Extension of the Havelock/Dawson Method to Include Nonlinear Free-Surface Boundary Conditions

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The Havelock/Dawson method of solving linearized free-surface problems was discussed by the author in 1999 [1]. A combination of Rankine singularities distributed on the surface of the ship hull and Havelock singularity distributions placed over the undisturbed free surface can be used to impose either the classic Kelvin linearized free-surface condition or Dawson's linearized boundary condition. With this approach we have been able to generate numerical solutions that combine the near-field accuracy of Rankine codes with the far-field accuracy of Havelock codes. We have found this method to be both robust and computationally efficient. An extension of the basic Havelock/Dawson approach to include fully nonlinear free-surface boundary conditions is discussed here.

The Kelvin and the Dawson linearized free-surface boundary conditions are special cases of the general expansion of the nonlinear free-surface conditions about an arbitrary basis function. Taking the velocity potential Φ as the sum of a known basis function ϕ_0 and an unknown perturbation potential ϕ_1, we can write the free-surface boundary condition in the following form:

$$\tfrac{1}{2}\nabla\phi_1 \cdot \nabla(\nabla\phi_0 \cdot \nabla\phi_0) \;+\; \nabla\phi_0 \cdot \nabla(\nabla\phi_0 \cdot \nabla\phi_1) \;+\; g\,\tfrac{\partial}{\partial z}\phi_1 \;=\; -\tfrac{1}{2}\nabla\phi_0 \cdot \nabla(\nabla\phi_0 \cdot \nabla\phi_0) \;-\; g\,\tfrac{\partial}{\partial z}\phi_0 \;+\; \mathrm{O}(\nabla\phi_1)^2 \;.$$

Assuming that the perturbation potential is "small", $\nabla\phi_1 \ll \nabla\phi_0$, we hope to be able to neglect the higher order terms in $\nabla\phi_1$. Of course, the nonlinear boundary condition should be satisfied on the exact position of free surface, $z = \zeta$. However, consistent with our intent to retain only the first order terms in the perturbation potential, we can expand this boundary condition in a Taylor series about the position of the free surface corresponding to the basis flow, $z = \zeta_0$. Such an expansion leads to first order terms in $\Delta\zeta = \zeta_1 - \zeta_0$, that should properly be retained under the assumption that they can be of the same order as the first order terms in ϕ_1; see Nakos and Sclavounos, 1991 [2], for details. However, Raven, 1996 [3], pointed out that the $\Delta\zeta$ terms can be quite irregular and suggested that the convergence of an iterative scheme may actually be improved by leaving these terms out. We decided to follow Raven's approach in order to take advantage of the numerical simplification, and we apply the boundary condition directly on $z = \zeta_0$.

In our iterative scheme, we use the Havelock/Dawson solution as the initial basis flow (or zeroth iteration). We then attempt to solve for the perturbation potential by applying the linearized expansion about the basis flow using

$$\tfrac{1}{2}\nabla\phi_{i+1}\cdot\nabla(\nabla\phi_i\cdot\nabla\phi_i) \;+\; \nabla\phi_i\cdot\nabla(\nabla\phi_i\cdot\nabla\phi_{i+1}) \;+\; g\,\tfrac{\partial}{\partial z}\phi_{i+1} \;\approx$$
$$-\tfrac{1}{2}\nabla\phi_i\cdot\nabla(\nabla\phi_i\cdot\nabla\phi_i) \;-\; g\,\tfrac{\partial}{\partial z}\phi_i \;, \quad \text{on } z=\zeta_i \,.$$

The new free-surface elevation ζ_{i+1} is calculated by applying the Bernoulli equation on $z = \zeta_i$. It was not obvious that such an iterative scheme would necessarily be convergent. However, by first extending our existing Havelock/Dawson code to include the perturbation expansion about an arbitrary basis flow, and then nesting the code within an iterative loop, we are able to investigate the convergence numerically.

With the basic Havelock/Dawson method, Rankine source panels are distributed over the surface of the hull $S(x, z)$, and Havelock source panels are distributed over a local region of the undisturbed free surface $Z(x, y) = 0$. It has been found that the Havelock singularity density necessary to satisfy the free-surface boundary condition tends to zero rapidly as the distance from the hull increases, and therefore the number of free-surface panels required can be quite small relative to other methods. For the nonlinear problem, we wish to distribute the free-surface panels on $\zeta_i(x, y) \neq 0$, and the accuracy achieved by panelizing a small near-field region of the free surface needs to be demonstrated. Furthermore, positioning Havelock singularities at locations above the mean free surface presents mathematical difficulties that will be addressed in the following section. For the moment, imagine that we distribute N Rankine panels over the surface of the hull $S(x, z)$, and M Havelock panels over the near-field region of the free surface $\zeta_i(x, y)$, to solve for ϕ_{i+1}. The determination of the source strengths σ^S and σ^Z necessary to satisfy the boundary conditions will involve solving a matrix equation of the form

