Waves Generated by Ship Motions

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1 Introduction

I am interested in seeing actual radiation waves or diffraction waves of ships at constant forward speed. One reason is that the wave pattern of those waves, let alone the quantitative information of the contour of the wave pattern, is not visible at usual tank test; the Kelvin wave pattern and the incident waves prevent us from seeing them. It is interesting not only for its own sake (for example Cao, Shultz and Beck (1994)) but practically; the damping of ship motions and added resistance in waves are direct results of those waves. My second interest is to know if our theoretical methods are capable of predicting accurately the contour of the wave pattern of radiation and diffraction waves. This will be a crucial issue in studies of ship motion theory.

Purpose of my study is to visualize instanteneous contour of the dynamic radiation wave pattern measured at tank test and to compare the measured contour with the one theretically predicted.

2 Computarized measurement of the wave contour

Our technique is able to visualiza the instanteneous contour of unsteady free surface elevation around a ship model moving at forward speed and oscillating at a frequency. Idea of this technique is briefly described below (see Ohkusu and Wen (1996) for the details). Wave is recorded with several wave probes continuously during a run of the experiment; their locations are fixed to the water tank and set on a line paralell to the ship model track with an equal distance between the neiboring probes. The wave probes move relatively in the reference frame fixed to the average position of the ship model (the ship reference frame) as it runs at a constant forward speed U. The wave probes reach an identical position P in the ship reference frame at different time instants. Adopt the wave record of each probe at the time instant when it reaches the location P, then we have wave records at a location P at sveral different time instants. Those wave records will determine $\zeta_0(x, y)$ (the Kelvin wave), $\zeta_1(x, y)$ (the fundamental harmonics of the radiation wave) and $\zeta_2(x, y)$ (the second harmonics) of the total wave elevation around the ship model $\zeta(x, y, t)$:

$$\zeta(x, y, t) = \zeta_0(x, y) + \zeta_1(x, y)e^{i\omega t} + \zeta_2(x, y)e^{i2\omega t} + \cdots \quad (1)$$

The wave probes record the wave elevation continuously and consequently the determination of $\zeta_{0,1,2}$ will be done at any location on the line i.e.we can determine the steady wave elevation and the amplitude and phase of the unsteady wave elevation everywhere on the line at a distance from the ship model's track. This process is feasible by the computer involvement.

The measurement is repeated at another line of the wave probes with different distance from the ship model's track. Repeatability of the measurement is confirmed and accurate contour of the waves $\zeta_1 e^{i\omega t}$ and $\zeta_2 e^{i\omega t}$ are constructed by the results obtained when we repeat this many times.

One example of snapshots of the contour of the first order wave elevation $\zeta_1 e^{i\omega t}$ and the second order's one $\zeta_2 e^{i2\omega t}$ (all are normalized by the amplitude of the vertical motion at FP)are shown on the last page of this report. They are for S175 model forced to pitch at $\tau = \omega U/g = 1.446$ and the amplitude of about $2.2°$ (the amplitude of the vertical motion at FP is about 30 percent of the draft) towed at the Froude number $F_n = 0.275$. It is concluded from the results of experiment at two different amplitude of pitch that ζ_1 is certainly the first order wave elevation and ζ_2 the secodn order wave (even if some other order effect is included, it will be very small). The upper part of the first order wave is the wave contour at $\omega t = 0$ and the lower part the wave contour at $\omega t = \pi/2$. For the second order wave, the upper is at $2\omega t = 0$ but the lower at $2\omega t = \pi/2$.

