Heave response of a semi-submersible near resonance

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The submerged surface of a typical semi-sub includes two submerged pontoons and several vertical columns extending up through the free surface, as shown in Figure 1. This configuration results in relatively small vertical exciting forces, and corresponding motions. Assuming for simplicity that the vessel is symmetrical about the midship plane $x = 0$, and also about the transverse centerplane $y = 0$, heave will be uncoupled from pitch and roll, and the response-amplitude-operator (RAO) is given by the ratio

$$\xi = \frac{X}{-\omega^2(a+m) + i\omega b + c}. \tag{1}$$

Here ξ and X are the complex heave amplitude and the exciting force, for a unit-amplitude incident wave, ω is the radian frequency, a is the added-mass coefficient, m the mass of the semi-sub, b the damping coefficient, and c is the hydrostatic restoring coefficient (equal to the product of the fluid density, gravity, and the waterplane area).

For vessels such as semi-subs, where the waterplane area is relatively small compared with the virtual mass, the natural frequency

$$\omega_n = \sqrt{\frac{c}{a+m}} \tag{2}$$

is small and the natural period $T_n = 2\pi/\omega_n$ is large. Since the damping due to wave radiation is small, for such frequencies, a highly-tuned resonant response will occur in the absence of viscous damping.

In the long-wavelength regime the heave exciting force can be estimated from the approximation derived by Newman (1977, pp 302-3):

$$X \simeq c + i\omega b - \omega^2(a + \rho V). \tag{3}$$

Here ρV is the mass of displaced fluid, which is equal to the body mass m for a freely floating body. If (3) is substituted in (1) it follows that the heave RAO is equal to unity. Thus the near-zero value of the denominator in (1) at resonance is matched by a corresponding near-zero value of the exciting force. It should be noted here that (3) is valid for all wave-heading angles, and it can be inferred from the derivation that the error in this approximation is of order ω^3 or higher.

A more detailed analysis has been given by Hooft (1972). Using simple approximations for the hydrodynamic coefficients based on separate consideration of the columns and pontoons, Hooft shows that the principal dimensions of a semi-sub can be determined such that the periods of resonance (T_n) and near-zero-excitation (T_0) are coincident, effectively cancelling the effect of resonance.

Faltinsen (1990) presents a more fundamental analysis, based on long-wavelength approximations of the hydrodynamic coefficients which are specifically adapted to a semi-sub. This analysis leads to the conclusion that, in beam seas, $T_0 < T_n$ (see Faltinsen, 1990, equation 3.67 and Figure 3.19).

In this work the heave response of a semi-sub is analyzed using the higher-order radiation/diffraction code HIPAN. The body surface is represented exactly, and cubic B-splines are used to represent the velocity potential. In most applications the hydrodynamic coefficients computed by HIPAN are accurate to several decimal places, much more than is generally required for engineering predictions. However for the semi-sub in the vicinity of resonance, much of this accuracy is lost due to cancellation errors in the numerator and denominator of (1), as shown below.

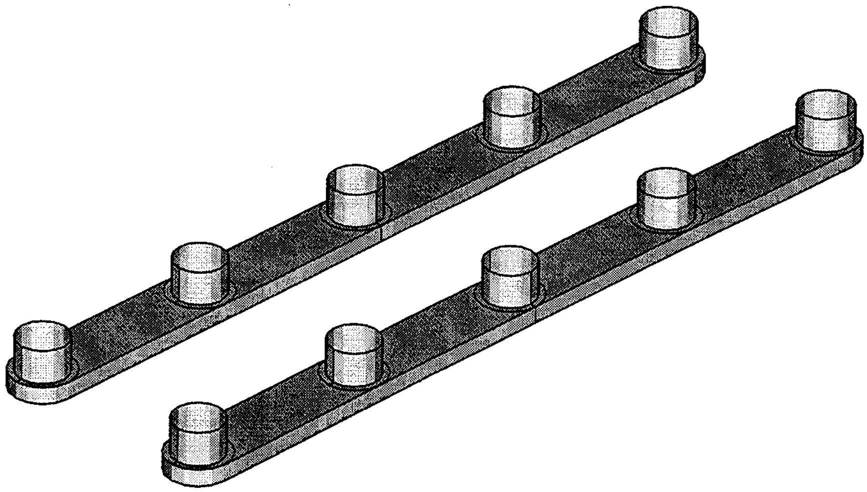

Figure 1: Perspective view of submerged portion of the semi-sub.

The computational results are for variants of the semi-sub shown in Figure 1. The principal dimensions of the pontoons are 260m length, 10m width, and 5m depth (vertical distance between the top and bottom of each pontoon). The spacing between the inner surfaces of the pontoons is 20m. Five equally-spaced circular columns of diameter 8m are situated on each pontoon. The first and fifth columns are coaxial with the semi-circular pontoon ends. The draft is varied to illustrate the resulting influence on the response near resonance. Different conditions of the draft will be identified in terms of the 'column depth', equal to the distance from the free surface to the upper deck of the pontoons.

