

DIFFRACTION OF WATER WAVES BY AN AIR CHAMBER

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The wave-induced loads on a very large floating structure can be reduced by using an air cushion to provide part of the static support. This idea has been advanced by Pinkster *et al* (1997, 1998), with computations and experimental measurements of the motions in waves and the air-cushion pressure. They consider a long rectangular barge with side walls and ends enclosing the air cushion. Further computations have been performed by Lee & Newman (1999) using a more complete description of the acoustic disturbance, with particular attention to the resonant modes of the air cushion which are analogous to the ‘cobblestone effect’ suffered by air-cushion vehicles (Ulstein & Faltinsen, 1996).

Here we consider a simpler two-dimensional diffraction problem, which provides qualitative insight into the coupling between the acoustic and water waves. In the plane (x, y) , water occupies the semi-infinite domain $-\infty < x < \infty$, $-\infty < y < 0$. Air is enclosed within a chamber bounded by vertical walls at $x = \pm a$, a horizontal lid at $y = b$, and by the free surface $-a < x < a$, $y = 0$, as shown in Figure 1. The walls and lid are fixed. Plane waves of amplitude A , frequency ω and wavenumber $k = \omega^2/g$ are incident from $x = -\infty$. The motions of the water and air are assumed sufficiently small to justify a linearized analysis.

With the time-dependence represented by the factor $e^{i\omega t}$, the velocities of the air and water are equal to the gradients of the complex potentials $\Phi(x, y)$ and $\phi(x, y)$. (Upper/lower case symbols or the subscripts a/w are used where appropriate to distinguish the air/water domains, respectively.) The governing equations are the Helmholtz and Laplace equations

$$\nabla^2\Phi + K^2\Phi = 0, \quad \nabla^2\phi = 0, \quad (1)$$

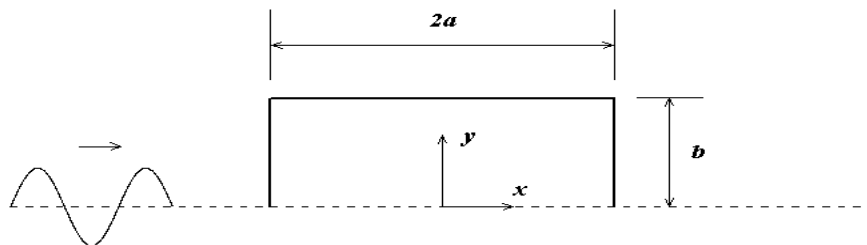


Figure 1: Sketch showing the air chamber and incident wave.

where $K = \omega/c$ is the acoustic wavenumber and c is the sound velocity. The corresponding pressures are given by the linearized Bernoulli equation:

$$P(x, y) = -i\rho_a\omega\Phi(x, y), \quad p(x, y) = -i\rho_w\omega\phi(x, y) - \rho_wgy, \quad (2)$$

where ρ is the density. The aerostatic pressure $-\rho_agy$ is neglected on the assumption that $c \gg g/\omega$.

Zero normal velocity is prescribed on the ends and lid of the chamber. On the air-water interface the kinematic and dynamic conditions are combined in the usual manner to give the linear free-surface condition

$$\rho_w(\omega^2\phi - g\phi_y) = \rho_a\omega^2\Phi. \quad (3)$$

The boundary-value problem for ϕ is completed by imposing the conventional conditions, including the homogeneous form of (3) on the the exterior free surface, the requirement that ϕ vanishes as $y \rightarrow -\infty$, and an appropriate radiation condition in the far field.

The potential in the air chamber can be expanded in the form

$$\Phi = i\omega A \sum_{m=0}^{\infty} \xi_m \Phi_m(x, y), \quad (4)$$

where

$$\Phi_m(x, y) = f_m(x) \frac{\cosh v_m(b-y)}{v_m \sinh v_m b} \quad (5)$$

and

$$f_m(x) = \cos(u_m(x-a)). \quad (6)$$

Here

$$u_m = \frac{m\pi}{2a}. \quad (7)$$

The normalizing factors are chosen such that $\Phi_{my}(x, 0) = f_m(x)$. The elevation of the interface is

$$\eta(x) = A \sum_{m=0}^{\infty} \xi_m f_m(x). \quad (8)$$

The eigenfunctions $\Phi_m(x, y)$ satisfy homogeneous Neumann conditions on the ends and top of the chamber. The coefficients v_m follow from the Helmholtz equation:

$$v_m^2 = u_m^2 - K^2. \quad (9)$$

These coefficients are either real or imaginary, and the eigenfunctions Φ_m are real.