One wave system significant in the first order wave pattern: the wave peaks are on a diagonal line emanating from the ship; the diagonal line makes about 21° with the ship's track. This angle as well as the " wave lenght " of the peaks is in agreement with that of the asymptotic wave pattern of the longer wave component (k_2-wave). Two wave systems will be seen in the second order wave pattern. One is on a diagonal line of the angle 21° which is the same as in the first order k_2-wave. Other is on a digonal line of 13.5° which is the angle of the asymptotic k_2 wave pattern if we assume the frequency of the motion as 2ω and the linear dispersion relation.This is true in terms of the wave length of the peaks on the diagonal line. The former wave system is interpreted by an analogy of the Stokes waves as the bound wave component mainly resulting from the nonlinerity in the free ayrface condition and the latter the free wave component due to the body nonlinearity. It must be discussed in the comparison with the theoretical prediction.

Another significant result seen in the wave contours is that the second order wave elevation is as large as 25percent of the first order wave elevation. This will be discussed in terms of the theoretical prediction.

3 Theoretical prediction

We assume that the steady flow around the ship is uniform flow U into the x direction. Another assumption is that two small parameters, the slenderness parameters ε and the motion amplitude δ, are independent of each other; when we are concererned with, for example, δ^2 terms, we retain the terms of the lowest order with respect to ε .

We may assume the derivative of any flow quantity f in the x direction is higher order than the derivatives into the y and z in terms of ε. Yet we retain $U\partial f/\partial x$ in the free surface conditions because of high speed U of the ship. As a consequence we may employ $2.5D$ approach. One way is to analyze fully nonlinear flow and to take ω and 2ω components to compare with each component of the measured wave field presented in the previous section. However here our analysis is rather by classical perturbation expansion in terms of δ. It is for better understanding of the behavior of the second order wave elevation separately and the convenience of direct comprison with the measured wave pattern of the second order. Velocity potentilal ϕ and wave elevation ζ will be written in the form

$$\phi = \phi_1 + \phi_2, \qquad \zeta = \zeta_1 + \zeta_2$$

Here we consider that the terms ϕ_2 and ζ_2 are of higher order than the first terms with respect to δ.

From now on we are concerned only with the oscillatory part of the flow i.e. we understand hereafter that $\phi_{1,2}$ and $\zeta_{1,2}$ represent the oscillatory parts of the flow. Our analysis is in the frequency domain and everything of the first order is oscillating at the frequency ω. Then it is readily shown that we may put

$$\phi_j(x,y,z) = \psi_j(x,y,z)e^{-ij\omega x/U + ij\omega t}, \quad \zeta_j(x,y,z) = \eta_j(x,y,z)e^{-ij\omega x/U + ij\omega t} \qquad j = 1,2 \tag{2}$$

Substitution of those expressions into the first and second order free surface conditions respectively will derive the conditons for $\psi_{1,2}$ and $\eta_{1,2}$ on $z = 0$

$$U\psi_{1x} = -g\eta_1, \quad U\eta_{1x} = \psi_{1z} \tag{3}$$

$$U\psi_{2x} = -g\eta_2 - \frac{U}{2}\eta_1\psi_{1zx} - \frac{1}{4}(\psi_{1y}^2 + \psi_{1z}^2), \quad U\eta_{2x} = \psi_{2z} - \frac{1}{2}\psi_{1y}\eta_{1y} - \eta_1\psi_{1zz} \tag{4}$$

Subscripts x,y and z denote the differentiation into the respective direction. In deriving (4) we have retained the lowest order terms of ε among the terms of δ^2. The same reasoning leads to the govering equation in fluid

$$\psi_{jyy} + \psi_{jzz} = 0 \quad j = 1,2 \tag{5}$$

The body boundary conditions are derived in the same manner based on the same assumptions.

$$\psi_{1n} = -\delta[(x - x_0)i\omega + F_n]n_z e^{i\omega x/U} \tag{6}$$

$$\psi_{2n} = 0.5\delta^2(x - x_0)n_z\psi_{1zz}e^{i2\omega x/U} \tag{7}$$

The body conditions (6) and (7) are both imposed on the surface of the ship at the equilibrium position. n is the normal directed to the fluid on the ship's sectional contour and n_z is the z component of the normal. x_0 is the x coordinate of the longituidinal center of mass around which the pitch motion is defined. The formulation above is under the condition that the ship form is wall sided i.e. the hull surface intersects vertically the $z = 0$ plane.

