Flow past a ship in restricted water

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1. Introduction

Several methods can be used to handle the potential flow due to steady forward motion of a ship in finite water depth. A linear method based on thin ship theory with the Green function of steady motion in finite constant water depth and a nonlinear theory based on Boussinesq-type equations are discussed here. The thin ship theory is robust in dealing with wash and wave resistance while nonlinearities, unsteady effects, a general sea bottom topography and solitary waves generated by a ship in a channel near critical depth Froude number can be described by nonlinear Boussinesq-type equations.

2. Green function of steady motion in finite constant water depth

The Green function G of steady motion in finite constant water depth satisfying the classical linear free surface condition can be expressed in alternative ways. We have found the following decomposition to be most convenient, i.e

$$G=\frac{1}{r}-\frac{1}{r_1}+\sum_{j=1}^{\infty}(-1)^j\left(-\frac{1}{r_{1,j}}+\frac{1}{r_{2,j}}-\frac{1}{r_{3,j}}+\frac{1}{r_{4,j}}\right)-\mathrm{Re}\left\{\lim_{e\to 0}\frac{2}{p}\int_{-\frac{p}{2}}^{\frac{p}{2}}dq\int_0^{\infty}dk\,\frac{\cosh(k(1+z))\cosh(k(1+z))\exp(ik(|x|\cos q+y\sin q))-f(q,k)}{\cosh^2 k(kF_h^2\cos^2 q-\tanh k+ie\cos q)}\right\}$$

$$-\mathrm{Re}\left\{4iH(-x)\int_{K_0}^{\infty}dk\,\frac{\cosh(k(1+z))\cosh(k(1+z))}{2kF_h^2\cos q_0\sin q_0\cosh^2 k}\cdot\left(e^{ik(|x|\cos q_0+y\sin q_0)}+e^{ik(|x|\cos q_0-y\sin q_0)}\right)\right\}\qquad\cdots\cdots\cdots(1)$$

Here the source point is at $(0,0,z)$ and the field point is at (x,y,z) where z is positive upwards with $z=0$ in the mean free surface and x is in direction of forward motion. Further $r^2=x^2+y^2+(z-z)^2$, $r_1^2=x^2+y^2+(z+z)^2$, $r_{1,j}^2=x^2+y^2+(z+z+2j)^2$, $r_{2,j}^2=x^2+y^2+(z-z+2j)^2$, $r_{3,j}^2=x^2+y^2+(z+z-2j)^2$, $r_{4,j}^2=x^2+y^2+(z-z-2j)^2$, $K_0=0$ if $F_h>1$ and K_0 is the positive root of $K_0F_h^2-\tanh K_0=0$ if $F_h<1$. $F_h=U/\sqrt{gh}$ is the depth Froude number, h is the water depth and $q_0=\arccos((\tanh k/k)^{0.5}/F_h)$. Length dimensions are nondimensionalized by h. $H(x)$ is the Heaviside step function. e is a small positive number proportional to the Rayleigh viscosity. The latter takes care of the radiation condition. i is the complex unit and Re means the real part. Further $f(q,k)$ removes the singularity at $k=0$. This decomposition of the Green function is similar as Newman (1987) used for infinite water depth. The numerical calculation of the set of Rankine singularities and the downstream wave part (the single integral in Eq. (1)) is fast in general while the calculation of the double integral in Eq. (1) is slow. The double integral does not influence wave resistance and the far field wash, but is important for trim and sinkage predictions. The sequence of integration matters in the double integral. At subcritical speeds, we must first integrate with respect to k while at supercritical speeds, we must first integrate with respect to q. At subcritical speeds, the integration with respect to k is divided into two parts. One is integration from 0 to a small value a. The other is integration from a to ∞. No singularity exists between 0 and a. Here we select $f(q,k)=\cosh^2 k$ for $k<a$ and $f(q,k)=0$ for $k\geq a$. By this selection, the singularity at $k=0$ is removed and the integration from 0 to a behaves like $\log R$ for large R. Here $R^2=x^2+(1-F_h^2)y^2$. The integration with respect to k from a to ∞ can be taken as a principle value integral plus a residue contribution. This integration domain is divided into subelements. A Taylor expansion around the midpoint of the subelement is made separately for the denominator and the nonoscillatory part of the numerator of the integrand. If the singularity of the integrand is within the subelement, the singularity has to be at the midpoint of the subelement. The integration from a large k to infinity can be solved by asymptotic expansion of the integrand at large k. Another way to deal with the integration from $k=a$ to ∞ is to take k as a complex variable and use Cauchy's theorem to find the steepest descent path. This is the most efficient method. At supercritical speeds, we can either use the principle value integral plus a residue contribution when integrating with respect to q or take q as a complex variable and use Cauchy's theorem to find several alternative integration paths. However, the alternative integration paths are not always better than the original path. We have also alternatively chosen e as a small positive value. Here, we select $f(q,k)=\cosh k$. Using this method, the singularity along the original

