Comparison between three steady flow approximations in a linear time-domain model

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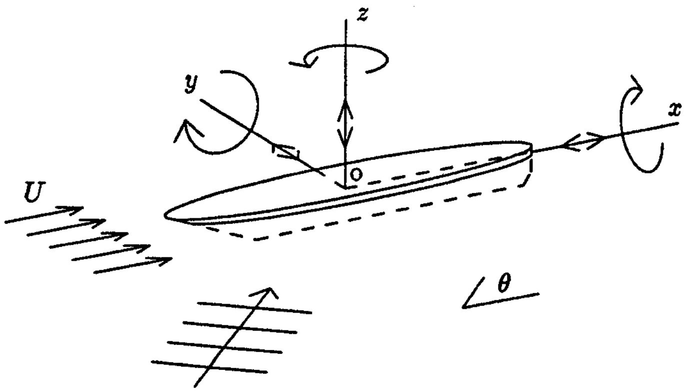
1 Introduction

The interaction between water waves and a sailing ship is a non-linear phenomenon that is very hard to solve. Under certain simplifying conditions, a linearization can be carried out that enables the time dependent part of this problem (the ship motions and the unsteady wave pattern) to be solved. This requires however a description of the steady flow around the ship on which this linearization is based. For this steady flow a number of choices can be made, depending on the application of interest. The simplest option is to use uniform flow, which goes straight through the ship like it doesn't exist. A better approximation is double body flow, that bends around the ship, but still doesn't satisfy the correct free surface condition. Both of these approximations are only suitable for low speeds of the ship. When the speed is increased, a better description of the steady flow is probably required that takes into account the correct steady wave pattern and the steady trim and sinkage of the ship. At the MARIN a method has been developed, RAPID [1], that calculates the potential flow satisfying the non-linear steady free surface condition, a zero flux condition on the hull, and that can determine the steady trim and sinkage. This flow is used to linearize the time dependent flow. In this abstract we will show the relevancy of the use of this flow by using the three different approximations for the steady flow in our calculations of the behaviour of an LNG-carrier sailing at $Fn = 0.2$. We will compare the computed results of motion amplitudes, motion phase shift and drift forces for the three methods with measurements done at the MARIN.

2 Mathematical and numerical model



We consider a ship sailing at a constant velocity U in incoming waves that enter the ship at an angle of incidence θ. The ship is free to translate along or rotate around any of its axis. We use potential flow to describe the hydrodynamics of the flow. When the unsteady waves and ship motions are small, the velocity potential can be perturbated according to

$$\Phi = \Phi_s(\vec{x}) + \epsilon\phi_u^{(1)}(\vec{x},t) + \mathcal{O}(\epsilon^2)$$

Φ_s is the steady part of the potential and $\phi_u^{(1)}$ the first order part of the unsteady potential. When the exact steady flow is used, satisfying the non-linear steady free surface condition and a Neuman condition on the hull, it can be shown that the free surface condition for the time dependent flow can be linearized into

$$\frac{\partial^2\phi_u^{(1)}}{\partial t^2} + 2\vec{\nabla}\Phi_s\cdot\vec{\nabla}\frac{\partial\phi_u^{(1)}}{\partial t} + \vec{\nabla}\Phi_s\cdot\vec{\nabla}\left(\vec{\nabla}\Phi_s\cdot\vec{\nabla}\phi_u^{(1)}\right) + \vec{\nabla}\phi_u^{(1)}\cdot\vec{\nabla}\left(\vec{\nabla}\Phi_s\cdot\vec{\nabla}\Phi_s\right) + g\frac{\partial\phi_u^{(1)}}{\partial z}$$

$$+\zeta_u^{(1)} \frac{\partial}{\partial z} \left(\frac{1}{2} \vec{\nabla} \Phi_s \cdot \vec{\nabla} \left(\vec{\nabla} \Phi_s \cdot \vec{\nabla} \Phi_s \right) + g \frac{\partial \Phi_s}{\partial z} \right) = 0 \quad \text{on } z = \zeta_s \tag{1}$$

