DIFFRACTION OF SURFACE WAVES AT FLOATING ELASTIC PLATE

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SUMMARY

Plane problem of a finite plate behavior in waves is considered. The new numerical method based on the Wiener-Hopf technique is presented. The boundary value problem is reduced to the infinite linear algebraic system. Three short-wave approximations are developed, which are in good agreement with results for general system in domains of their applicability. Explicit formulae are obtained for both the one-mode and uniform approximations.

1. FORMULATION OF THE PROBLEM

We assume that the liquid is ideal incompressible and occupies the region $-H_0 < y < 0$. The upper boundary is covered partly with a thin homogeneous plate $(y = 0,\ 0 < y < L_0)$ of thickness h. Plane progressive waves of a small amplitude are incident normal to the plate. In the linear approach the fluid motion is described by the velocity potential φ which satisfies the Laplace equation. We assume also that the wave length is much greater than the plate thickness. The plate draft is neglected.

The time dependence of unknown functions is expressed by the factor $e^{-i\omega t}$. To reduce the number of free parameters, we introduce scaled variables as follows $\varphi' = \varphi/A\sqrt{gl},\ \ x' = x/l,\ \ y' = y/l,\ \ H = H_0/l,\ \ l = g/\omega^2$, where A is the incident wave amplitude, l is the characteristic length, g is the gravity acceleration. Hereafter primes are omitted. Let us represent the potential φ as follows

$$\varphi = (\varphi_0 + \varphi_1)e^{-i\omega t},\ \ \varphi_0 = e^{i\gamma x}\cosh(\gamma(y + H))/\cosh(\gamma H),$$

where φ_0 is the incident wave potential, φ_1 is the scattered potential. The value γ satisfies the dispersion relation $\gamma\tanh(\gamma H) - 1 = 0$ for surface waves in water of the depth H. In non-dimensional variables we derive the following boundary-valued problem

$$\Delta\varphi_1 = 0,\ (-H < y < 0),\quad \frac{\partial\varphi_1}{\partial y} = 0,\ (y = -H), \tag{1}$$

$$\frac{\partial\varphi_1}{\partial y} - \varphi_1 = 0,\ (y = 0,\ x \in (-\infty, 0) \cup (L, \infty)), \tag{2}$$

$$(\beta\frac{\partial^4}{\partial x^4} + 1 - \delta)\frac{\partial\varphi_1}{\partial y} - \varphi_1 = Be^{i\gamma x},\ (y = 0,\ 0 < x < L), \tag{3}$$

$$\frac{\partial^2}{\partial x^2}\frac{\partial\varphi}{\partial y} = \frac{\partial^3}{\partial x^3}\frac{\partial\varphi}{\partial y} = 0,\ (y = 0,\ x = 0, L). \tag{4}$$

$$L = L_0/l,\ \ H = H_0/l,\ \ \beta = D/\rho g l^4,\ \ \delta = \rho_0 h/\rho l,\ \ B = \delta - \beta$$

Here L, H, β and δ are the non-dimensional parameters of the problem, D is the flexural rigidity of the plate, ρ and ρ_0 are fluid and plate densities, respectively. Furthermore, the radiation condition for $|x| \to \infty$ and the regularity condition in a vicinity of the plate edge should be satisfied. The latter condition means that the flow energy in a vicinity of the edge is finite. According to the above assumptions the parameter $\delta << 1$, and we take $\delta = 0$ below.

2. THE SYSTEM OF INTEGRAL EQUATIONS

The problem is solved by the Wiener – Hopf technique. We introduce the functions of the complex variable α as follows

$$\Phi_+(\alpha, y) = \int_L^\infty \mathrm{e}^{i\alpha(x-L)}\varphi_1(x, y)dx,\quad \Phi_-(\alpha, y) = \int_{-\infty}^0 \mathrm{e}^{i\alpha x}\varphi_1(x, y)dx,\quad \Phi_1(\alpha, y) = \int_0^L \mathrm{e}^{i\alpha x}\varphi_1(x, y)dx, \tag{5}$$

$$\Phi(\alpha, y) = \Phi_-(\alpha, y) + \Phi_1(\alpha, y) + e^{i\alpha L}\Phi_+(\alpha, y).$$

