STEADY AND UNSTEADY FLOW IN WAVE-INDUCED BOUNDARY LAYERS

D. Mouazé, Y. Ourmières and J. R. Chaplin,
Department of Civil and Environmental Engineering, University of Southampton, Southampton SO17 1BJ, UK
E-mail: d.p.m.mouaze@soton.ac.uk; y.ourmieres@soton.ac.uk; j.r.chaplin@soton.ac.uk

SUMMARY

This paper describes measurements of velocities inside the seabed boundary layer in conditions where the boundary is slightly rough. Agreement with linear and non-linear theory is found to deteriorate as the higher harmonic components of the waves become more important. In all cases the steady streaming at the edge of the boundary layer is considerably weaker than predicted.

1. INTRODUCTION

Wave-induced boundary layers on the seabed and on the surface of floating bodies have some remarkable properties that are fully understood only over smooth boundaries and in waves of moderate amplitude at small scale. Owing to its progressive and oscillatory characteristics, the boundary layer at the seabed generates steady streaming in the direction of wave propagation that may be responsible for material transport over large distances [1]. Secondary flow similarly generated around floating and submerged bodies can have surprising effects on their loading and response [2].

For the simple harmonic case in which the wave-induced flow just outside the boundary layer is the real part of $u = u_1 \exp[i(kx - \omega t)]$, a solution to second order in the expansion parameter ($u_1 k^2 \nu/\beta\omega$), where β^{-1} is the boundary layer length scale, $\beta = (\omega/2\nu)^{1/2}$ and ν is the kinematic viscosity, was obtained by Longuet-Higgins [3]. The first order part consists of the oscillatory flow in the boundary layer

$$u^{(1)} = u_1 \exp[i(kx - \omega t)]\,\{1 - \exp[-(1+i)\beta y]\} \qquad (1)$$

while the second order component has a steady Eulerian velocity

$$u^{(2s)} = \frac{u_1^2 k}{4\omega}[3 - 2(\beta y + 2)\exp(-\beta y)\cos(\beta y) - 2(\beta y - 1)\exp(-\beta y)\sin(\beta y) + \exp(-2\beta y) \qquad (2)$$

where y is measured from the surface. As y tends to infinity, $u^{(2s)}$ approaches $3u_1^2 k/4\omega$, which is the steady streaming velocity that might be expected at the outer edge of the boundary layer. Higher order solutions were developed by Sleath [4] for the bed boundary layer and by Riley [5] for the related case of the viscous flow around a cylinder driven around a circular path without rotation. Comparisons are made here with the former, to second order, in conditions where the amplitude of second harmonic component of the wave-induced flow is as much as 30% of that of the fundamental.

Previous measurements of the flow inside wave-induced boundary layers include those by Beech [6], Sleath [7], Hwung & Lin [8] and Liu, Davis & Downing [9]. These are small scale experiments over smooth beds, and generally reveal good agreement with the oscillatory flow predicted by equation (1), and moderate agreement with the steady flow predicted by equation (2). The importance of scale effects, transition to turbulence, and wave non-linearities are however not known. In this paper we present measurements of velocities inside the seabed boundary layer in conditions where the boundary is slightly rough. Agreement with equations (1) and (2), and with the non-linear theory, is found to deteriorate as the higher harmonic components of the waves become more important.

2. EXPERIMENTAL ARRANGEMENTS

These velocity measurements were made in advance of separate investigations of the flow around a horizontal cylinder at two different scales [10]. They were performed by Laser Doppler Velocimetry in two wave flumes. The larger one, at the University of Hanover, is 120m long, 2.2m wide, and has a still water depth of 1.0m. Other tests were carried out in a smaller flume at the University of Caen, 22m long, 0.8m wide and with a still water depth of 0.5m. Both wavemakers have active absorption control; reflections from the beach were around 10% in the Hanover flume, but much smaller in the Caen flume. The fibre-optic based LDV system provided a spatial resolution of about 0.15mm, and was positioned in turn with the centre of its measurement volume at elevations between 0.1mm and 300mm above the floor of the tank. The roughness of the floor was estimated at 0.8mm and 0.15mm for the Hanover and Caen flumes respectively. Elevations y given below are measured from the top of the roughness elements. A wave gauge was placed directly over the LDV system.

