# AN EXPERIMENTAL VERIFICATION OF THE WAVE DRIFT DAMPING FORMULA

J.A.P.Aranha[(1)] & S. da Silva[(2)] & M.R.Martins[(1)] & A.J.P.Leite[(3)]

(1) Dep. of Naval Eng., EPUSP, S.Paulo, S.P.,Brazil
(2) Dep. of Naval Eng., IPT , S.Paulo , S.P. , Brazil
(3) E&P, PETROBRAS, Rio de Janeiro, R.J., Brazil

## 1.INTRODUCTION

A weathervane ship (turret) free to rotate around a vertical axis is being used by the oil industry as a floating production unit. To minimize the influence of the pitch motion on the risers dynamic it has become attractive to place the turret near the midship, at a distance $a$L of the order of 0.2L or 0.3L from this section, L being the ship length. In this case the *trivial* equilibrium position, represented by the yaw angle $\psi = 0^0$ between the ship axis and the ocean current, becomes unstable, the stable equilibrium positions being given by non-zero values of the yaw angle. Typically $\psi \approx 5^0$ when $a = 0.3$ and $\psi \approx 20^0$ when $a = 0.2$, independent of the current intensity. It is certainly of importance to know how these non-zero equilibrium values of the yaw angle are affected by a simultaneous action of sea waves.

In Campos Basin the major ocean current goes south (current of Brazil), with estimated deccenary value of 1.58 m/s, and the centenary wave goes north, with significant wave height of 7.6 m and zero up-crossing period of 9.2 s. If the sea state is represented by the Pierson-Moskowitz spectrum, the following values are theoretically predicted for a typical VLCC (L = 320 m; B = 54.5 m; T = 21.6 m; $C_B = 0.832$) with a turret placed at 0.2L: $\psi = 74.1^0$ when the wave-current interaction is ignored while $\psi = 28.8^0$ when this effect is incorporated in the analysis by means of the formula proposed in Aranha (1996; J. of Fluid Mechanics vol.313, pp. 39-54). These numbers show dramatically the importance that the wave-current interaction may have in a practical problem and they motivated a closer analysis of this problem.

The particular centenary wave/deccenary current condition in Campos Basin can be simulated in a wavetank by towing a model with constant velocity U while being subjected to a following sea. Using a 1:90 model of the actual VLCC, tests run at IPT wavetank, under the same condition described above, indicated that the experimental value of the steady yaw angle was $\psi = 28^0$, in good agreement with the predicted theoretical value. Besides the practical relevance, this study discloses also a clear experimental set up to verify the adherence of the wave drift damping formula. The main purpose of the present work is to present this verification in a more systematic way. One last point is worth to be observed here: while Wichers decay test is concerned only with the longitudinal drift force, the proposed experiment is the first, to the authors knowledge, to test the influence of the wave-current interaction on the lateral drift force and steady yaw moment.

## 2. THEORETICAL BACKGROUND AND EXPERIMENTAL RESULTS

Suppose a ship being displaced with velocity Ui in the x-direction while subjected to the action of a harmonic wave, with amplitude A and frequency $\omega$, incident in a direction that makes an angle $\beta$ with the x-direction. If $c = g/\omega$ is the wave celerity, in the frame of reference that moves with the ship one observes a frequency $\omega_e$ (*Doppler shift*) and an incident direction $\beta_1$ (*aberration effect*) given by

$$\omega_e = \left(1 - \frac{U}{c}\cos\beta\right)\cdot\omega;$$
$$\beta_1 = \beta + 2\frac{U}{c}\sin\beta. \qquad (1a)$$

Let $\mathbf{D}_0(\omega,\beta)$ be the generalized second order steady wave vector in the horizontal plane in the standard problem, where the advacing velocity is zero ($U = 0$), and $\mathbf{D}_U(\omega,\beta)$ be the related vector when $U \neq 0$. Specifically, the **i** and **j** components of **D** are the two horizontal drift forces components while the **k** component is the steady yaw moment. From some plausible assumptions and conservation of "wave action" the following formula can be derived relating these two force vectors:

$$\mathbf{D}_U(\omega,\beta) = \left(1 - 4\frac{U}{c}\cos\beta\right)\cdot\mathbf{D}_0(\omega_e,\beta_1). \qquad (1b)$$

Furthermore, formula (1b) has been shown to be *exact* within the context of the pertinent theory, where terms of order $U^2$ are ignored, but this conclusion has been questioned based on observed discrepancies with some numerical results (although in some others numerical experiments the agreement was exact). The intention here is to use the experimental set up described in the introduction to check the adherence of formula (1b) to the experimental results.

