A New Direct Method for Calculating Hydroelastic Deflection of a Very Large Floating Structure in Waves

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1. Introduction

Very large floating structures with shallow draft, considered as an airport, are featured in that the hydroelastic responses are more important than the rigid-body motions due to relatively small flexural rigidity. Several methods for calculating hydroelastic responses have been developed; those are categorized roughly into the mode-expansion method[1)~3)] and the direct (FEM-BEM combined) method.[4)5)]

In the mode-expansion method, the deflection of a structure is represented generally by a superposition of so-called dry eigenmodes. Then the amplitude of each mode is determined by solving the vibration equation of a thin plate, with the added mass and damping force corresponding to specified mode shapes computed in advance. One problem in this method is that an analytical solution of the dry eigenmode, satisfying the free-end boundary condition along the periphery of a structure, is not yet known. However, it has been recently confirmed that an orthogonal system of mathematical functions can be used to represent the elastic deflection, and the free-end boundary conditions can be satisfied subsequently as natural boundary conditions in the process of partial integrations in solving the vibration equation with a Galerkin scheme.

If our interest is placed not on the contribution of each mode function but on the elastic deflection as a whole, the direct method is more lucid than the mode-expansion method. However, the direct method is generally time consuming, because the vibration equation must be solved simultaneously with the integral equation for the pressure distribution beneath a structure. In most prior works[4)5)] based on the direct method, the vibration equation has been solved using a commercial software of FEM, and the pressure at nodal points used in FEM analyses has been determined by means of BEM. Therefore, the relation between the direct method and the mode-expansion method seems not clear, from a viewpoint of numerical calculation scheme.

The present paper is intended to develop a new direct solution method, which does not rely on the FEM, and to make clear the relation of the new method with Kashiwagi's numerical scheme[1)] based on the mode-expansion method.

2. Mathematical Formulation

Cartesian coordinates are defined with $z = 0$ as the plane of the undisturbed free surface and $z = h$ as the horizontal sea bottom. The incident regular wave comes from the negative x-axis with incidence angle β.

Time-harmonic motions of small amplitude are considered, with the complex time dependence $e^{i\omega t}$ applied to all first-order oscillatory quantities. The boundary conditions on the body and free surface are linearized, and the potential flow is assumed. The plan view of the structure is rectangular with length L and width B, and the draft is regarded as zero because of its very small value relative to L and B.

We express the velocity potential, $\phi(x, y, z)$, the pressure distribution, $p(x, y)$, the vertical displacement of the free surface, $\zeta(x, y)$, and the elastic deflection of a structure, $w(x, y)$, in nondimensional form as follows:

$$\left.\begin{aligned} &\phi(x,y,z) = i\,\omega a(L/2)\,\phi'(x,y,z), \quad p(x,y) = \rho g a\, p'(x,y) \\ &\zeta(x,y) = a\,\zeta'(x,y), \quad w(x,y) = a\,w'(x,y) \end{aligned}\right\} \tag{1}$$

where a is the amplitude of incident wave, ω the circular frequency, ρ the fluid density, and g the gravitational acceleration. The prime denotes nondimensional quantities, but it will be omitted for brevity in what follows.

The coordinates (x, y, z) are also made nondimension in terms of $L/2$, and thus the structure exists in the region of $|x| \leq 1$ and $|y| \leq b \equiv B/L$ on $z = 0$.

Hydrodynamically, the disturbance due to the presence of a structure can be expressed by the pressure applied on the free surface. Then the dynamic and kinematic free-surface boundary conditions are given by

$$p = K\phi + \zeta, \quad \frac{\partial \phi}{\partial z} = \zeta \quad \text{on } z = 0 \tag{2}$$

where $K = \omega^2/g$. Note that $p = 0$ outside of a structure and $\zeta = w$ beneath a structure.

Since the velocity potential can be given by the convolution integral of the pressure, $p(x, y)$, and the Green function, $G(x, y, z)$, satisfying (2) with $p = 0$, it is of relative ease to show that the integral equation for the unknown pressure takes the form

$$p(x,y) - K \iint_{S_H} p(\xi,\eta)\, G(x-\xi, y-\eta, 0)\, d\xi d\eta = w(x,y) \tag{3}$$

where S_H denotes the bottom of a structure with zero draft.

The body boundary condition can be satisfied by writing the deflection of a structure in the following form:

$$w(x,y) = w_S(x,y) + w_R(x,y) = -\zeta_I(x,y) + w_R(x,y) \tag{4}$$

where

$$\zeta_I(x,y) = \exp\{-ik_0(x\cos\beta + y\sin\beta)\} \tag{5}$$

is the elevation of incident wave, and subscripts S and R mean the scattering and radiation components, respectively.

Substituting (4) in (3) gives the equation to be solved:

$$p(x,y) - K \iint_{S_H} p(\xi,\eta)\, G(x-\xi, y-\eta, 0)\, d\xi d\eta - w_R(x,y) = -\zeta_I(x,y) \tag{6}$$

The radiation component of the deflection, $w_R(x, y)$, is unknown and subject to the vibration equation of a thin plate:

$$-MK\Lambda w_R(x,y) + D\Big(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}\Big) w_R(x,y) = -p(x,y) \tag{7}$$

where M is the mass of a structure (divided by ρLBd), D is the flexural rigidity (divided by $\rho g(L/2)^4$), and $\Lambda = 2d/L$ with d being the draft.