where $\zeta_u^{(1)}$ is the first order wave elevation and ζ_s the steady wave elevation

$$\zeta_u^{(1)} = -\frac{1}{g} \left(\frac{\partial \phi_u^{(1)}}{\partial t} + \vec{\nabla} \Phi_s \cdot \vec{\nabla} \phi_u^{(1)} \right) / \left(1 + \frac{1}{2g} \frac{\partial}{\partial z} \left(\vec{\nabla} \Phi_s \cdot \vec{\nabla} \Phi_s \right) \right)$$

$$\zeta_s = -\frac{1}{2g} \left(\vec{\nabla} \Phi_s \cdot \vec{\nabla} \Phi_s - U^2 \right)$$

The non-linear condition on the hull states that the normal velocity of the hull should equal the normal velocity of the water. When this condition is linearized about the mean position of the ship we get

$$\frac{\partial \phi_u^{(1)}}{\partial n} = \frac{\partial \vec{\alpha}}{\partial t} \cdot \vec{n} + \left(\left(\vec{\nabla} \Phi_s \cdot \vec{\nabla} \right) \vec{\alpha} - \left(\vec{\alpha} \cdot \vec{\nabla} \right) \vec{\nabla} \Phi_s \right) \cdot \vec{n}$$

where α is the total displacement vector. This potential flow is solved by using a boundary integral formulation that describes the potential as an integral of Rankine sources over the free surface and the hull of the ship. Outgoing waves are absorbed by using a damping zone on the free surface.

We discretize this model by dividing the hull and steady free surface in panels, on which the source strength is constant. Collocation points are in the middle of a panel and the appropriate boundary condition is applied there. The tangential derivatives in the free surface condition are modelled as differences between potential values in the collocation points. In these difference schemes, only points are used that are upstream of the collocation point of interest (and that point itself) to assure numerical stability, as shown on the previous workshop in Alphen aan den Rijn. A set of equations is now obtained for the unknown source strengths, that is solved in the time domain with Gaussian elimination. This eventually gives us the first order motion of the ship and the second order forces.

3 Uniform flow and double body flow

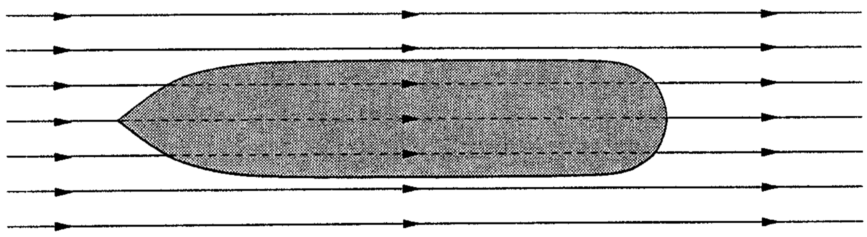


Figure 1: Topview of uniform flow

In stead of the relatively difficult use of the non-linear steady flow, also simpler approximations can be used to capture some aspects of the steady flow around the ship. It is expected that for low speeds of the ship these approximations give satisfactory results, but that for moderate and high speeds the results get worse. The simplest approximation of the steady flow is uniform flow, that goes straight through the ship like it doesn't exist, as shown in figure 1. This means that this approximation, in theory, is only valid for slender ships and low speeds. Substitution of $\Phi_s = Ux$ in (1) gives the well known Kelvin condition. In the linearized body boundary condition, the so called m-terms, consisting of velocity derivatives, disappear, which are often a source of errors.

A better approximation is the double body flow. It is obtained by reflecting the ship in the calm water plane and calculating the flow around this double body, as shown in figure 2. The flow now satisfies the no-flux condition on the hull, but by reflecting the ship, the free surface disappears so the wave pattern is predicted incorrect. This means that this approximation is again valid for low speeds only.