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \sigma^S \\ \sigma^Z \end{bmatrix} = \begin{bmatrix} \mathbf{B}^S \\ \mathbf{B}^Z \end{bmatrix} , \quad \text{where}$$

$\mathbf{A}_{11}$ contains the influence of the Rankine hull panels on the hull collocation points,

$\mathbf{A}_{12}$ contains the influence of the Havelock free-surface panels on hull collocation points,

$\mathbf{A}_{21}$ contains the influence of the Rankine panels on free-surface collocation points, and

$\mathbf{A}_{22}$ contains the influence of the Havelock panels on the free-surface collocation points.

The vectors $\mathbf{B}^S$ and $\mathbf{B}^Z$ contain the boundary conditions to be satisfied on the hull and free surface respectively.

There is no difficulty calculating the $\mathbf{A}_{11}$ and $\mathbf{A}_{21}$ sub-matrices since the influence of Rankine panels is well defined for arbitrary field points. However, evaluation of the $\mathbf{A}_{12}$ and $\mathbf{A}_{22}$ sub-matrices can be problematic since either the source point, the field point, or both, might occur at positions above the level of the undisturbed free surface, $z = 0$. The $e^{k(z+z')}$ term in the Havelock Green function represents the attenuation of free-surface waves due to either the depth of the source point z or the field point z', and generally both z and z' are ≤ 0. With positive values of the source and field points, the influence of the

Havelock wave potential will grow exponentially. One method of avoiding this problem is to use a coordinate system fixed relative to the local free surface. Then the position of both the field point and the source point can be taken as the local depth and the $e^{k(z+z')}$ term will always be ≤ 1.

In the calculation of the $\mathbf{A}_{22}$ sub-matrix, we have employed an *ad hoc* assumption that since both the source point and the field point are located on the free surface at zero depth, the depth attenuation term is equal to unity. In the calculation of the $\mathbf{A}_{12}$ sub-matrix, the Havelock source point is always located at zero depth and the depth of the field point on the hull is calculated relative to the local dynamic waterline. Otherwise, the subroutines that we use to calculate the influence of Havelock singularity distributions are similar to those used in the linear Havelock/Dawson code.

As always, the Wigley hull was the initial geometry used to investigate the convergence of the iterative scheme. Since the wetted area of the hull will change with each iteration, one should repanelize at every step. However, for this investigation, the hull was assumed to be fixed in sinkage and trim and we could then simply panelize the hull to some distance above the design waterline, and assign a source strength of zero to each panel that is not submerged at any particular iteration. We allowed the iterative scheme to proceed until the Rankine and Havelock singularity strengths associated with the perturbation potential were two orders of magnitude less than the singularity strengths calculated for the basis