If the turret is placed at a distance $aL$ from the midship and T is the ship draft, the moment due to the current can be written as

$$\hat{N}_C(\psi) = \frac{1}{2}\rho U^2 T L^2 [C_{6C}(\psi) - a\cdot C_{2C}(\psi)], \qquad (2)$$

where $C_{6C}(\psi)$ is the coefficient of the yaw moment and $C_{2C}(\psi)$ of the lateral force. Heuristic models, based on the low aspect wing theory, can be used to express these coefficients in terms of the main ship dimensions, the adherence with experimental results being quite reasonable in general, see Leite at al (1998; Applied Ocean Research vol. 20,n.3, pp. 145-156).

In *following sea* $\beta = 0^o$ and the incidence angle with respect to the ship axis is $\psi$. Let $N_Z(\omega,\psi)$ be the standard steady yaw moment normalized by $1/2\rho g A^2 L^2$ and $D_y(\omega,\psi)$ be the lateral drift force normalized by $1/2\rho g A^2 L$. The total moment in the turret is the sum of the moment due to the current, given in (2), with the moment due to the wave; normalizing this value by $1/2\rho U^2 T L^2$ one obtains

$$N^{(wo)}(\omega,\psi)=\left[C_{6C}(\psi)-a\cdot C_{2C}(\psi)\right]+\frac{gT}{U^2}\left(\frac{A}{T}\right)^2\left[N_Z(\omega,\psi)-a\cdot D_y(\omega,\psi)\right], \tag{3a}$$

if the wave-current interaction is ignored. In the other hand, if the wave-current interaction is accounted for by means of the formula (1), the following expression can be derived ($\beta = 0$):

$$N(\omega,\psi)=\left[C_{6C}(\psi)-a\cdot C_{2C}(\psi)\right]+\frac{gT}{U^2}\left(\frac{A}{T}\right)^2\left(1-4\frac{U}{c}\right)\left[N_Z(\omega_e,\psi)-a\cdot D_y(\omega_e,\psi)\right]. \tag{3b}$$

Using the expressions for $\{C_{2C}(\psi);\ C_{6C}(\psi)\}$ proposed in Leite et al (1998) and computing the steady force coefficients $\{N_Z(\omega,\psi);\ D_y(\omega,\psi)\}$ by means of a standard linear frequency domain program one can determine, for a wave with amplitude A and frequency $\omega$, the function $N(\omega,\psi)$; the *stable* equilibrium position $\psi_E$ is defined by the conditions

$$\begin{aligned} &N(\omega,\psi_E)=0;\\ &\left(\frac{\partial N}{\partial \psi}\right)_{\psi=\psi_E}<0. \end{aligned} \tag{4}$$

Keeping the wave amplitude A and the towing velocity U constant the equilibrium angle $\psi_E$ is a function of the wave frequency KT, with $K = \omega^2/g$. The function $\psi_E(KT)$ can be determined either from (3a) and (4), if the wave-current interaction is ignored, or else from (3b) and (4) when this effect is considered, and can be compared with the experimentally determined value of $\psi_E(KT)$. Figure (1) displays this result for two turret positions: one at $a = 0.3$ and the other at $a = 0.2$. The agreement is quite good in the whole range of frequencies analysed ($3 < KL < 16$).

By comparing the curves obtained from (3a) and (3b) one can check the dramatic decrease of the equilibrium angle when the wave-current interaction is incorporated in the analysis, a result consistent to the one obtained in random sea, as described in the Introduction. Furthermore, the close adherence between formula (1) and the experiments may, perhaps, motivate a more open look to the demonstration given for the formula, based in essence on Green's Theorem and the Method of the Stationary Phase.