Since the structure is freely floating, $w_R(x, y)$ must satisfy the free-end boundary conditions along the periphery of the structure. Those conditions can be written as

$$\frac{\partial^2 w_R}{\partial n^2} + \nu \frac{\partial^2 w_R}{\partial s^2} = 0, \quad \frac{\partial}{\partial n}\Big\{\frac{\partial^2 w_R}{\partial n^2} + (2-\nu)\frac{\partial^2 w_R}{\partial s^2}\Big\} = 0 \tag{8}$$

where n and s denote the normal and tangential directions, respectively, and ν is Poisson's ratio.

In the case of a rectangular plate, a concentrated force, stemming from the replacement of the torsional moment with an equivalent shear force, acts at four corners, which must be also zero. Namely

$$R = 2D(1-\nu)\frac{\partial^2 w_R}{\partial x \partial y} = 0 \quad \text{at} \quad x = \pm 1,\ y = \pm b \tag{9}$$

In summary, (6) and (7) are the simultaneous equations for the two unknowns: the pressure distribution $p(x, y)$ and the vertical elastic deflection $w_R(x, y)$. Solving (6) and (7) at the same time while satisfying (8) and (9) is referred to as the direct method.[5)6)]

If $w_R(x, y)$ is expressed in terms of a system of appropriate known functions, $w_j(x, y)$ $(j = 1, 2, \cdots)$, in the form

$$w_R(x,y) = \sum_{j=1}^{\infty} X_j\, w_j(x,y), \tag{10}$$

the corresponding pressure can be sought from (6) in the form

$$p(x,y) = p_S(x,y) + \sum_{j=1}^{\infty} X_j\, p_j(x,y) \tag{11}$$

Here the amplitude X_j is unknown, but can be determined subsequently by solving (7) with free-end boundary conditions, (8) and (9), satisfied in an appropriate manner. This solution method is referred to as the mode-expansion method and featured in solving (6) and (7) separately.

3. A New Numerical Method

In the mode-expansion method developed by Kashiwagi[1)], the pressure distribution is represented using bi-cubic B-spline functions. It may be natural in the direct method to express the elastic deflection, $w_R(x,y)$, with the same B-spline functions. Therefore we try to obtain numerical solutions in the following form:

$$\left.\begin{aligned} p(x,y) &= \sum_{k=0}^{NX+2}\sum_{\ell=0}^{NY+2} \alpha_{k\ell}\, B_k(x)B_\ell(y) \\ w_R(x,y) &= \sum_{k=0}^{NX+2}\sum_{\ell=0}^{NY+2} \gamma_{k\ell}\, B_k(x)B_\ell(y) \end{aligned}\right\} \tag{12}$$

where $B_k(x)$ and $B_\ell(y)$ are the cubic B-spline functions. NX and NY are the number of panel division in the x- and y-directions, respectively. Since one cubic spline function extends its influence over four panels, the number of total unknowns in each of $p(x,y)$ and $w_R(x,y)$ is $(NX+3)*(NY+3)$.

Substituting (12) into (6) and (7) and applying a Galerkin scheme with $B_p(x)B_q(y)$ ($p = 0 \sim NX+2$, $q = 0 \sim NY+2$) as the weight function, we obtain a linear system of simultaneous equations, in the form

$$\sum_{k=0}^{NX+2}\sum_{\ell=0}^{NY+2}\left[\alpha_{k\ell}\left\{\mathcal{L}^{(1)}_{pq,k\ell} - K\,\mathcal{L}^{(2)}_{pq,k\ell}\right\} - \gamma_{k\ell}\,\mathcal{L}^{(1)}_{pq,k\ell}\right] = \mathcal{R}_{pq} \tag{13}$$

$$\sum_{k=0}^{NX+2}\sum_{\ell=0}^{NY+2}\left[\alpha_{k\ell}\,\mathcal{L}^{(1)}_{pq,k\ell} + \gamma_{k\ell}\left\{-MK\Lambda\,\mathcal{L}^{(1)}_{pq,k\ell} + D\,\mathcal{L}^{(3)}_{pq,k\ell}\right\}\right] = 0 \tag{14}$$

where

$$\mathcal{L}^{(1)}_{pq,k\ell} = \iint_{S_H} B_p(x)B_q(y)\, B_k(x)B_\ell(y)\, dxdy \tag{15}$$

$$\mathcal{L}^{(2)}_{pq,k\ell} = \iint_{S_H} B_p(x)B_q(y)\left[\iint_{S_H} B_k(\xi)B_\ell(\eta)\, G(x-\xi, y-\eta, 0)\, d\xi d\eta\right] dxdy \tag{16}$$

$$\mathcal{L}^{(3)}_{pq,k\ell} = \iint_{S_H} B_p(x)B_q(y)\, \nabla^4\{B_k(x)B_\ell(y)\}\, dxdy \tag{17}$$