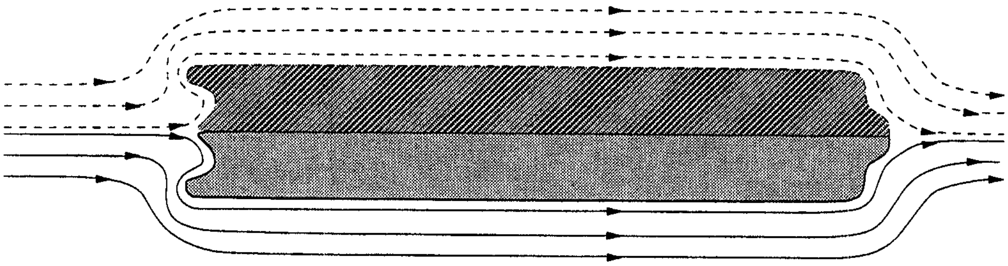


Figure 2: Sideview of double body flow

4 LNG-carrier

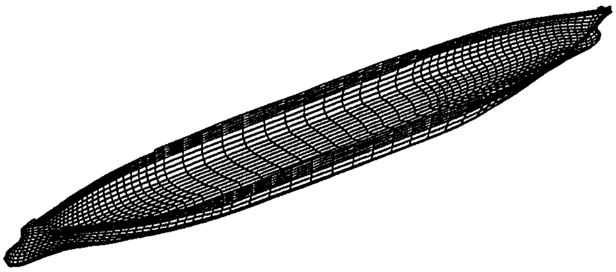


Figure 3: Panels on the hull

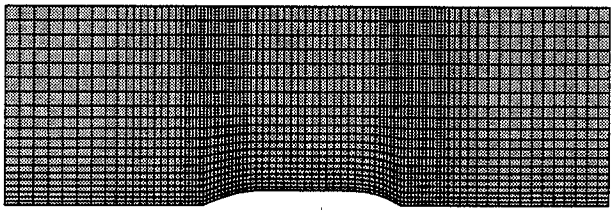


Figure 4: Topview of free surface panels

Designation	Symbol	Unit	
Length	L	m	273
Breadth	B	m	42
Draught	T	m	11.50
Displacement	Δ	m^3	98,740
Centre of gravity above waterline		m	2.20
Longitudinal gyradius	k_{yy}	%L	24
Transverse gyradius	k_{xx}	%B	35

We applied the described model to a LNG-carrier, sailing at a Froude number of 0.2, where we used successively the non-linear steady flow, the double body flow and uniform flow as a basis for the linearization of the time dependent flow. In figure 3 the panel distribution on the hull is shown. In the calculations only the starboard side, divided in 1190 panels, was used because we made use of symmetry relations. On the free surface 1800 panels were used, which size ranges from very small near the ship to quit large far from the ship, as shown in figure 4. To compare the three different methods, we calculate the ship motions and drift forces and compare them with measurements done at the MARIN. It can be shown, that just like the potential, the force on the ship can be perturbated like

$$\vec{F} = \vec{F}^{(0)} + \epsilon \vec{F}^{(1)} + \epsilon^2 \vec{F}^{(2)}$$

The first-order force is responsible for the first order motions of the ship. The average value of the second order force can cause the ship to drift away from its mean position and an accurate prediction of this force is therefore important. Some very long and difficult formulas can be derived for the drift force, which will not be mentioned here.

In figure 5 the heave amplitude is shown in headwaves. In the low frequent region (long waves), the prediction is not very good in all three cases, and especially the use of uniform flow leads to a large over estimation. This is probably because these long waves don't fit on the computational free surface that we used. For increasing frequency, we see the use of the RAPID velocities gives the best result. In the high frequent region, where the motions are small, no large differences are found. In figure 6 the pitch amplitude is shown. Again low frequent motions are predicted incorrectly and the high frequent motions are predicted best using the RAPID velocities.