Results given below were obtained from velocity records that had been phase-averaged over a sequence of similar waves. The wave conditions for the 8 test cases discussed below are set out in Tables 1 and 2, in which u_2/u_1 is the ratio of the amplitudes of the first two harmonic components of the wave-induced flow just outside the boundary layer.

Case	C1	C2	C3	C4
T (s)	1.33	1.50	1.80	2.20
H (m)	0.08	0.13	0.16	0.16
L (m)	2.39	2.82	3.57	4.53
β (m^{-1})	1439	1355	1237	1119
u_2/u_1	0.021	0.049	0.199	0.269

Table 1. Wave conditions for the Caen tests

Case	H1	H2	H3	H4
T (s)	2.00	3.00	3.50	5.00
H (m)	0.15	0.15	0.22	0.18
L (m)	5.21	8.69	10.36	15.24
β (m^{-1})	1174	958	887	742
u_2/u_1	0.035	0.064	0.197	0.300

Table 2. Wave conditions for the Hanover tests

3. DISCUSSION OF THE MEASUREMENTS

Figures 1 and 2 show the oscillatory flow through the thickness of the boundary layer at eight phases equally separated over one wave period. For the mildest cases (C1, C2, H1, H2) the data generally agree very well with the second order theory [4], but in more non-linear conditions there are significant differences. These occur when the amplitude of the second harmonic component of the wave-induced flow becomes an appreciable proportion of that of the fundamental.

The mean Eulerian velocities through the boundary layer are plotted in figure 3, and compared with the results of the same analytical solution. Here the agreement is much less satisfactory, and in each case the steady streaming at the outer edge of the boundary layer is considerably weaker than predicted. Some of the differences can be attributed to the very sensitive nature of the measurements, but similar observations were made by Liu, Davis & Downing [9]. They put forward as possible explanations the effects of wave reflections, secondary currents in the tank, and higher order non-linearities. In the case of the present series C tests, wave reflections were very small. We propose to explore further the importance of non-linear contributions through a full application of Sleath's solution, and by a fully non-linear numerical model outlined below.

4. NUMERICAL COMPUTATIONS

The chief difficulty in formulating a Navier Stokes code for this flow is in specifying the boundary conditions. If the computational domain covers only a thin region close to the seabed, a method has to be found for setting the appropriate steady streaming velocity at its upper boundary. Since the steady streaming is created by the flow inside, it seems unlikely that a suitable boundary condition can be formulated explicitly. In numerical solutions to be presented later, we follow the approach adopted by Riley [11]. The computation is repeated with different steady outer velocities superimposed on the wave-induce oscillatory flow, until a solution is found in which the vorticity tends uniformly to zero at large distances from the boundary layer. This has been found to give results that are in close agreement with second order analytical solutions in appropriate conditions.

5. CONCLUSIONS

Velocity measurements in the seabed boundary layer beneath waves are compared with non-linear analytical solutions. For the oscillatory flow, agreement is generally very good, but it deteriorates as the harmonic components of the wave-induced flow become more important. In all cases the mean flow at the outer edge of the boundary layer is found to be considerably weaker than the predictions. This is consistent with earlier observations [2] that the steady streaming around a circular cylinder beneath waves is weaker than might be expected from boundary layer theory.

6. REFERENCES

[1] Batchelor, G.K., *An Introduction to Fluid Dynamics* Cambridge University Press, 1967.

[2] Chaplin, J.R., 'Non–linear forces on a horizontal cylinder beneath waves' *J. Fluid Mech.* **147**, 449–464, 1984.

[3] Longuet-Higgins, M.S. 'Mass transport in water waves' *Phil. Trans. Roy. Soc. Lond.* **A245**, 535-581.

[4] Sleath, J.F.A., 'A second approximation to mass transport by water waves' *J. Marine Res.* **30**, 295-304, 1972.

[5] Riley, N. 'Stirring of a viscous fluid' ZAMP **22**, 645-653, 1971.

[6] Beech, N.W., 'Laser Doppler measurements in the oscillatory boundary layer beneath water waves' *DISA Information*, No. 23, 6-10, 1978.

[7] Sleath, J.F.A., 'Measurements of mass transport over a rough bed' *19th Conf. Coastal Engrg*, 1149-1160, 1984.

[8] Hwung, H.H & Lin, C., 'The mass transport of wave propagating on a sloping bottom' *22nd Conf. Coastal Engrg*, 544-556, 1990.