integration path is avoided and it is unnecessary to consider the sequence of integration. However, since e should not be finite, the accuracy cannot be guaranteed. Table 1 compares the double integral by different methods at subcritical speed while Table 2 shows similar comparisons of the x-derivative of the double integral at supercritical speed. The results by principle value integral plus a residue contribution and by using Cauchy's Theorem agree well. At subcritical speed, a constant difference exists between the results by keeping e a small positive value and the results by other methods. This is due to the different choices of $f(q,k)$ which have no influence on the derived physical variables like fluid velocity.

1	Method	x=0	x=0.1	x=0.3	x=0.5	x=1.0
2	Rayleigh's viscosity	1.323445	-0.661246	-2.191790	-3.105782	-4.760434
3	P.V.integral+Residue	13.917151	11.938006	10.418638	9.515693	7.887826
4	Cauchy's Theorem	13.917142	11.937978	10.418633	9.515693	7.887816
5	Line 4 – Line 2	12.593697	12.599225	12.610423	12.621475	12.648250
6	Line 3 – Line 4	0.000009	0.000028	0.000005	0.000000	0.000010

Table 1: The double integral in Eq. (1) calculated by different methods at subcritical speeds.
($F_h = 0.9, y = 0, z = 0, z = -0.1, a = 0.1, e = 0.001$)

1	Method	x=0	x=0.1	x=0.3	x=0.5	x=1.0
2	Rayleigh's viscosity	-9.571250	-3.189684	-0.854762	-0.460710	-0.181828
3	P.V.integral+Residue	-9.579987	-3.195378	-0.858562	-0.463533	-0.183305
4	Line 3 – Line 2	-0.008737	-0.005694	-0.003800	-0.002823	-0.001477

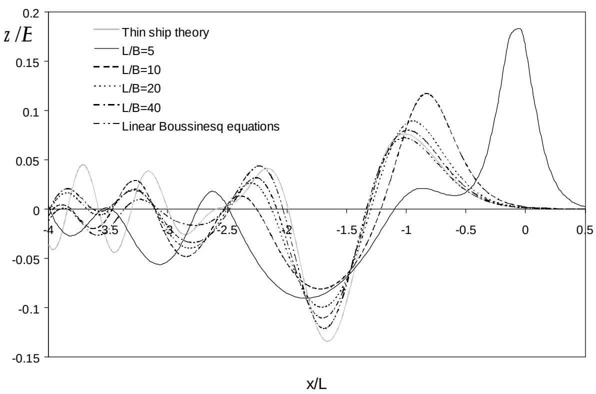
Table 2: The *x*-derivative of the double integral in Eq. (1) by different methods at supercritical speeds.
($F_h = 1.5, y = 0, z = 0, z = -0.1, a = 0.1, e = 0.001$)

The integrand of the downstream wave part can be highly oscillatory. We have used the method of repeated averaging of partial sums (Dahlquist et al 1974) to accelerate the convergence of the integral. This method is very efficient when the integrand is highly oscillatory and slowly decaying for large *k*. The essential steps of this method are to find the zeros of the integrand, change the integral into an alternating series and do the repeated averaging of the partial sums. Each term in the series is the integral between two successive zeros of the integrand. 10 terms are normally sufficient to have an accuracy of 10^{-6}. If no stationary phase exists for the integrand, this method can be applied directly while if stationary phases exist, this method can only be applied from the *k* corresponding to the largest stationary phase value.