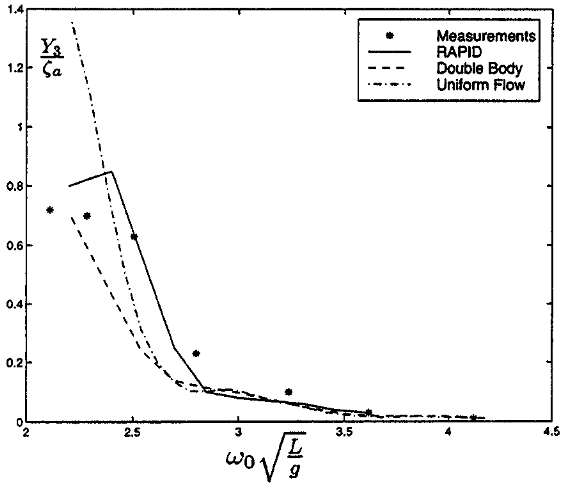


Figure 5: Amplitude of heave motion in head waves, Fn=0.2.

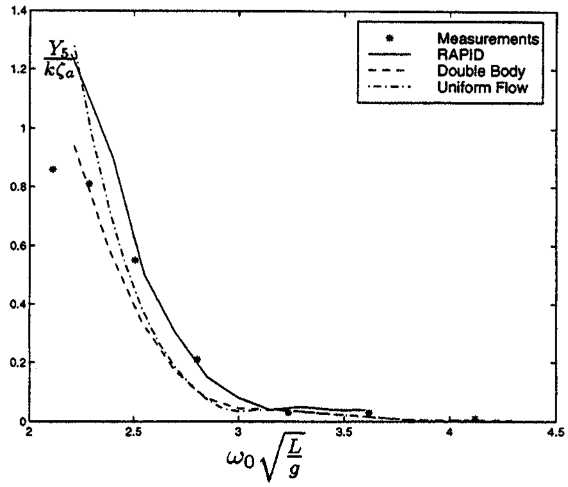


Figure 6: Amplitude of pitch motion in head waves, Fn=0.2.

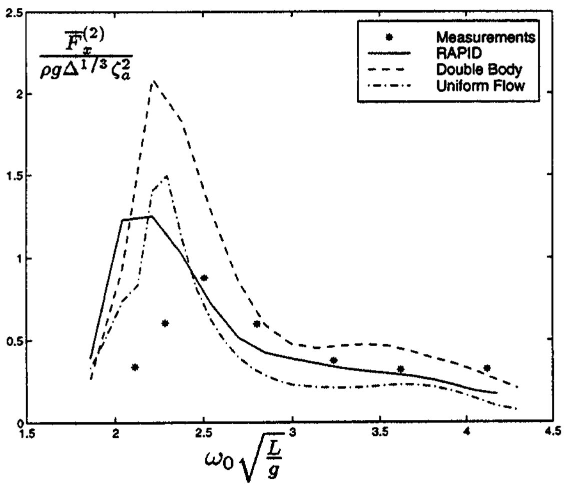


Figure 7: Drift force in head waves, Fn=0.2.

In figure 7 we show the first component of the drift force in head waves, again for Froude number 0.2. The contribution of second derivatives of the potential is neglected in the driftforces, because our first order panel method can not calculate these accurately. Again we see that in the low frequent region the prediction is not good, which can now also partially be explained by our neglection of the second derivatives. For increasing frequencies the prediction that uses the RAPID velocities is good, the one that uses uniform flow gives an under prediction. In the high frequent region the results are not reliable anymore because the wave length is then too short to fit accurately on our grid (we used the same grid in all our calculations).

References

[1] H.C. Raven. *A Solution Method for the Nonlinear Ship Wave Resistance Problem.* PhD Thesis, Delft University of Technology, 1996.