STEADY FLOW NEAR THE BOW OF A FLAT SHIP

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1.Introduction

In the analysis of ship-flow interaction or ship-wave interaction the $2D+t$ (or 2.5 Dimensional) approach has been proved reliable in giving stable solution even at weakly nonlinear situation. One ambiguity of this approach is, however, in the initial condition in the t integration which assumes wave elevation is zero at the bow of a ship. It appears to be a reason why the prediction is not accurate enough in the wave elevation at the bow part when relatively blunt bow ship forms are concerned. A resolution for this problem based on a thin ship approximation is given by Fontaine and Faltinsen (1997). Analysis of the bow flow with flat ship geometry was carried out by Dagan and Tulin (1972) and Ferdinandez (1981): the former is concerned with two dimensional problem and the latter is the analysis of the near bow flow as two dimensional asymptote to the actual three dimensional flow. Both are based on a nonlinear model of the flow in the bow near domain.

In this report we apply a flat ship approximation to obtain a correction to the wave elevation predicted by $2D+t$ approach. Our first analysis will proceed along a similar line (linear approach) as Fontaine and Faltinsen but with a flat ship assumption. The second analysis is based on a nonlinear model of the bow flow as Ferdinendez

2. Bow form of small flare angle

A ship advances on a calm water surface at a constant speed U; the ship is flat i.e. the draft is small compared with the width; when compared with the ship length the width is very small. The $x-y$ plane coincides with the still water surface and the z axis is take as vertically upward; the origin is at the apex of the ship bow and the x axis directs backward of the ship. The water plane is as shown in Figure in the last page.

In near bow domain of the order $O(b)$ it is assumed the ship bottom is a plane making very small angle α with the $x-y$ plane. At vanishing draft is imposed the body condition on $z=0$.

$$\frac{\partial \phi}{\partial z} = U\alpha \qquad \text{on } S,\ z=0 \tag{1}$$

Free surface condition at $z=0$ in virtue of high Froude number assumption will be

$$\phi = 0, \qquad U\frac{\partial \eta}{\partial x} = \frac{\partial \phi}{\partial z} \qquad \text{on } \ z=0 \tag{2}$$

Here ϕ is the velocity potential of the flow and η the wave elevation.

A solution of three dimensional Laplace equation satisfying the above conditions will be a solution for a lifting surface (see for example Newman (1973), Jordan(1974)) .

Therefore our solution ϕ_{NB} in the near bow domain will be

$$\phi_{NB} = \frac{U\alpha}{4\pi} \int dy_1 \int_S dx_1 \frac{\partial \tau}{\partial x_1} \frac{z}{(x-x_1)^2+z^2} \left[1 + \frac{x-x_1}{\sqrt{(x-x_1)^2+(y-y_1)^2+z^2}} \right] \tag{3}$$

$\tau(x,y) = (1 - f(x,y))\sqrt{b^2-y^2}$

$f(x,y) \to 0$ as $x \to \infty$ and $f(x,y) \to 1 + O\big((x - x_{LE}(y))^{1/2}\big)$ as $x \to x_{LE}(y)$
where x_{LE} is the x coordinate of the leading edge.

Bow domain of the order $O(b^{1/2})$ is the zone where $2.5D$ approach will be valid; two dimensional Laplace equation (in the $y-z$ plane) is imposed with the body condition at the exact body surface and the free surface condition

$$U^2 \frac{\partial^2 \phi_B}{\partial x^2} + g \frac{\partial \phi_B}{\partial z} = 0 \qquad \text{on} \quad z = 0 \tag{4}$$

The initial condition to be imposed at $x = 0$ will be

$$\phi_B(0,y,0) = 0, \quad \eta_B(0,y) = 0 \tag{5}$$

Matching of the solution ϕ_{NB} with the solution ϕ_B in the bow domain will be readily confirmed. (3) will be at $x \to \infty$

$$\phi_{NB} = \frac{U\alpha}{2\pi} \int_{-b}^{b} \tau(x,y_1) \frac{z}{(y-y_1)^2+z^2} dy_1 \tag{6}$$

This satisfies $2D$ Laplace condition and is appropriate to be matched with $2.5D$ solution; obviously it provides the initial conditions of (5) (the wave elevation at $x = \infty$ due to ϕ_{NB} may be selected to be zero as an initial condition of the integration of (2)).

Outer solution ϕ_{BO} of ϕ_B in the bow domain will be given by vertical doublet distribution of the strength $M(x)$ along the centerline of the ship and its asymptotic behavior as $\sqrt{y^2+z^2}$ approaches zero will be

$$\phi_{BO} \sim -\frac{M(x)}{2\pi} \frac{z}{y^2+z^2} \tag{7}$$

This will be an appropriate condition for $2.5D$ solution at $\sqrt{y^2+z^2} \to \infty$. Consequently $M(x)$ will be determined by $2.5D$ solution.