The potential in the water can be derived by superposition of the incident-wave potential

$$\phi_I = \frac{igA}{\omega} \exp(ky - ikx) \quad (10)$$

with the solution for an oscillatory pressure imposed on the free surface (Wehausen & Laitone, 1960, equation 21.17). After adapting to the present notation it follows that

$$\phi = \phi_I - \frac{i\omega}{\pi\rho_w g} \int_{-a}^a P(\xi, 0) d\xi \int_0^{\infty} \cos \kappa(x - \xi) e^{\kappa y} \frac{d\kappa}{\kappa - k}. \quad (11)$$

In the integral with respect to κ the contour of integration passes below the pole $\kappa = k$, in accordance with the radiation condition. After substituting (2) and (4-5),

$$\phi = \phi_I + i\omega A \sum_{m=0}^{\infty} \xi_m \phi_m, \quad (12)$$

where

$$\phi_m = -\frac{k\rho_a}{\pi\rho_w v_m \tanh v_m b} \int_{-a}^a f_m(\xi) d\xi \int_0^{\infty} \cos \kappa(x - \xi) e^{\kappa y} \frac{d\kappa}{\kappa - k}. \quad (13)$$

After imposing the kinematic boundary condition $\phi_y = i\omega\eta$, multiplying by $f_n(x)/a$, and integrating over $(-a, a)$, a linear system of equations is derived for the unknown coefficients ξ_m in the form

$$\sum_{m=0}^{\infty} \xi_m C_{mn} = D_n, \quad (14)$$

where

$$C_{mn} = \frac{1}{a} \int_{-a}^a f_n(x) \left[f_m(x) - \frac{k\rho_a}{\pi\rho_w v_m \tanh v_m b} \int_{-a}^a f_m(\xi) d\xi \int_0^{\infty} \cos \kappa(x - \xi) \frac{\kappa d\kappa}{\kappa - k} \right] dx \quad (15)$$

and

$$D_n = \frac{1}{a} \int_{-a}^a f_n(x) e^{-ikx} dx = -\frac{2i^n ka}{(u_n^2 - k^2)a^2} \sin\left(ka + \frac{n\pi}{2}\right). \quad (16)$$

The integrals in (15) with respect to x and ξ are elementary, and the remaining integral with respect to κ can be evaluated in terms of sine and cosine integrals.

The simplest physical parameters to consider are the vertical exciting force and pitch moment, due to the acoustic pressure P acting on the ends and lid of the air chamber. Neglecting the contribution to the pitch moment from the ends, the force and moment are given by

$$\begin{pmatrix} X_3 \\ X_5 \end{pmatrix} = \int_{-a}^a \begin{pmatrix} 1 \\ -x \end{pmatrix} P(x, 0) dx. \quad (17)$$

After normalizing by the long-wavelength (hydrostatic) limits of the force and moment for a flat plate of the same width, it follows that

$$\frac{X_3}{2\rho_w g a A} = \frac{\rho_a k \xi_0}{\rho_w v_0 \sinh v_0 b} \quad (18)$$

and

$$\frac{X_5}{\frac{2}{3}\rho_w g k a^3 A} = -\frac{12\rho_a}{\pi^2 \rho_w} \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} \frac{\xi_n}{n^2 v_n \sinh v_n b}. \quad (19)$$

For asymptotically large wave periods the normalized heave force tends to a value less than the usual hydrostatic limit of 1.0, due to the compressibility of the air. The pitch moment tends to zero more quickly than the corresponding hydrostatic moment for a conventional floating body, due to equalization of the pressure throughout the chamber when the period is very large.

Preliminary computations indicate a very large pitch exciting moment in relatively short waves, at the resonant frequency of the first anti-symmetric acoustic mode ($Ka = \pi/2$). The maximum heave force occurs in longer waves. More complete computations of the heave force, pitch moment, and modal amplitudes ξ_n will be presented at the Workshop.

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