We denote through $D_\pm(\alpha), D_1(\alpha)$ integrals of the form (5), where the integrand is the left-hand side of (2), and through $F_\pm(\alpha), F_1(\alpha)$ the integrals, where the integrand is the left-hand side of (3). The functions with subscript $\pm$ are regular in the upper, $\mathrm{Re}\alpha > 0$ and lower, $\mathrm{Re}\alpha < 0$ half-planes, respectively. The function $\Phi(\alpha, y)$ is the classical Fourier transform of φ_1. From (1) we have

$$\Phi(\alpha, y) = C(\alpha)\cosh(\alpha(y+H))/\cosh(\alpha H).$$

We introduce also dispersion functions $K_1(\alpha) = \alpha\tanh(\alpha H) - 1$ for open water and $K_2(\alpha) = (\beta\alpha^4 + 1)\alpha\tanh(\alpha H) - 1$ for the liquid under the plate. The function $K_1(\alpha)$ has two real roots $\pm\gamma$ and a countable set of imaginary roots, $K_2(\alpha)$ has two real roots $\pm\alpha_0$, a countable set of imaginary roots $\alpha_n,\ n = 1, 2, ...$ and four complex roots $\pm\alpha_{-1}$ and $\pm\alpha_{-2}$. The boundary conditions (2) and (3) provide

$$D_-(\alpha) = D_+(\alpha) = 0, \quad D_1(\alpha) = D(\alpha) = C(\alpha)K_1(\alpha),$$

$$F_1(\alpha) = \frac{B[e^{i(\alpha+\gamma)L} - 1]}{i(\alpha+\gamma)}, \quad F_-(\alpha) + F_1(\alpha) + e^{i\alpha L}F_+(\alpha) = C(\alpha)K_2(\alpha).$$

Combining this equations, we obtain

$$F_-(\alpha) + \frac{B[e^{i(\alpha+\gamma)L} - 1]}{i(\alpha+\gamma)} + e^{i\alpha L}F_+(\alpha) = D_1(\alpha)K(\alpha), \quad K(\alpha) = \frac{K_2(\alpha)}{K_1(\alpha)}. \tag{6}$$

The function K is factorized as $K(\alpha) = K_+(\alpha)K_-(\alpha)$. Multiplying (6) by $e^{i\alpha L}[K_-(\alpha)]^{-1}$, we transform it to the form

$$\frac{F_+(\alpha)}{K_+(\alpha)} + \frac{Be^{i\gamma L}}{i(\alpha+\gamma)K_+(\alpha)} + U_+(\alpha) - V_+(\alpha) = D_1(\alpha)K_-(\alpha)e^{-i\alpha l} - U_-(\alpha) + V_-(\alpha), \tag{7}$$

$$U_+(\alpha) + U_-(\alpha) = \frac{e^{-i\alpha L}F_-(\alpha)}{K_+(\alpha)}, \quad V_+(\alpha) + V_-(\alpha) = \frac{Be^{-i\alpha L}}{i(\gamma+\alpha)K_+(\alpha)}.$$

Now we divide (6) by $K_-(\alpha)$ and rewrite it in the form

$$\frac{F_-(\alpha)}{K_-(\alpha)} + R_-(\alpha) - S_-(\alpha) - \frac{B}{i(\alpha+\gamma)}\left[\frac{1}{K_-(\alpha)} - \frac{1}{K_-(-\gamma)}\right] = D_1(\alpha)K_+(\alpha) - R_+(\alpha) + S_+(\alpha) + \frac{B}{i(\alpha+\gamma)K_-(-\gamma)} \tag{8}$$

$$R_+(\alpha) + R_-(\alpha) = \frac{e^{i\alpha L}F_+(\alpha)}{K_-(\alpha)}, \quad S_+(\alpha) + S_-(\alpha) = -\frac{Be^{i(\alpha+\gamma)L}}{i(\gamma+\alpha)K_-(\alpha)}.$$

