimated by applying the vertical distribution of fluid velocity and acceleration under the wave to Morison's formula [1] or something similar. An accurate approximation of both the wave elevation and the flow kinematics is clearly crucial to the success of this technique.

Wave flow kinematics can be determined from measurements, calculations, or a combination of the two. Measured wave elevation records are readily obtainable, but the associated kinematics are rarely measured and must usually be computed. Many methods exist for approximating the kinematics from a known wave elevation time series. The widely adopted "Wheeler stretching" method [2] and subsequent variations, many of which are reviewed by [3], attempt to correct the linear Airy theory result by introducing a local scaling which maps the free surface to the still water level. The resulting expressions violate the boundary value problem, but are widely used in practice. A number of rational approaches also exist, which are based on satisfying the exact boundary value problem over some portion of the time record. Sobey [4] discusses many of these methods, which differ mainly in the assumption made for the (unknown) spatial variation of the elevation. Trulsen *et al* [5] have also developed a rational method, but based on approximate equations.

Although stretching methods are widely used for predicting wave kinematics, they can be expected to give significant errors when the waves are highly nonlinear. This is clear by comparison with semi-analytic solutions for steady nonlinear waves. Figure 1 compares the surface velocities, and the vertical distribution of velocity under an intermediate water depth wave with $kh = 1$ and $H/h = 0.55$ which is about 90% of the stable limit. Here $H, L, k = 2\pi/L$, and $c = L/T$ are the wave height, length, wavenumber, and celerity respectively, with T the wave period. It is clear that the stretched linear approximation gives large errors for a wave of this nonlinearity. The picture is qualitatively the same in deep and shallow water. The reference solution was computed using stream-function theory [6] with $N = 32$ modes which gives $O(10^{-12})$ as the ratio between the first and the last Fourier coefficient and hence can be safely assumed to be spectrally converged. The corresponding stretched linear approximation is obtained by taking the wave to be a linear superposition of 1st-order Stokes waves. Thus,

$$\eta(x,t) = \sum_{j=1}^{N} A_j \cos(k_j x - j\omega_0 t), \tag{1}$$

where the primary frequency $\omega_0 = 2\pi/T$. The wavenumbers k_j are obtained from ω_0 via the dispersion relation $j\omega_0 = \sqrt{g k_j \tanh(k_j h)}$. This mimics what would be done in practice

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using a measured time series of the surface elevation. The corresponding velocity potential is then

$$\phi(z,t) = \sum_{j=1}^{N} \frac{g}{j\omega_0} \frac{\cosh\left[k_j(z+h)\right]}{\cosh\left(k_j h\right)} A_j \sin\left(k_j x - j\omega_0 t\right). \tag{2}$$

All flow quantities can be obtained by taking derivatives of (2). In particular, the horizontal and vertical components of velocity are $u = \phi_x$ and $w = \phi_z$, while a further time derivative gives the acceleration. The simplest stretching method is applied by making the coordinate transformation $(z+h) \rightarrow h(z+h)/(\eta+h)$ in each linear expression.