To complete the matching it is required that ϕ_{BO} at $\sqrt{y^2+z^2} \to 0,\ x \to 0$ must be equal to ϕ_{NB} at $\sqrt{y^2+z^2} \to \infty,\ x \to \infty$. It will be satisfied if

$$M(0) = -U\alpha \int_{-b}^{b} \tau(\infty, y_1) dy_1 \tag{8}$$

Once we have established the matching of all solutions, we may evaluate three dimensional correction to the wave elevation at the bow computed with $2.5D$ approach.It will be a composite expression of the wave elevation in the near bow and the bow domains; it is given by

$$\eta(x,y) = \frac{\alpha}{4\pi} \int \int_S \frac{\partial \tau}{\partial x_1} dx_1 dy_1 \left[-(x-x_1) + \sqrt{(x-x_1)^2 + (y-y_1)^2}\right] \tag{9}$$

If τ is obtained somehow (numerical approach must be employed for an arbitrary water plane configuration), then we are able to evaluate the wave elevation at the bow part. Here we apply this approach to a slender rectangular form whose τ is analytically given (Wieghardt (1939), Newman (1973)) though this from is not plausible for the bow form of a ship. Then three dimensional correction to the wave elevation is evaluated as

$$\eta(x,y) = \frac{\alpha}{4\pi} \int_{-b}^{b} dy_1 \int_0^{\infty} dx_1 \frac{\sqrt{\pi}(b^2 - y_1^2)^{1/2}}{\sqrt{x}_1 + 2.4x_1^3} \left[-(x-x_1) + \sqrt{(x-x_1)^2 + (y-y_1)^2}\right] \tag{10}$$

Numerical results will be presented at the Workshop.

3.Bow form of finite flare angle

If the bow flare of a flat ship intersects the water surface with a finite angle, we have to employ another approach for the near bow flow; we follow the approach by Ferdinandez (1981). The flow very close to the bow is approximated by two dimensional flow within the planes normal to the water line of the bow; the plane is defined at every location of the water line. The free surface condition based on high Froude number assumption will be

$$\left(\frac{\partial \phi}{\partial \xi}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2 = U_i^2, \qquad \frac{\partial \phi}{\partial z} = \frac{\partial \phi}{\partial \xi}\frac{\partial \eta}{\partial \xi} \qquad \text{on } z = \eta \tag{11}$$

The flow satisfying these conditions will be modeled by a jet flow of thickness q along the bow flare (see Dagan and Tulin (1972), Ferdinandez (1981)).

The outer expansion of this flow model is given by

$$\phi_\xi - i\phi_z \sim -U_i - \frac{2}{3} i U_i \left[\left(\frac{\pi}{\alpha}\right)^2 - 1\right]\left(\frac{q}{\pi Z}\right)^{\frac{3}{2}} - 3iU_i\left[\left(\frac{\pi}{\alpha}\right)^2 - 1\right]\left(\frac{q}{\pi Z}\right)^{\frac{5}{2}} \log\left(\frac{q}{\pi Z}\right) + RZ^{-\frac{5}{2}} \tag{12}$$

where $Z = \xi + iz$. The singularity is a little unusual and its legitimacy must be confirmed by investigating if it is to be matched with appropriate outer solution.

For a preliminary stage of our study we consider here two dimensional case: two dimensional outer flow around a body of small draft at high Froude number is computed to the third order and then the limit is taken as its draft approaching zero. After some algebra we have the 1-term inner expansion of the outer solution which will be, for example at $\alpha = \pi/2$,

$$\phi_\xi - i\phi_z \sim -1 + iA_1\left(\frac{1}{Z}\right)^{\frac{3}{2}} + i\frac{3TA_1}{2\pi}\left(\frac{1}{Z}\right)^{\frac{5}{2}} \log Z + iC_2\left(\frac{1}{Z}\right)^{\frac{5}{2}} \tag{13}$$

where A_1 and C_2 are the unknowns to be determined in the process of matching with (12). T is the draft of the body.

Certainly the matching determines all the unknowns q, A_1 and C_2 in (12) and (13). Wave elevation at the bow is computed immediately with the solution for two dimensional case.

Now we know the singularity appearing in (12) is rather natural. We should notice, however, that in order to achieve the matching perfectly we need the outer solution to the third order.

Outer solution for three dimensional case will be similar to (2). Its inner expansion, however, has a singularity of the order $z^{-1/2}$ and does not match with (2). Perhaps the derivative of (2) with respect to x will be an alternative of the first order outer solution whose singularity at the leading edge will be of $z^{-3/2}$. As mentioned above the solution to the third order will be required to achieve perfect matching. Numerical approach must be introduced to carry out the computation along this scheme. The details and the result will be discussed at the Workshop.

References

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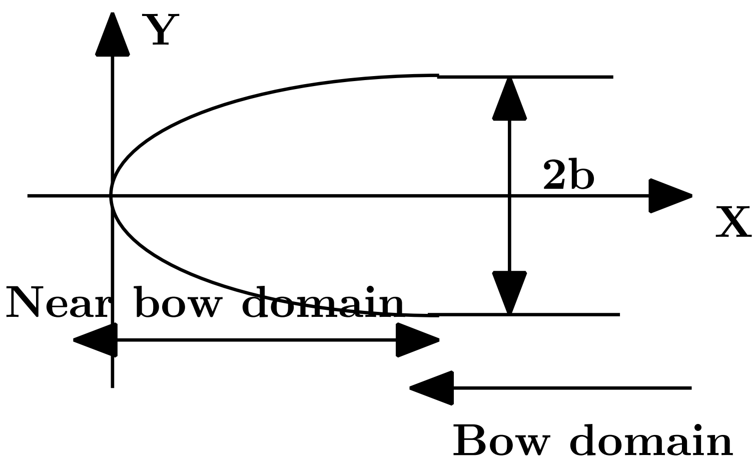

Figure Water plane of a flat ship