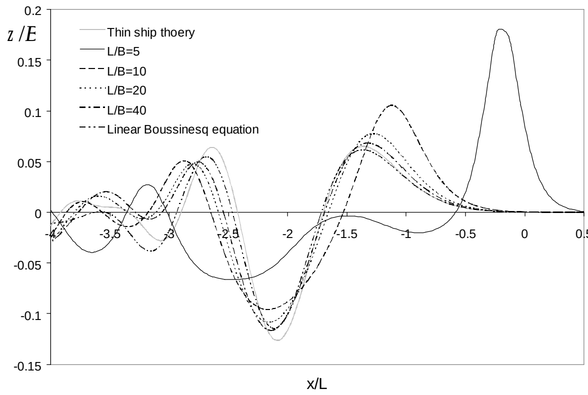
3. Boussinesq-type equations

The extended Boussinesq-type equations by Nwogu (1993) and the Predictor-Corrector model by Wei and Kirby (1995) are used. The Boussinesq-type equations require that a local measure of the wave length is large relative to the water depth. If 2-D regular waves are considered, the phase and group velocities are predicted with a relative error <5% when $h/L<0.4$. Here L is wave length. This means that the wave number $kh<2.51$. We have used the downstream wave part of the Green function in combination with the 2-D regular wave criterion to study the applicability of the theory for our problem. The wave numbers for the transverse wave system are almost the same at a given depth Froude number. This requires that $F_h > 0.716$. The wave numbers for the divergent wave system change substantially as a function of angular position $a = \arctan(|y/x|)$ of the field point. When $a \to 0, k \to \infty$. This means that Boussinesq-type equations are not valid at small a and requires for instance that $a > 15.5^0$ at $F_h = 1.2$. When considering the source distribution representing a ship, the previous criteria for a single source is nonconservative. Another limitation is how the body boundary condition is introduced into the problem. Two models have been used. One is a hybrid method using the linear steady thin ship theory in an inner domain near the ship and using horizontal velocities and free surface elevations along a longitudinal cut at some distance away from the ship as the input to the Boussinesq-type equations. The other is based on slender body theory in shallow water by Tuck (1966). The boundary condition at the centreplane is chosen as the outer expansion of the inner solution. However, both of them have limitations. Using linear thin ship theory to represent the ship is not compatible with using nonlinear free surface condition in the Boussinesq-type equations. Moreover, the flow must be steady and the reflection from for instance channel walls cannot be handled. Using Tuck's slender body theory in shallow water implies a rigid wall free surface condition in the near field. The length Froude number must then be small or moderate.

A ship travelling in an open area of constant finite water depth is first studied. Steady conditions are considered. Wigley's parabolic hulls with same length *L* and draft *T* (*L/T*=16) but different beam B are examined We choose *h/T*=1.6 and $F_h = 1.2$. Fig 1 shows wave elevation at two longitudinal cuts at *y=L* and *y=1.5L* by linear thin ship theory and using Tuck (1966)'s theory as body boundary condition in nonlinear and linear Boussinesq-type equations for different *L/B*. The two linear theories agree well at the first and second wave crests and the first wave trough after the ship bow. One reason to the difference after the second wave crest may be that the length Froude number is large (0.38) and is inconsistent with Tuck's assumptions. Another reason may be that the angular position a of some of the calculated area is less than 15.5^0, i. e that the wave lengths are inconsistent with the Boussinesq-type equations. The nonlinear theory causes higher wave crest and smaller wave trough relative to linear theory. Some phase differences exist between the linear and nonlinear theories. The nonlinear results converge to the results by linear Boussinesq-type equations when $B/L \to 0$. We note that at L/B=5, the difference between linear and nonlinear theories is large and there are waves ahead of the ship. We have also used the thin ship theory to calculate velocities and the free surface elevations along a longitudinal cut *0.5L* away from the ship as input to the Boussinesq-type equations and calculated the wave elevation at *y=L* and *y=1.5L*. Here, only the downstream wave part of the Green function is used. The two linear theories are now in good agreement. This implies that it is the body boundary condition that is the major error source.