One quadrant ($x > 0, y > 0$) of the submerged surface is defined by exact parametric mappings on 10 separate patches, which are outlined in Figure 1. 'Coarse', 'fine', and 'extra-fine' discretizations of the unknown potential are achieved by subdividing the patches into panels, with B-spline knots on the boundaries of adjacent panels. The total number of unknown B-spline coefficients is 344, 734, and 1850, respectively.

Figure 2 shows the heave RAO and exciting force for a broad range of wave periods. Two column depths (7m and 10m) are included. The RAO's are similar for these two depths, with the exception of the resonant regime near 23 seconds. For wave periods below 20 seconds the fluctuations of the RAO's are due primarily to the corresponding fluctuations of the exciting forces, resulting from longitudinal interference of the incident-wave diffraction field along the length of the pontoons. For wave periods greater than 25 seconds the RAO's are practically equal to one, as in the long-wavelength limit. In the intermediate regime near 23 seconds two adjacent features are apparent, where the resonance and near-zero exciting force occur. For the 7m column depth $T_0 > T_n$, and for the 10m depth $T_0 < T_n$. This suggests that, at a certain intermediate draft, the two periods coincide.

Figure 3 shows the RAO's for two intermediate column depths, 8.7m and 8.8m. Three curves are shown in each figure, corresponding to the base, fine, and extra-fine panel subdivisions as defined above. It is evident from these results that the level of subdivision is significant, and there is some uncertainty in the results associated with lack of convergence. This can be attributed to the fact that, within this regime of period and draft, the RAO is effectively the ratio of two small numbers, both of which are subject to substantial cancellation errors.

For the semi-subs considered here, the dominant component of the exciting force is real, and this component varies linearly through zero at the period T_0. In this regime the imaginary component is small, on the order of 0.1, or four orders of magnitude smaller than the maxima shown in Figure 2. Thus there is a loss of about four decimals accuracy associated with the cancellation errors near resonance.

Figure 4 shows the RAO and exciting force for the 7m column depth in a range of wave headings between head and beam seas. As the heading angle increases, T_0 decreases. While this downshift is small, the effect on the RAO is to reverse the relative positions of T_0 and T_n. Thus, for oblique and beam seas, $T_0 < T_n$. At reduced column depths this difference is smaller, but in beam seas and column depths greater than about one meter it is impossible for the two periods to coincide, at least for semi-sub geometries similar to that shown in Figure 1. At column depths on the order of one meter the standing-wave phenomenon noted by Newman *et al* (1984) gives rise to negative values of the added mass, and this can result in coincidence of the two periods. Except for this unusual event at very small column depths, we confirm Faltinsen's (1990) conclusion that, in beam seas, $T_0 < T_n$. Figures V-7A,B,C of Hooft (1972) show experimental and computational results for three wave headings which are consistent with this trend.

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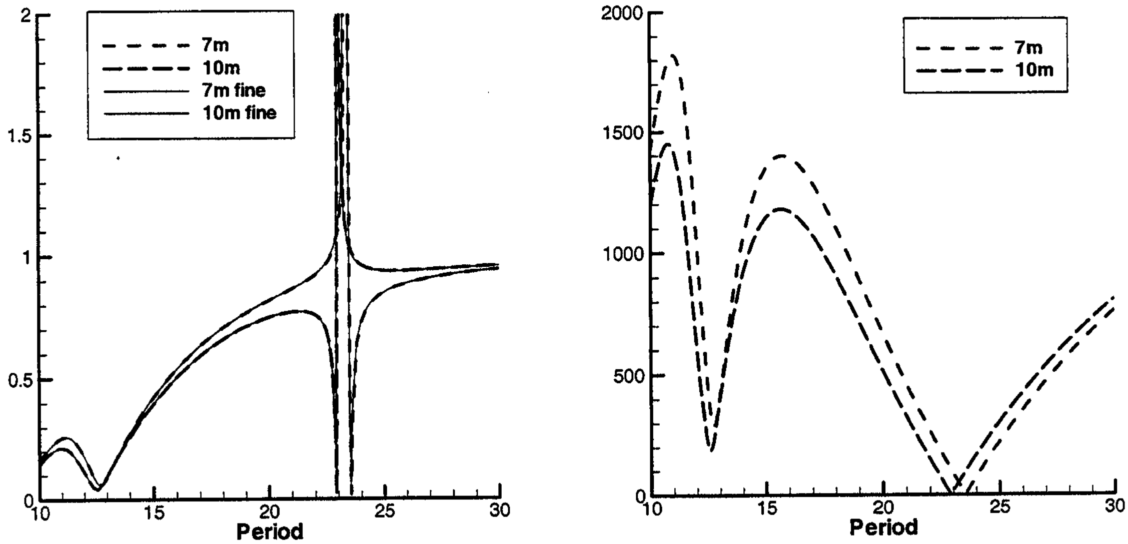

Figure 2: Heave RAO (left) and exciting force (right) in head seas, for column depths 7m and 10m.