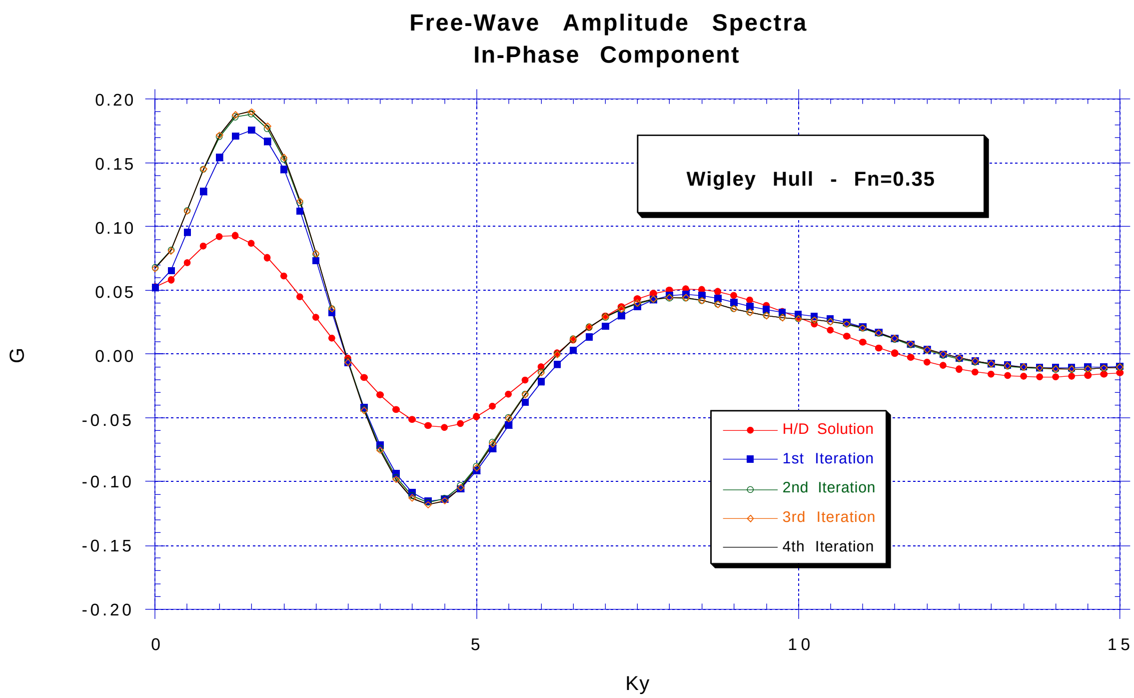

Figure 1 – Free-Wave Spectra Calculated for First 4 Iterations

flow (Havelock/Dawson solution). We found that this convergence criterion was satisfied in just 4 iterations for the Wigley hull test case at Fn = 0.35. Moreover, the radiating waves had converged after only 2 iterations, as can be seen from the plots of the free-wave spectra. The hull was panelized with 604 Rankine panels and a similar number of Havelock panels were distributed on the free surface. These computations required about 22 minutes to run on a 400 MHz Macintosh G3.

We ran a second test case using a more complex hull geometry – a high-speed naval combatant hull form. This geometry required 890 Rankine panels to define the hull shape. The local free-surface domain was panelized with 975 Havelock panels. For an 18-knot case (Fn = 0.23), the convergence criterion was met in 7 iterations. However, as was observed with the Wigley hull test case, the calculated free-wave spectra are quite close to the final nonlinear results even after the first two iterations. In the near-field, the dominant effects of the nonlinear free-surface conditions appear as a deepening of the wave troughs, an increase in the wave slopes, and a minor phase shift in the bow wave. This computation took about 100 minutes on the same Macintosh G3 computer.

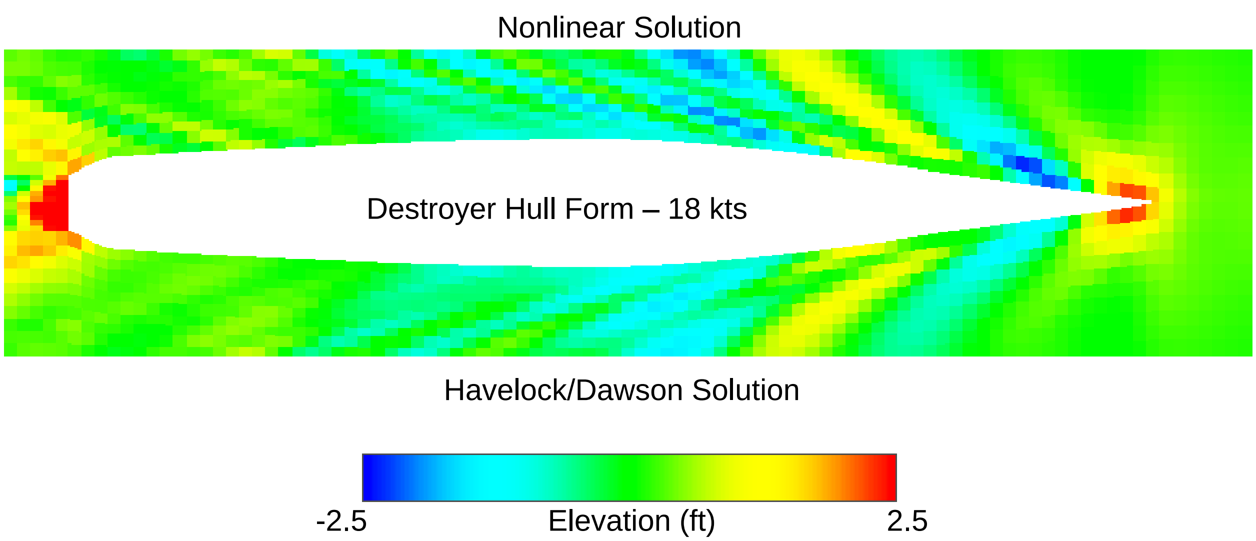

Figure 2 – Nonlinear vs. Linearized Free-Surface Waves

[1] Scragg, C.A. "On the Use of Free-Surface Distributions of Havelock Singularities," 14th International Workshop on Water Waves and Floating Bodies, Port Huron, Michigan, 1999, pp 133-37.

[2] Nakos,D. and P.Sclavounos "Ship Motions by a Three-Dimensional Rankine Panel Method," 18th Sym. on Naval Hydrodynamics, Ann Arbor, 1991. pp 21-40

[3] Raven, H. "A Solution Method for the Nonlinear Ship Wave Resistance Problem," Thesis, Technische Universiteit Delft, 1996, 220 pages.