Using the analytical continuation onto the whole complex plane and the Liouville theorem, we obtain from (7) and (8)

$$\frac{F_+(\alpha)}{K_+(\alpha)} + \frac{Be^{i\gamma L}}{i(\alpha+\gamma)K_+(\alpha)} + U_+(\alpha) - V_+(\alpha) = a_1\alpha + b_1, \tag{9}$$

$$\frac{F_-(\alpha)}{K_-(\alpha)} + R_-(\alpha) - S_-(\alpha) - \frac{B}{i(\alpha+\gamma)}\left[\frac{1}{K_-(\alpha)} - \frac{1}{K_-(-\gamma)}\right] = a_2\alpha + b_2, \tag{10}$$

where a_1, a_2, b_1, b_2 are unknown constants to be determined from conditions (4).

It is convenient to introduce new unknown functions

$$\Psi_+(\alpha) = F_+(\alpha) + \frac{Be^{i\gamma L}}{i(\alpha+\gamma)}, \quad \Psi_-^*(\alpha) = F_-(\alpha) - \frac{B}{i(\alpha+\gamma)}.$$

The star indicates that the function $\Psi_-^*(\alpha)$ has the pole at the point $\alpha = -\gamma$. Substituting the expressions for $U_\pm, V_\pm, R_\pm, S_\pm$ [1] into (9) and (10), we obtain

$$\frac{\Psi_+(\alpha)}{K_+(\alpha)} + \frac{1}{2\pi i}\int\limits_{-\infty-i\sigma}^{\infty-i\sigma} \frac{e^{-i\zeta L}\Psi_-^*(\zeta)d\zeta}{K_+(\zeta)(\zeta-\alpha)} = a_1\alpha + b_1,$$

$$\frac{\Psi_-^*(\alpha)}{K_-(\alpha)} + \frac{B}{i(\alpha+\gamma)K_-(-\gamma)} - \frac{1}{2\pi i}\int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{e^{i\zeta L}\Psi_+(\zeta)d\zeta}{K_-(\zeta)(\zeta-\alpha)} = a_2\alpha + b_2,$$

It is found that

$$a_1 = \frac{1}{2\pi i}\int\limits_{C_-} \frac{e^{-i\alpha L}\Psi_-^*(\alpha)d\alpha}{\alpha^2 K_+(\alpha)}, \quad b_1 = \frac{1}{2\pi i}\int\limits_{C_-} \frac{e^{-i\alpha L}\Psi_-^*(\alpha)d\alpha}{\alpha K_+(\alpha)},$$

$$a_2 = -\frac{i\beta\gamma^2}{K_+(\gamma)} - \frac{1}{2\pi i}\int\limits_{C_+} \frac{e^{i\alpha L}\Psi_+(\alpha)d\alpha}{\alpha^2 K_-(\alpha)}, \quad b_2 = \frac{i\beta\gamma^3}{K_+(\gamma)} - \frac{1}{2\pi i}\int\limits_{C_+} \frac{e^{i\alpha L}\Psi_+(\alpha)d\alpha}{\alpha K_-(\alpha)}.$$

Here C_- and C_+ are the contours along the real axis from $-\infty$ to ∞, which pass the points $-\alpha_0, -\gamma$ above and points α_0, γ below, C_-/C_+ pass the zero below/above. We obtain the system

$$\frac{\Psi_+(\alpha)}{K_+(\alpha)} + \frac{\alpha^2}{2\pi i}\int\limits_{C_-} \frac{e^{-i\zeta L}\Psi_-^*(\zeta)d\zeta}{\zeta^2(\zeta-\alpha)K_+(\zeta)} = 0, \quad \frac{\Psi_-^*(\alpha)}{K_-(\alpha)} - \frac{\alpha^2}{2\pi i}\int\limits_{C_+} \frac{e^{i\zeta L}\Psi_+(\zeta)d\zeta}{\zeta^2(\zeta-\alpha)K_-(\zeta)} = -\frac{i\beta\gamma^2\alpha^2}{(\alpha+\gamma)K_+(\gamma)}. \tag{11}$$