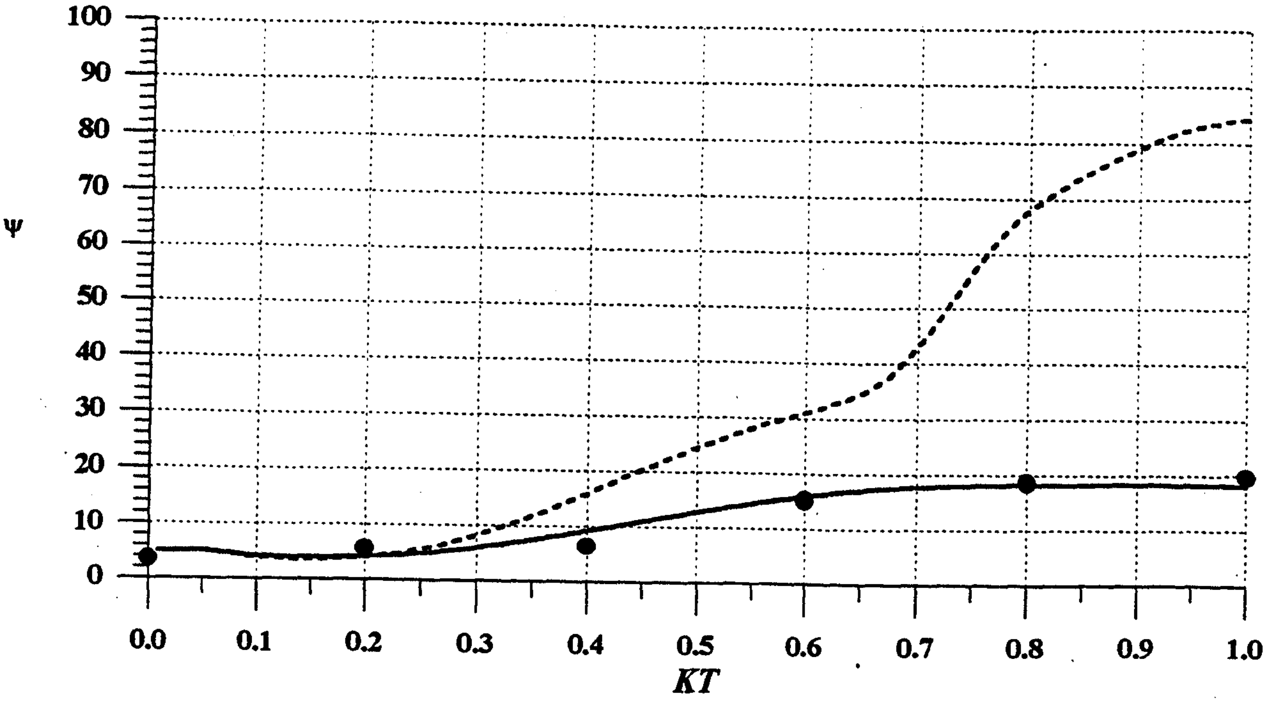


FIG.(1a): Yaw angle at the stable equilibrium as function of the wave frequency KT. Turret at 0.3L, A = 2.70 m, U = 1.90 m/s. Experiments: (•); Theory (4b): (——); Theory w/o wave-current interaction, see (4a): (------).

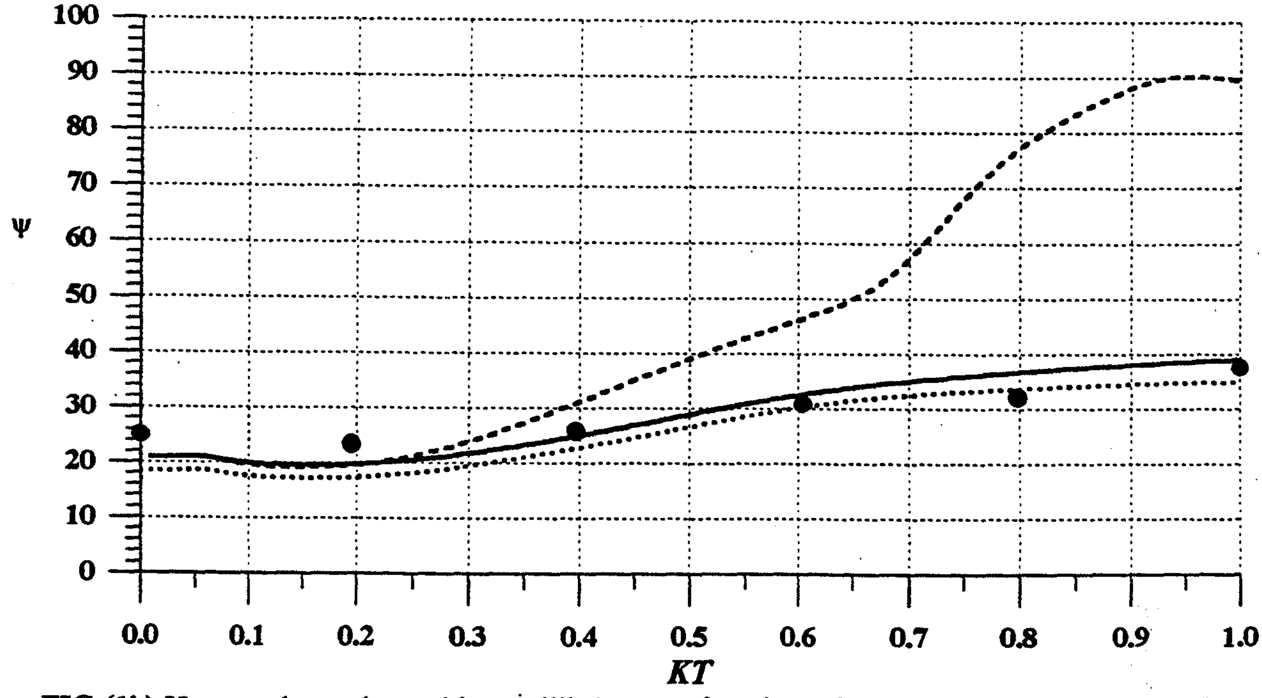


FIG.(1b):Yaw angle at the stable equilibrium as function of the wave frequency KT. Turret at 0.2L, A = 2.70 m, U = 1.90 m/s. Experiments: (•); Theory (4b): (——); Theory w/o wave-current interaction, see (4a): (------); Theory (4b) w/o ad hoc correction in (1a) (........).