[9] Liu, P.L.-F., 'Wave-induced boundary layer flows above and in a permeable bed' *J. Fluid Mech.* **325**, 195-218, 1996.

[10] Mouazé, D.P.M., 'Experimental study of the boundary layer around a cylinder subject to waves', PhD thesis, University of Caen, 2001.

[11] Riley, N. Circular oscillations of a cylinder in a viscous fluid *ZAMP* **29**, 439-449, 1978.

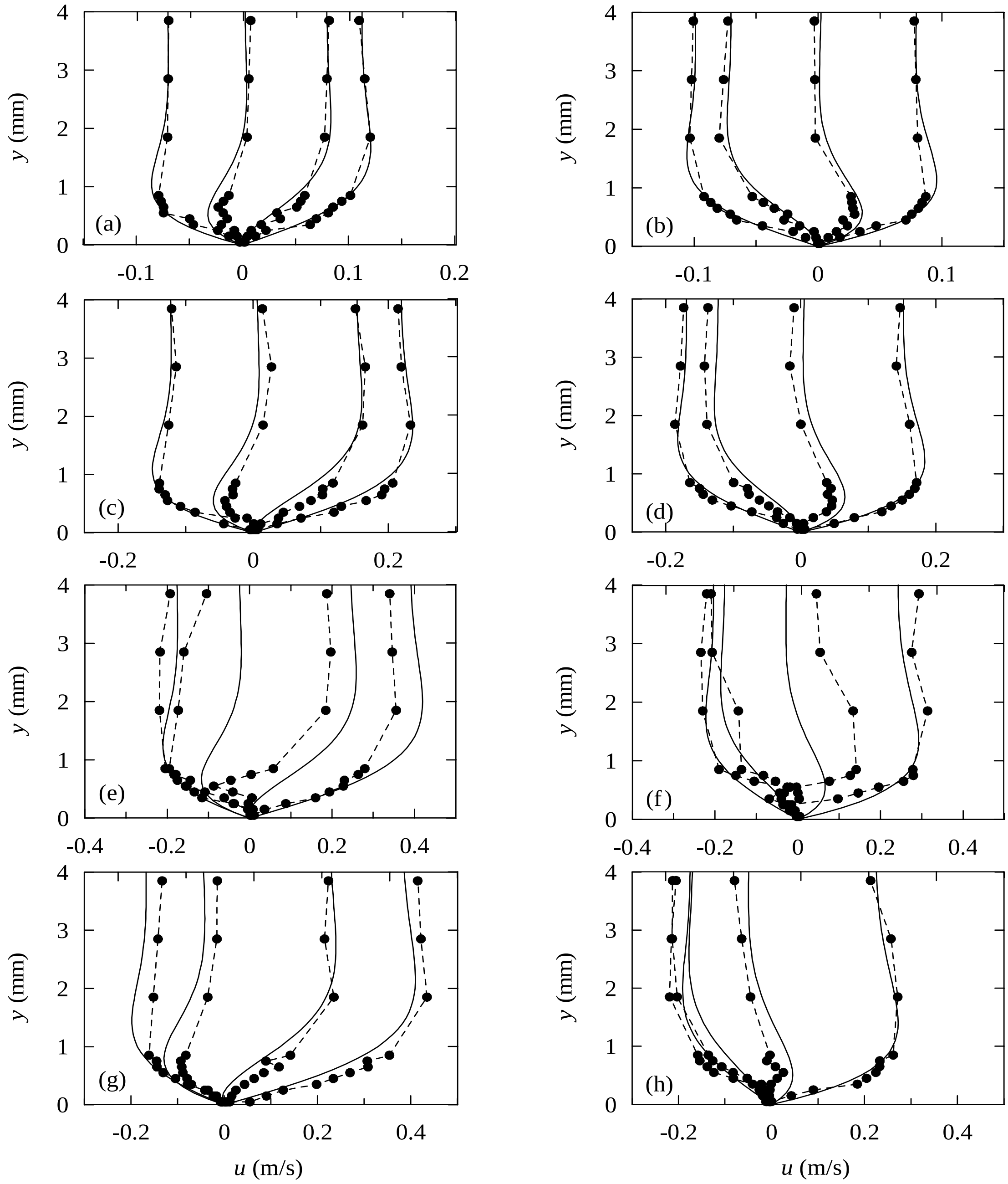

Figure 1. The oscillatory flow in cases C1 (a,b), C2 (c,d), C3 (e,f) and C4 (g,h). The right-most line in each left hand plot corresponds to the phase of the wave crest, and the other profiles (of which the last four are shown on the adjacent plot on the right) are at equal phase intervals through one wave period. Measurements are shown as points, and the lines represent second order theory [4].

