Fluid Motions in a Tank with Internal Structure

Renchuan Zhu and Kimio Saito
Div. of Development Science, Hiroshima University, JAPAN

1. INTRODUCTION

The linear and nonlinear fluid motions has been widely studied by using Boundary Element Method (BEM)*(Brebbia (1978))*, which has a merit of reducing the dimensions by one and it is applicable even for the infinite domain problem. When BEM is used for the calculations of the velocity potential and the hydrodynamic forces, it is important to get a precise value in the boundary integral of the functions of $1/r$, in which r is the distance between elements. It is difficult to get the analytical solution for the three-dimensional (3D) problem and the numerical method such as Gaussian integration will be applied to the integral of the function $1/r$, in which the singular integration on the boundary should be paid attention. In a tank with the internal structure, the distance r between elements on both sides of the internal structure will tend to 0, when the thickness of the plate t tends to 0. So it is very difficult to avoid the numerical error in the integral of the function $1/r$. It has shown the errors between the analytical solution and the numerical integral in the paper of Nishino et al.*(1999)*.

In this paper, the sloshing in 3D tank with the internal structure and the vibration of the internal structure in 3D tank are discussed by extending the basic BEM to the multiple domain problems. The fluid motions in a tank with the internal structure that is subjected to the forced oscillations and the dynamic pressure distributions on the thin vibrating internal structure are shown. Some of the computed results are compared with the ones in the published paper. It indicates that they agree well each other and the present method is effective.

2. MULTIPLE DOMAIN "BEM"

As shown in Fig.1, we assume that the whole domain composes of the two connected domains, I and II with the imaginary boundary Γ_i. When the outside boundaries of the domain I and II are expressed by Γ_1 and Γ_2 respectively, the domain I is surrounded by Γ_1 and Γ_i, and the domain II by Γ_2 and Γ_i.

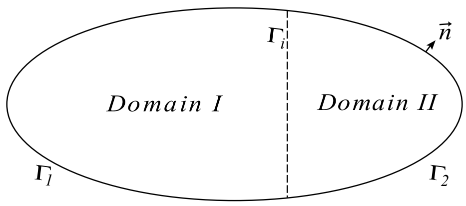


Fig.1 Multiple domain system

We assume the fluid is incompressible and the flow irrotational so that there exists a velocity potential ϕ that satisfies the Laplace equation, in the whole fluid domain, and the velocity potential is defined as $\nabla\phi = \vec{q}$ ($\vec{q}$ is the fluid velocity).

On the outside boundaries there are two kinds of conditions. One is expressed by the value of the velocity potential on the part of the outside boundary, i.e. ϕ*=known*. Another is the value of the normal differentiation of the velocity potential on the remaining outside boundary, $\partial\phi/\partial n$*=known*.

On the imaginary boundary, the flow velocity and its potential at any point is the same as the one in the adjacent domain, so that we obtain the following conditions on the imaginary boundary,

$$\phi\Big|^{II} = \phi\Big|^{I}, \qquad \frac{\partial\phi}{\partial n}\Big|^{II} = -\frac{\partial\phi}{\partial n}\Big|^{I}, \qquad on \;\; \Gamma_i \tag{1}$$

According to the Green's formula, the velocity potential ϕ can be written as,

$$c_P \phi_P = \oint_S \frac{\partial \phi}{\partial n} ln\frac{1}{r} ds - \oint_S \phi \frac{\partial}{\partial n}\left(ln\frac{1}{r}\right) dS \tag{2}$$

where r is the distance between the source and field point, c_P and ϕ_P is the radian measure and the velocity potential at the node P on the boundaries, S is the whole boundaries in the domain.

By using the expression of vector and matrix, it can be written as:

$$[H]\{\phi\} = [G]\left\{\frac{\partial \phi}{\partial n}\right\} \qquad \text{in domain I, II} \tag{3}$$

where matrices [H] and [G] consist of constants determined by Green's function and the mesh system of the boundary discretization method, vectors $\{\phi\}$ and $\{\partial\phi/\partial n\}$ are the values at the nodes on the boundaries.

By using the domain name for the superscript and the variables on the boundary for the subscript, the following matrix equation will be obtained,

$$\begin{Bmatrix} H_o^I & H_i^I & -G_i^I & 0 \\ 0 & H_i^{II} & G_i^{II} & H_o^{II} \end{Bmatrix} \Phi = \begin{Bmatrix} G_o^I & 0 \\ 0 & G_o^{II} \end{Bmatrix} V \tag{4}$$

where the subscript "i" means the variable of the nodes on the imaginary boundary Γ_i, and "o" denotes the variable of the nodes on the outside boundaries Γ_1 and Γ_2 and

$\Phi = \left\{\phi_o^I, \phi_i^I, \partial\phi/\partial n\big|_i^I, \phi_o^{II}\right\}^T$, $V = \left\{\partial\phi/\partial n\big|_o^I, \partial\phi/\partial n\big|_o^{II}\right\}^T$.