Equations (4) and (7) suggest that ψ_2 and η_2 will be decomposed into two parts, one part satisfying the homogeneous free surface conditions corresponding to (3) and the body condition (7), and other part sastisfying the free surface condition (4) and the homogeneous body condition (7) with the right hand side replaced by 0. The former will be understood as the free wave part due to the body nonlinearity and the latter the bound part due to the free surface nonlinearity.

Numerical implementation of this approach for ϕ_1 is straightforward and well known. The 2nd order Runge-Kutta scheme was used to forward the solution from one section to next section; behavior of ψ_1 on $z = 0$ away from the body surface ($|y| > 0.5L$) is approximated by vertical dipole behavior; intial condition at the bow $x = 0$ for the marching is $\psi_1 = \eta_1 = 0$ (This must be improved in future while it doee not have a serious effect with a slender hull form of S175).

After ψ_1 and η_1 are obtained, we solve for ψ_2 and η_2 in almost similar manner. Equations (4) is integrated to forward ψ_2 and η_2. We need to evaluate the forcing terms due to ψ_1 and η_1 on the right hand side. We must rely on numerical differentiation for evaluating ψ_{1zx} for ψ_{1z} given on $z = 0$. We evaluated ψ_{1zz} by solving a new boundary value problem for ψ_{1z} when ψ_{1z} is prescribed on both the ship section and the free surface.

The condition imposed for ψ_2 at large $|y|$ is determined as follows. The forcing terms bahavior at large $|y|$ are known because ψ_1 bahves as a vertical dipole. For ψ_2 we assume the slowest attenuation expected from the forcing terms on the right hans side of (4) at large $|y|$ i.e. the same behavior as ψ_1.

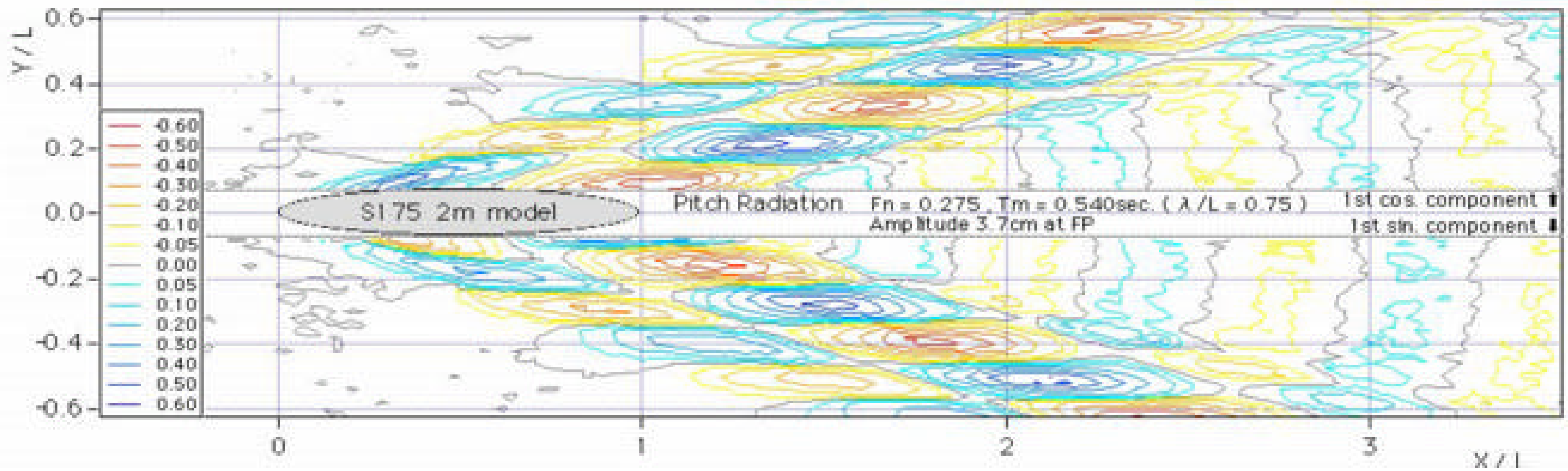
Numerical treatment of the singularity of the velocity potential at the intersection of the body surface and $z = 0$ will be crucial for the computation of the second order component, which will be discussed at the Workshop.