3. NUMERICAL METHOD

The integrals in (11) can be evaluated with the help of the residue theory. Then we have the infinite linear algebraic system with respect to the new unknown quantities ξ_j and η_j

$$\xi_j - \sum_{m=-2}^{\infty} c_{jm}\eta_m = 0, \quad \eta_j - \sum_{m=-2}^{\infty} c_{jm}\xi_m = q_j, \tag{12}$$

$$\xi_j = \frac{\Psi_+(\alpha_j)}{\alpha_j^2 K_+(\alpha_j)}, \quad \eta_j = \frac{\Psi_-^*(\alpha_j)}{\alpha_j^2 K_-(\alpha_j)}, \quad q_j = -\frac{i\beta\gamma^2}{(\gamma-\alpha_j)K_+(\gamma)}, \quad c_{jm} = \frac{e^{i\alpha_m L}K_+(\alpha_m)}{(\alpha_m+\alpha_j)K_-'(\alpha_m)} = \frac{e^{i\alpha_m L}K_+^2(\alpha_m)}{(\alpha_m+\alpha_j)K'(\alpha_m)}.$$

We can explicitly express ξ_j and obtain the matrix equation for the vector $\boldsymbol{\eta}$

$$(E-D)\boldsymbol{\eta} = \boldsymbol{q}, \quad D = C^2, \tag{13}$$

where E is the unit matrix. Once equation (13) has been solved, the amplitudes of the reflected and transmitted waves and plate deflection are determined by formulae

$$R = \frac{\gamma^2}{K_1'(\gamma)K_+(\gamma)}\left[\frac{\beta\gamma}{2K_-(-\gamma)} + i\sum_{j=-2}^{\infty}\xi_j\frac{e^{i\alpha_j L}K_+(\alpha_j)}{(\alpha_j-\gamma)K_-'(\alpha_j)}\right], \quad T = \frac{ie^{-i\gamma L}}{K_+(\gamma)K_1'(\gamma)}\sum_{j=-2}^{\infty}\eta_j\frac{e^{i\alpha_j L}K_+^2(\alpha_j)}{(\alpha_j-\gamma)K'(\alpha_j)},$$

$$w(x) = -\sum_{j=-2}^{\infty}\frac{\alpha_j^3\tanh(\alpha_j H)K_+(\alpha_j)}{K_2'(\alpha_j)}\left[e^{-i\alpha_j(x-L)}\xi_j + e^{i\alpha_j x}\eta_j\right].$$

4. SHORT-WAVE APPROXIMATIONS

We consider the case when $L >> 1$.

4.1. FOUR-MODE APPROXIMATION

This approximation is obtained if we keep in (13) four less decaying modes which correspond to the roots $\alpha_0, \alpha_{-1}, \alpha_{-2}, \alpha_1$.

4.2. ONE-MODE APPROXIMATION

This approximation is obtained if we keep in (13) only the mode corresponding to the root α_0. In this case we have analytical formulae

$$\eta_0 = -\frac{i\beta\gamma^2}{(\gamma-\alpha_0)K_+(\gamma)(1-c^2)}, \quad \xi_0 = -\frac{i\beta\gamma^2 c}{(\gamma-\alpha_0)K_+(\gamma)(1-c^2)}, \quad c = c_{00} = \frac{e^{i\alpha_0 L}K_+^2(\alpha_0)K_1(\alpha_0)}{2\alpha_0 K_2'(\alpha_0)},$$

$$w(x) = \frac{i\beta\gamma^2\alpha_0^3\tanh(\alpha_0 H)K_+(\alpha_0)(e^{i\alpha_0 x} + e^{-i\alpha_0(x-L)})}{(\gamma-\alpha_0)K_+(\gamma)K_2'(\alpha_0)(1-c^2)}, \quad R = \frac{\beta\gamma^3}{2K_+^2(\gamma)K_1'(\gamma)(1-c^2)}\left(1 - c^2\frac{(\gamma+\alpha_0)^2}{(\gamma-\alpha_0)^2}\right).$$