y=L

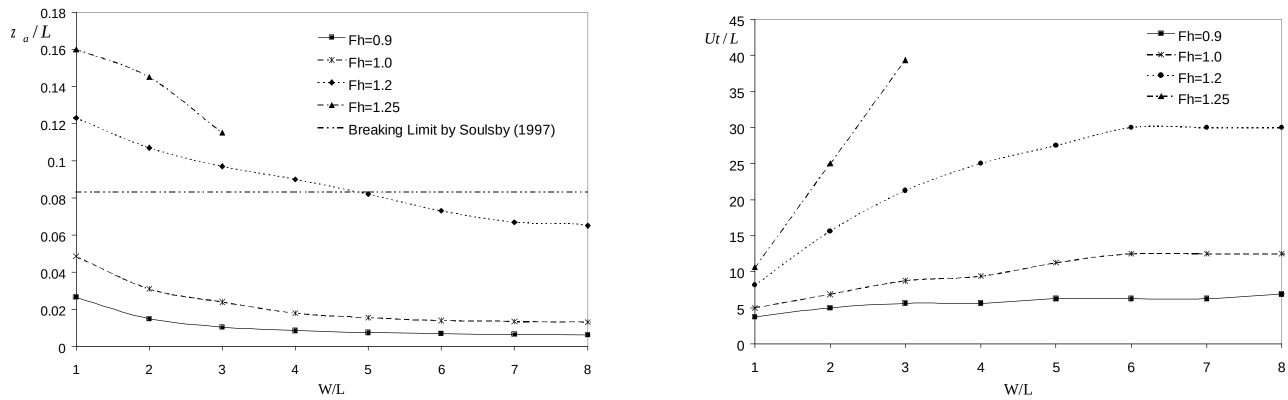


y=1.5L

Figure 1: The convergence of nonlinear Boussinesq-type equations to linear theory as a function of *L/B* for Wigley hulls. $F_h = 1.2$. $h/T = 1.6$. The body boundary condition is represented by Tuck's slender body theory in shallow water. The ship is located at y=0 and –1<x<0.

The solitary waves generated by a ship travelling along the center line of a channel with constant rectangular cross-section near critical depth Froude number are studied. Reflective boundary conditions at the channel walls and absorbing boundary conditions at some distance before and after the ship are imposed. A Series 60 ship with $C_b = 0.6$ is studied for different F_h and channel width *W*. The ship starts from rest and obtains the required speed instantaneously. Since we want to study the time required for the solitary waves to generate, we compare the amplitudes of the solitary waves at the time when the first crest leaves the bow. In a narrow channel (*W/L*<~4), at that time, the solitary wave crests are almost orthogonal to the channel wall and the wave elevation is almost the same along the crest, i.e the associated flow is nearly 2-D in a longitudinal cross-section. The larger *W/L* is, the more 3-D the associated flow is. In such cases, we use the average wave amplitude across the channel. Fig 2 shows the amplitudes of the first crest of the solitary waves and the time required for it to generate. Generally speaking, the amplitudes of the solitary waves increase with increasing depth Froude number at a given channel width and decrease with increasing channel width for a given depth Froude number. The time required for the solitary waves to form increases with increasing depth Froude number at a given channel width and increases with increasing channel width at a given depth Froude number. At a given supercritical depth Froude number, the solitary waves cannot be generated if the channel width is too large. For example, if $F_h = 1.25$, the solitary waves cannot be generated if *W/L*>~4. One reason is that at supercritical speeds, the group velocity of the solitary waves which depends on the wave amplitude must be larger than the ship speed. Fig 3 shows the lower limit of the amplitudes by using 2-D solitary wave with single crest. Fig 3 can in combination with that the amplitude decreases with increasing *W/L* be used qualitatively to explain why there is a limiting value *W/L* for solitary waves to be generated at supercritical Froude number. For instance, it is required from Fig 3 that $z_a / L > 0.048$ at $F_h = 1.2$ and $z_a / L > 0.062$ at $F_h = 1.25$.

This is consistent with the results in Fig 2(a). When *W/L*>~4 and $F_h = 1.25$, solitary waves cannot be generated while solitary waves still exist for *W/L*=8 and $F_h = 1.2$. The cases at critical speed and subcritical speeds are different. The amplitudes seem to decrease to a small finite value and the generation time of the solitary waves seems to increase to a large finite value at large channel width. However, the amplitudes at large channel width are small and the channel wall plays an important role in the formation of the solitary waves. The theoretical breaking limit (Soulsby 1997) of the solitary waves is also shown in Fig 2(a). The amplitudes of the solitary waves for $F_h = 1.2$ and 1.25 with *W/L*<~5 are larger than this breaking limit. When they propagate, numerical problems exist which in an indirect way is likely to be associated with the physical fact that these waves have amplitudes larger than the breaking limit of solitary waves proposed by Soulsby.



(a) Wave heights

(b) Time required for the solitary waves to form

Figure 2: The influence of channel width *W* on the amplitudes z_a and the generation time *t* of the first solitary waves at different depth Froude number F_h. Series 60 hull with $C_b = 0.6$. $h/T = 2.0$.