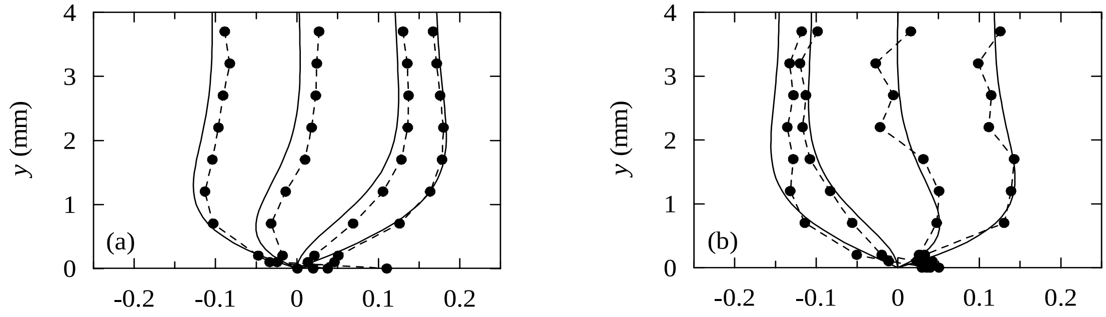

Figure 2(a) and (b). See caption below.

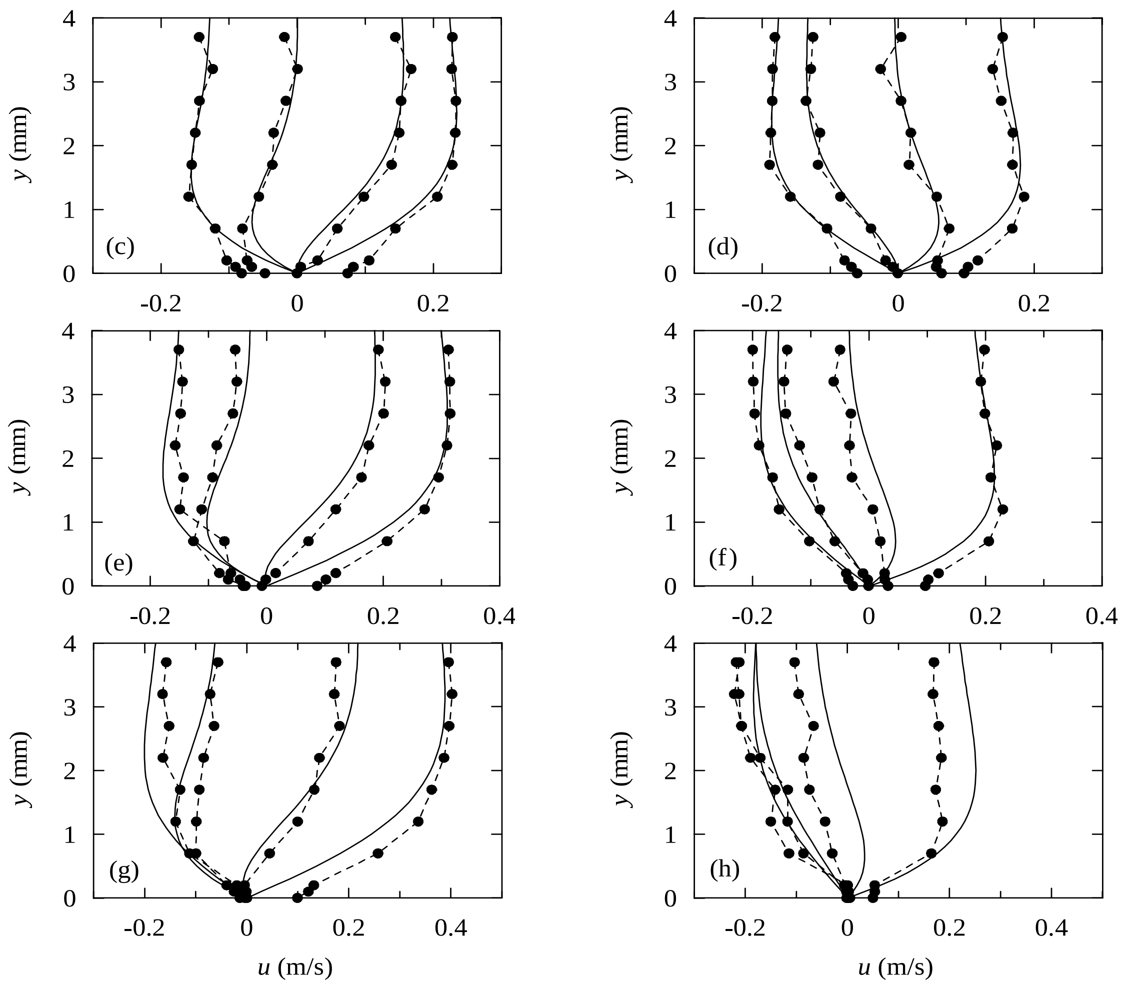

Figure 2. As for figure 1, cases H1 to H4.