The reflection coefficient $|R|$ is zero where the equality is hold

$$\alpha_0 L + \text{Arg}(K_+^2(\alpha_0)) = \pi k, \quad k = 1, 2, ...$$

4.3. UNIFORM APPROXIMATION

Note that in the case $L >> 1$ all elements of the matrix in (13) are small except for the elements at the diagonal and in the column corresponding to the root α_0. Keeping only the distinguished elements and replacing others with zero, we obtain the following explicit formulae

$$\eta_m = -\frac{i\beta\gamma^2}{(\gamma-\alpha_j)K_+(\gamma)} + cc_{m0}\eta_0, \quad \xi_m = c_{m0}\eta_0, \quad w(x) = \frac{\beta\gamma^2}{K_+(\gamma)(1-c^2)} \sum_{m=-2}^{\infty} \frac{\alpha_m^3 \tanh(\alpha_m H) K_+(\alpha_m)}{K_2'(\alpha_m)} \times$$

$$\times \left[\frac{2\alpha_0 c}{(\alpha_0+\alpha_m)(\gamma-\alpha_0)} e^{-i\alpha_m(x-L)} + \left(\frac{1}{\gamma-\alpha_m} + \frac{c^2(\alpha_0-\alpha_m)(\alpha_0+\gamma)}{(\gamma-\alpha_0)(\alpha_0+\alpha_m)(\gamma-\alpha_m)} \right) e^{i\alpha_m x} \right]$$

5. NUMERICAL RESULTS

Calculations were performed for the plate used in the experiments in [2]. The obtained results are in a good agreement with the results by other methods [3,4]. Numerical results for non-dimensional bending moments are shown in fig. 1-3 for wave periods 0.7 s, 1.429 s, 2.875 s. The experimental results are depicted be dots. Solutions of (13) are shown by solid lines, uniform approximations coincide with solid lines, four mode and one mode approximations are displayed by dashed and dotted lines. The reflection coefficients as a function of the plate length is shown in fig.4 for the ice of thickness 1 m and wave length 100 m in deep water. Solution of (13) is shown by solid line, one mode approximations is displayed by dashed line.

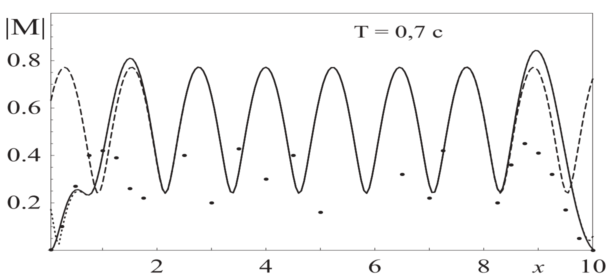

Fig.1

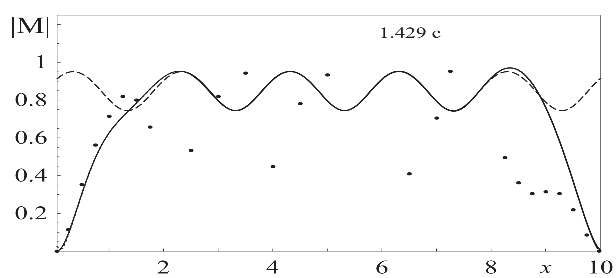

Fig.2

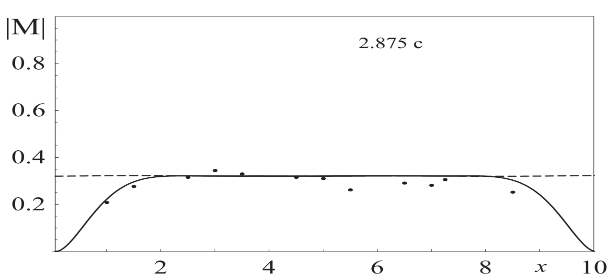

Fig.3

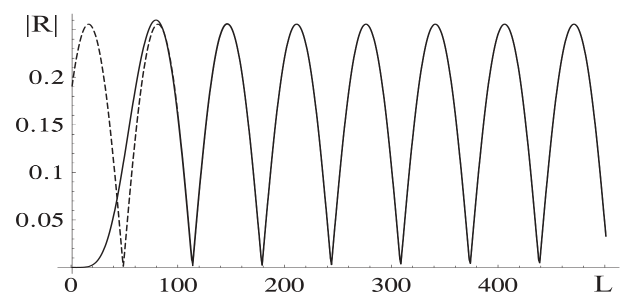

Fig.4

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6. REFERENCES

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