It is obvious that the velocity potential can be obtained by solving Eq. (4).

3. SLOSHING IN 3D TANK WITH INTERNAL STRUCTURE

We consider a 3D tank that is subjected to forced oscillations and assume that the fluid in a tank is inviscid and incompressible and the flow is irrotational in the whole domain.

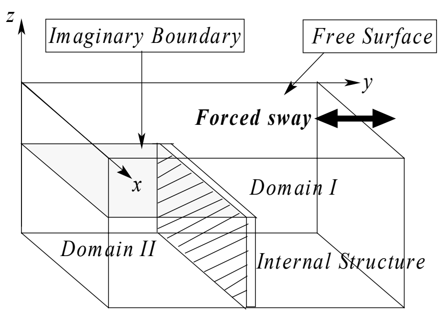


Fig.2 3D sloshing model and fluid domains

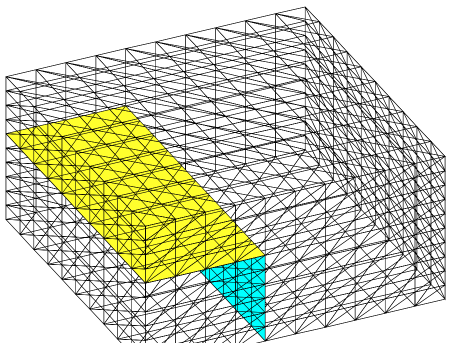


Fig.3 Triangle mesh system

As shown in Fig.2, for simplicity, we assume the tank is only a forced sway in y direction. The governing equation satisfies the 3D Lapalace equation. On the free surface, the dynamic and kinematic boundary conditions can be described by Eqs. (5) and (6),

$$\frac{\partial \phi}{\partial t} = -\frac{1}{2}\left\{\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2\right\} + a_y y + gz + \mu\phi \tag{5}$$

$$n_y \frac{\partial \varsigma}{\partial t} = \frac{\partial \phi}{\partial n} \tag{6}$$

where μ is the viscosity coefficient. On the rigid boundaries in the whole domain are all zero. The time-stepping technique and the center finite difference schemes with respect to time are used to obtain the velocity potential at the node on the nonlinear free surface:

$$\phi\Big|_{t+\Delta t} = \phi\Big|_{t-\Delta t} + \frac{d\phi}{dt}\Big|_t \cdot 2\Delta t, \qquad \varsigma\Big|_{t+\Delta t} = \varsigma\Big|_{t-\Delta t} + \frac{d\varsigma}{dt}\Big|_t \cdot 2\Delta t \tag{7}$$

where $d\phi/dt = \partial\phi/\partial t + |\nabla\phi|^2$ and $d\varsigma/dt$ is substituted by $\partial\varsigma/\partial t$. From Eq. (7), the free surface profile and its cooresponding velocity potential at every time step will be obtained.

In order to verify the numerical method and the program of this study, the numerical calculations for the sloshing problem have been carried out and the results are compared with the one of Shinkai et al.*(1987)* which was computed by MAC (Mark and Cell) method. We used the same tank as Shinkai's having the breadth and length: a=40cm, the water depth of 14cm fitting with the 4cm internal structure on the centerline of the bottom. The tank is subjected to the forced sway oscillations in which the period is 1.0 second and the amplitude is 1.0cm. For the start of the computation, it is assumed that the tank is set to be horizontal and the water is at rest. The computed free surface elevations are shown in Fig. 4 and Fig. 5. The former gives the free surface profile in the tank at time t=2.40 second and the latter illustrates the time histories of the free surface elevation at the tank side on the centerline ($x=a/2$). In Fig. 5 the solid line denotes the result of present method, symbol is Shinkai's which is compared by using two dimensional method *(Shinkai et al.(1987))*. The periodic line is close to the symbols and it indicates the present method is effective.