Wave elevation computed by this approach is compared with the measured wave elevation presented in the previous section. The predicted of the first order wave contour is apprently in good agreement with the measured but not quantitatively with the second order wave contour. The details of the results will be presented at the Workshop

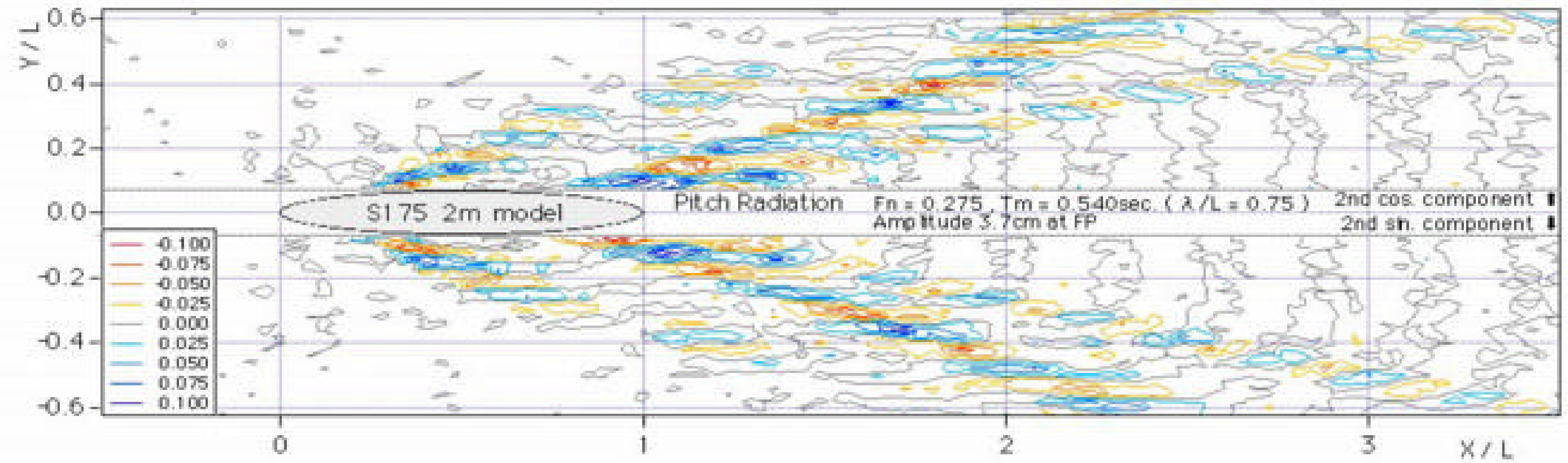
References

(1) Cao YS, Shultz W and Beck R (1994) Inner-angle Wavepackets in an Unsteady Wake, Proc.19th ONR Symposium, Seoul

(2) Ohkusu M and Wen G (1996) Radiation and Diffraction Waves of a Ship at Forward Speed, Proc.22nd ONR Symposium,Trondheim



S175 2m model
Pitch Radiation
Fn = 0.275, Tm = 0.540sec. (λ/L = 0.75)
Amplitude 3.7cm at FP
1st cos. component
1st sin. component
Y / L
X / L



S175 2m model
Pitch Radiation
Fn = 0.275, Tm = 0.540sec. (λ/L = 0.75)
Amplitude 3.7cm at FP
2nd cos. component
2nd sin. component
Y / L
X / L