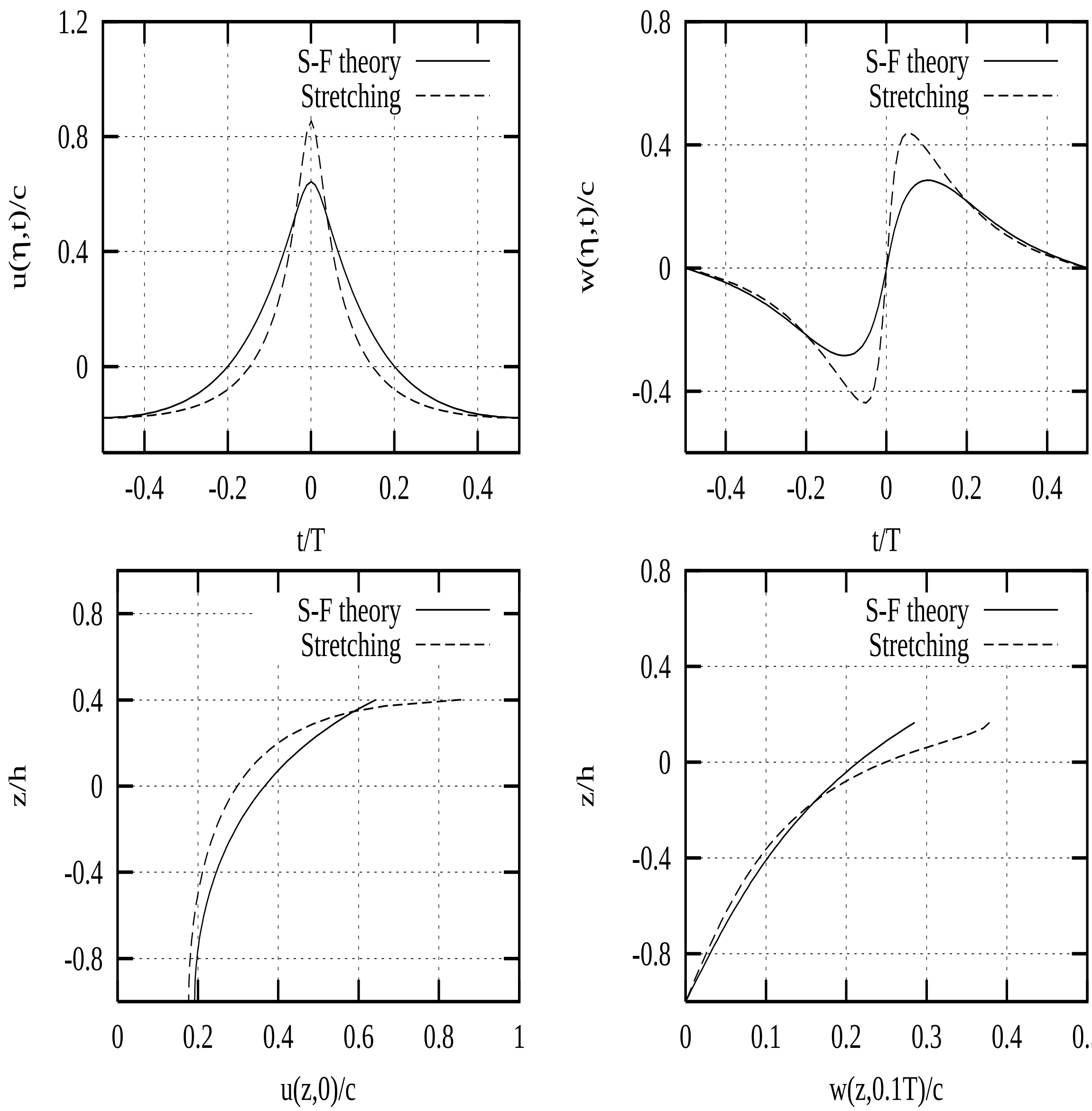

Figure 1: Surface velocities (top plots) and vertical distribution of (near) peak velocities (bottom plots) under a nonlinear wave with $kh = 1$, and nonlinearity $H/h = 0.57$. Comparison between the exact value and stretched linear theory.

This example leads us to expect a similar situation for highly nonlinear irregular waves. To look into it further, we apply a high-order Boussinesq method to irregular waves shoaling up a two-dimensional beach, and compare the resultant vertical distribution of fluid velocity with stretched linear theory. The Boussinesq method employed is described in [7] and [8], where it is shown to produce highly accurate solutions on mildly sloping bathymetries for for waves right up to the stable limit. The method has been extensively validated and shown to be capable of propagating nonlinear waves accurately in relative water depths of $0 \leq kh <\approx 20$. The kinematics (vertical distribution of velocity and acceleration) are accurate up to $kh \approx 10$. The ability to treat waves with very large kh is important for predicting the evolution of irregular waves propagating into near-shore regions. This is because a significant portion of the wave spectrum may be at large kh in deep water and have important contributions to the result in shallow water. Figure 2 compares the Boussinesq calculations for a JONSWAP spectrum wave with $T_p = 9.2$s, $H_s = 4$m generated at $x = 0$ and allowed to shoal up the beach shown in the first plot. The second plot shows