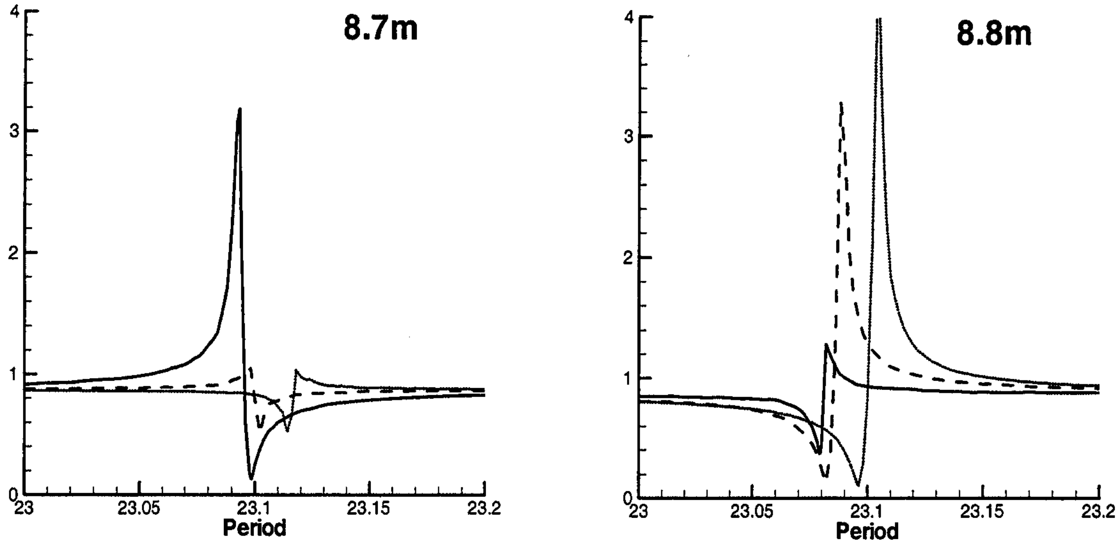

Figure 3: Heave RAO in head seas, for column depths 8.7m and 8.8m in the resonant regime. The three curves correspond to the base (dotted), fine (dashed) and extra-fine (solid) levels of panel subdivision.

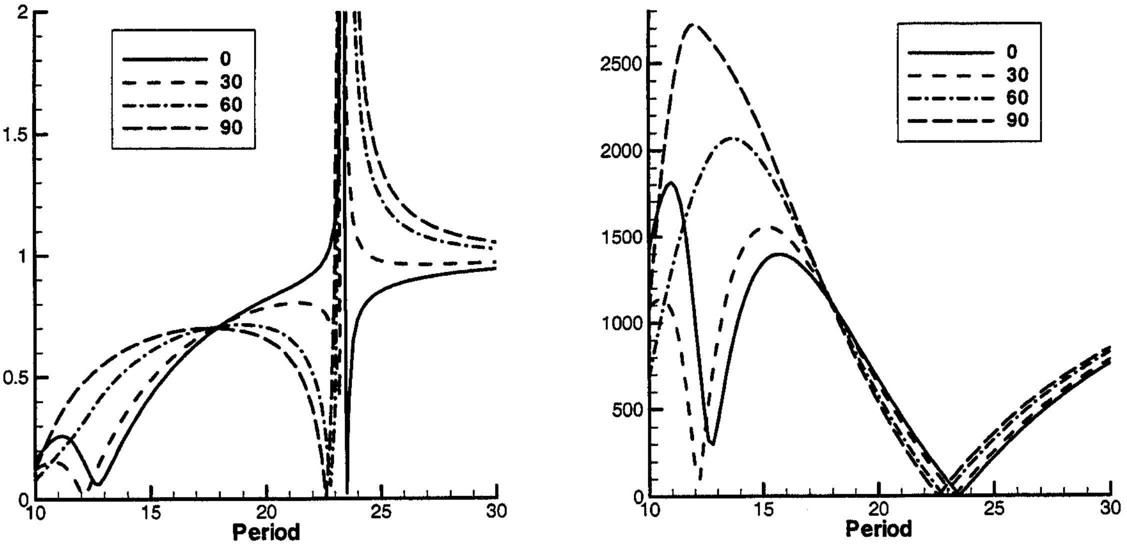

Figure 4: Heave RAO (left) and exciting force (right) for column depth 7m at wave headings $0°, 30°, 60°, 90°$.