The effect of changing the ship beam has also been studied. The ship lengths and drafts are the same as a Series 60 with $C_b = 0.6$. We choose *W/L*=2 and $F_h = 1.2$. The solitary wave amplitudes for different beam values are almost the same but the generation time varies a lot. If the beam is the same as the original beam of the Series 60 ship, the nondimensional generation time Ut/L=15.625 while if the beam is equal to 0.70 times the original beam, then Ut/L=51.25. The relative difference between the amplitudes is only ~2%. If the beam is less than ~0.65 times the original beam, no solitary waves can be generated for $F_h = 1.2$.

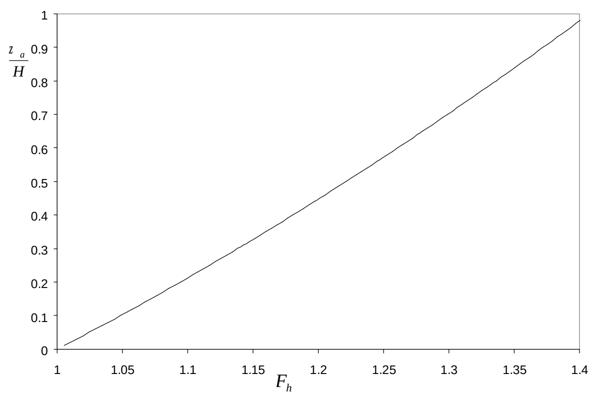


Figure 3: The lower limit for the wave amplitudes of the 2-D solitary waves at different supercritical depth Froude numbers.

Reference:

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Discussion Sheet

Abstract Title :	Flow past a ship in restricted water		
(Or) Proceedings Paper No. :	51	***Page :***	199
First Author :	Q.Z. Yang		
Discusser :	Xiao Bo Chen		

Questions / Comments :

I'm particularly interested in your way to evaluate the finite-depth G.F. by taking a small but finite value of ε representing Rayleigh coefficient. In reality, ε is in fact not zero but how big is it?. Could you design an experimental test to determine it?.

Author's Reply :
(If Available)

In this paper, ε is used to determine the integration path over the singularity. In the limiting case, it should go to zero mathematically.
Since ε is a small value, it is difficult to determine it through experiments. The influence of other effects like free surface tension, viscous effects of basin walls may be at the same order as ε.

Discussion Sheet

Abstract Title :	Flow past a ship in restricted water		
(Or) Proceedings Paper No. :	51	***Page :***	199
First Author :	Q. Yang		
Discusser :	Alexander H. Day		

Questions / Comments :

I would like to congratulate the authors on an extremely interesting piece of work. I have one small query with regard to the use of the repeated averaging method, since I have used this in a related problem with only partial success.

In the reference quoted (Dahlquist et. al. section 3.2.1), it is stated that for alternating series "if the absolute value of the j^{th} term (considered as a function of j) has a k^{th} derivative which approaches zero monotonically for j>n0, then every other value of in column Mk is larger than the sum and every other value is smaller". (The column Mk referes to a figure in the original reference).

It is not immediately obvious to me how this condition is satisfied in the context of the current study.

Author's Reply :
(If Available)

We refer to Sec. 7.4.3 page 297 (Dahlquist et. al.). This section refers back to section 3.2.1. It deals with the integration with the following form $\int_{K_0}^{\infty} f(k)\sin(g(k))dk$. The requirements for using the method of repeated averaging of partial sums are

1. g(k) is an increasing/decreasing function.
2. Both g(k) and f(k) can be locally approximated by a polynomial

The above requirements can be proved to be satisfied. However, we can not prove that the condition mentioned in the question is always satisfied. From our experience, if g(k) is increasing/decreasing monotonically and the derivative of g(k) doesn't change a lot, the method works. However, near the boundary of the wave system, g(k) may increase/decrease very slowly during a interval from k=k1 to k=k2. In such cases, this method can be used from k2.

We apply this method on the wave part of the Green function in which the integrand is clearly in the form of $f(k)\sin(g(k))$ or $f(k)\cos(g(k))$. We don't think this method can be applied to an arbitrary oscillating integrand.

Questions from the floor included; Laurie Doctors, Touvia Miloh, Howell Peregrine, Marshall Tulin & Bill Webster.