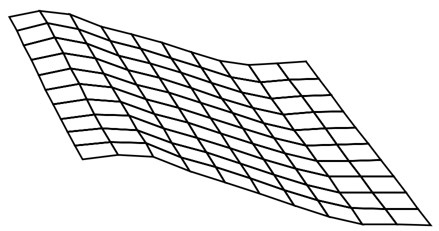


Fig. 4 Free surface profile in the tank at t=2.40

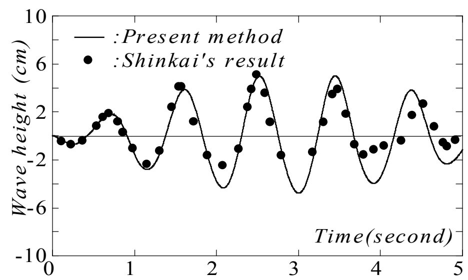
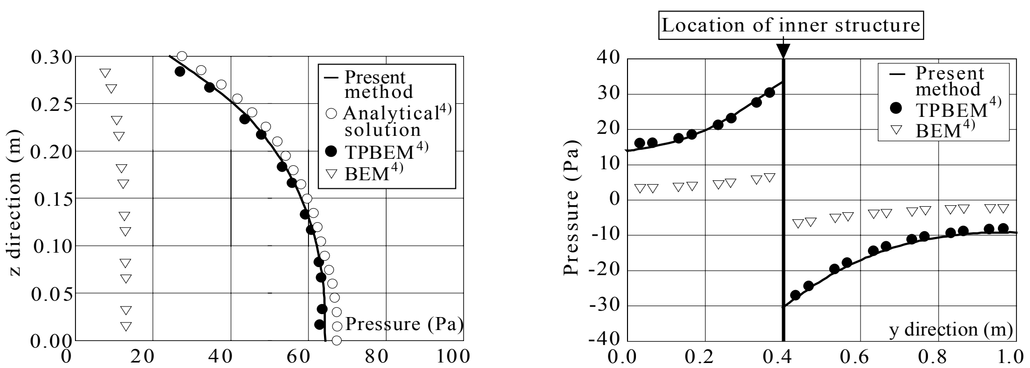


Fig. 5 Comparison of the free surface elevations at the tank side on the centerline ($x=a/2$)

4. VIBRATION OF INTERNAL STRUCTURE IN 3D TANK

We consider a 3D tank with an internal structure that is subjected to a periodical forced oscillation with an infinitely high frequency. It is assumed that the fluid is incompressible and the flow irrotational so that there exists a velocity potential ϕ that satisfies the Laplace equation. Since the frequency $\omega \to \infty$,

$$\phi = 0 \quad \text{on the free surface.} \tag{8}$$



(a) Pressure gap distribution on thin plate

(b) Pressure distribution on bottom

Fig. 6 Pressure distribution due to a unit acceleration of the internal structure

On the tank wall and internal structure, $\partial\phi/\partial n=-V_y$ and on the tank bottom $\partial\phi/\partial n=0$ (n is the normal to the boundaries).From the Bernoulli's equation, the pressure can be expressed as,

$$p = -\rho\frac{\partial\phi}{\partial t} \quad (\rho\text{: fluid density}) \qquad (9)$$

By the integration of Eq. (9) on the rigid boundaries, the hydrodynamic forces acting on the boundaries can be obtained.

For the numerical calculations, we choose the case of Nishino et al. *(1999)*, in which the dimensions of the tank and the internal structure are as shown in Fig. 2. Both the breadth and length of the tank: a=1.0m and the fluid depth is 0.50m. The internal structure locates at y=0.40m and it has a height of 0.30m. Here we also used the method of the triangle element mesh system to discretize two domains of the tank, which is similar to section **3**. It is assumed that the internal structure oscillates in the fluid with unit acceleration in its normal direction. Fig. 6 shows the pressure distributions on the centerline ($x=a/2$) due to unit acceleration computed by using the triangle element with linear shape function. It also gives the pressure gap distribution on the thin internal structure and the pressure on the bottom of the tank. In the figure, the analytical solution, the results of the basic BEM and TPBEM (Thin Plate BEM by Nishino et al. *(1999)*) are shown by the symbols of circle and triangle respectively. The result of the present method is very close to the analytical solution and the one of TPBEM.

5. DISCUSSIONS

Nonlinear simulations have been carried out for the fluid motions in a tank with internal structure that is subjected to the forced sway oscillation. In order to avoid the numerical error which will be appeared in the integral on the internal structure, the basic BEM has been extended to the multiple domain problems. The computer program has been developed and applied to the simulation of the sloshing phenomena. Some of the computational results are compared with the ones in the published paper and it indicates that they agree well each other and the present method is effective.

The multiple domain BEM has been also applied to the vibration of the internal structure in contact with the water. From the comparison of the solutions among the basic BEM, TPBEM and the analytical ones, the present method agrees well with TPBEM and analytical solutions. It is also expected that the present method is a useful tool for the evaluation of the added mass in the fluid-structure interactive vibration.

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