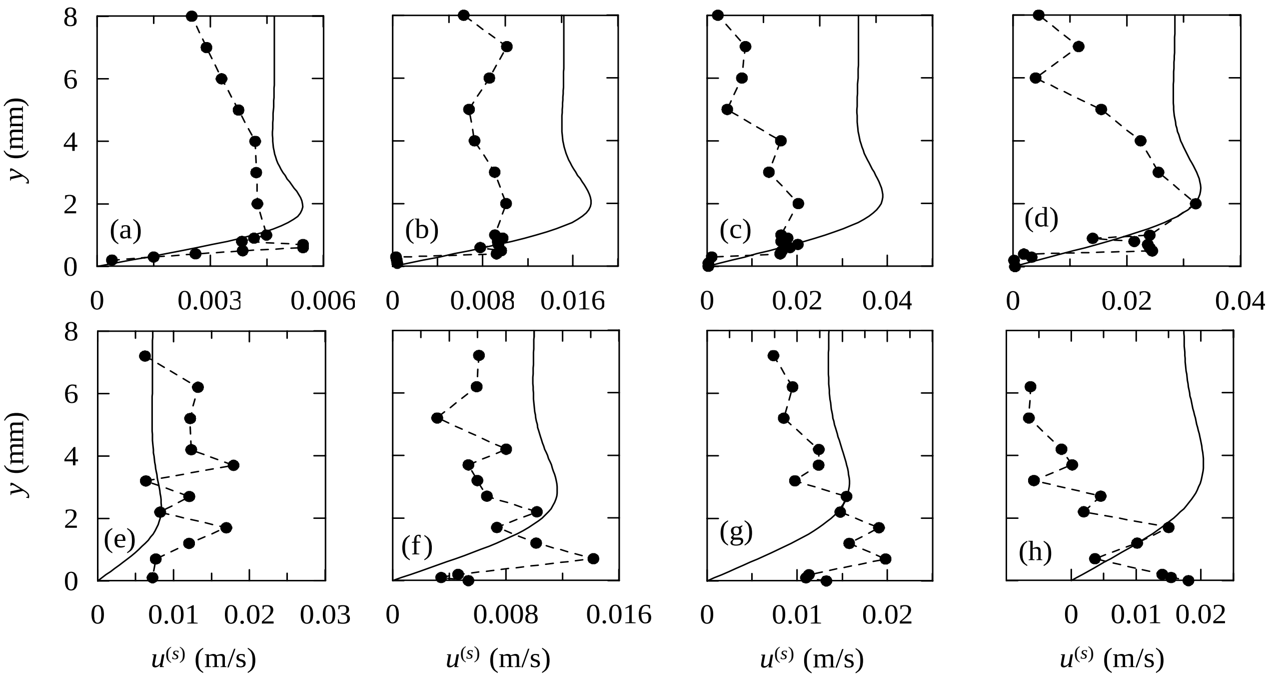

Figure 3. The Eulerian mean flow for cases C1 (a) to C4 (d), and H1 (e) to H4 (h). Measurements are shown as points, and the lines represent theory [1,4].