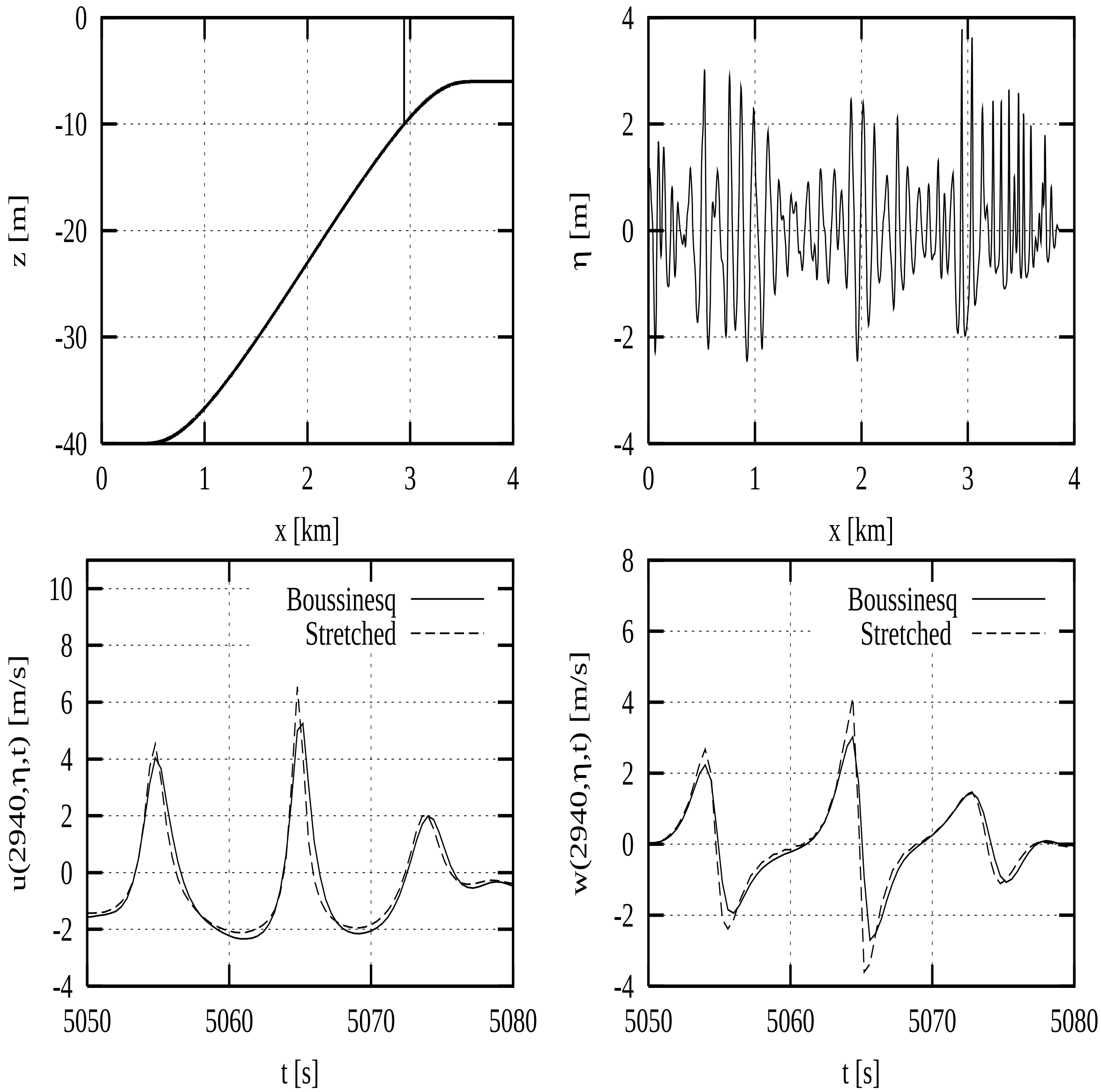

Figure 2: A JONSWAP spectrum wave $T_p = 9.2$s, $H_s = 4$m, shoaling up a beach computed using a Boussinesq model. Beach profile and a snapshot of the surface elevation (top), time series of surface velocities at $h = 10$m ($x = 2.94$km) compared to stretched linear theory (bottom).

a snapshot of the surface elevation $t = 5065s$, while the bottom plots compare the surface velocities at a depth of 10m ($x = 2.94$km) with the predictions of stretched linear theory.

Here the surface elevation computed by the Boussinesq model is taken as a measured signal and processed as discussed above to obtain the stretched velocities. The local wave length of the large wave at $t = 5065$s is around 75m corresponding to $kh \approx 1$, and the local nonlinearity is $H/h \approx .55$ which is roughly the same conditions as that used in the steady wave example discussed above; and we see the same trend here.

The discrepancies between stretched linear theory and the exact result are even larger for the fluid acceleration (as will be discussed at the workshop), so it appears likely that stretched linear theory when used in Morison's equation will contain large errors when the waves are highly nonlinear. The high-order Boussinesq method discussed here provides an attractive alternative.

Acknowledgements

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Question by : B. Molin
In deep water, Wheeler stretching is known to underpredict the kinematics in the crest. You seem to obtain the opposite. Is it due to the reduced waterdepth?

Author's reply:
No. I found the same behaviour in deep water for waves near the limiting steepness. I'm not sure what the explanation is.