$$\mathcal{R}_{pq} = -\iint_{S_H} B_p(x)B_q(y)\, \zeta_I(x,y)\, dxdy \tag{18}$$

The stiffness matrix, (17), must be transformed by partial integrations to incorporate the free-end boundary conditions, (8) and (9). The procedure is the same as that used in the mode-expansion method of Kashiwagi[1)], and the result takes the form

$$\begin{aligned} \mathcal{L}^{(3)}_{pq,k\ell} &= \iint_{S_H} \nabla^2 B_{pq}\, \nabla^2 B_{k\ell}\, dxdy \\ &\quad -(1-\nu)\iint_{S_H}\left\{\frac{\partial^2 B_{pq}}{\partial x^2}\frac{\partial^2 B_{k\ell}}{\partial y^2} + \frac{\partial^2 B_{pq}}{\partial y^2}\frac{\partial^2 B_{k\ell}}{\partial x^2} - 2\frac{\partial^2 B_{pq}}{\partial x \partial y}\frac{\partial^2 B_{k\ell}}{\partial x \partial y}\right\} dxdy \end{aligned} \tag{19}$$

where $B_{pq} = B_p(x)B_q(y)$ and $B_{k\ell} = B_k(x)B_\ell(y)$.

The mass matrix, $\mathcal{L}^{(1)}_{pq,k\ell}$, serves also as the cross-coupling matrix between the pressure and elastic deflection, which has been computed using Clenshaw-Curtis quadrature with absolute error less than 10^{-7} required. The integral $\mathcal{L}^{(2)}_{pq,k\ell}$ given by (16) is the most time-consuming part but the same as that appearing in Kashiwagi's mode-expansion method. Therefore, by taking advantage of 'relative similarity relations', it can be computed with less computational time.

4. Numerical Results

It is confirmed that the present method gives substantially the same results as those by the mode-expansion method using products of one-dimensional free-free beam modes to represent the elastic deflection.

Since the present method uses only the B-spline functions as a basis function, the computer code is simpler than the mode-expansion method. However, in the present method, the symmetry relation is not used, and thus the number of unknowns and the computational time are greater than that in the mode-expansion method of Kashiwagi.[1)]

Various computations have been performed, including the comparison with the experiments conducted at Ship Research Institute in Japan using a 1/30.8 scale model for a floating structure of $L \times B \times d = 300\text{m} \times 60\text{m} \times 0.5\text{m}$. Those results will be presented at the Workshop. Here, instead, we show one example of the wave profile around a structure of $L/B = 4$. Since the pressure is zero on the free surface, the total wave elevation can be computed from (2) and (5) by the equation:

$$\zeta_T(x,y) = \zeta_I(x,y) - K \iint_{S_H} p(\xi,\eta)\, G(x-\xi, y-\eta, 0)\, d\xi d\eta \tag{20}$$

Figure 1 is the result computed for $L/\lambda = 10$ and $\beta = 30°$ in deep water, with $NX = 40$ and $NY = 10$. The flexural rigidity was taken equal to 1.875×10^{-6}, which might be stiffer than a realistic floating airport. For comparison, Fig. 2 shows the wave profile around a rigid structure with the same dimensions. We can see that the wave reflection from an elastic plate is small near the bow and the transmitted wave is visible even downstream. The pattern of elastic deflection on the plate is different from that of water wave both in the wave length of fluctuation and the propagation angle.

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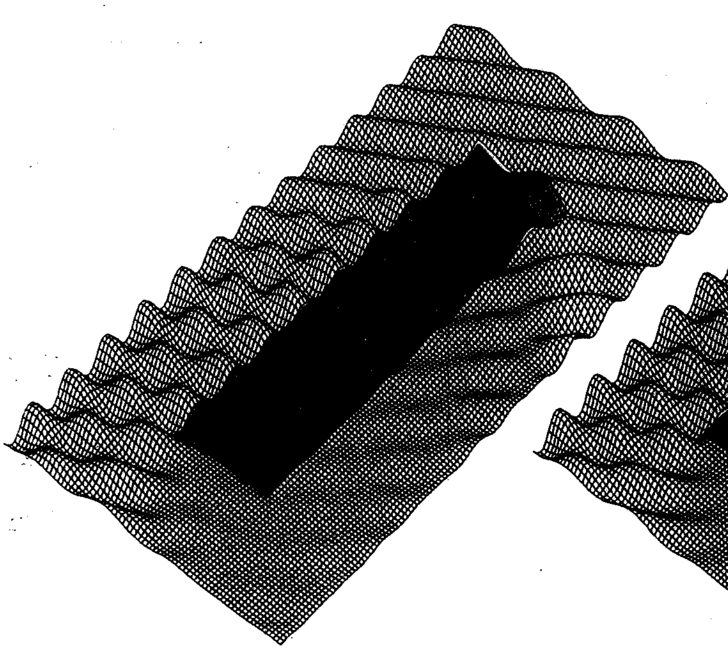

Fig. 1: Wave pattern around an elastic plate of $L/B = 4$ and $D = 1.875 \times 10^{-6}$. $L/\lambda = 10$ and $\beta = 30°$ in deep water.

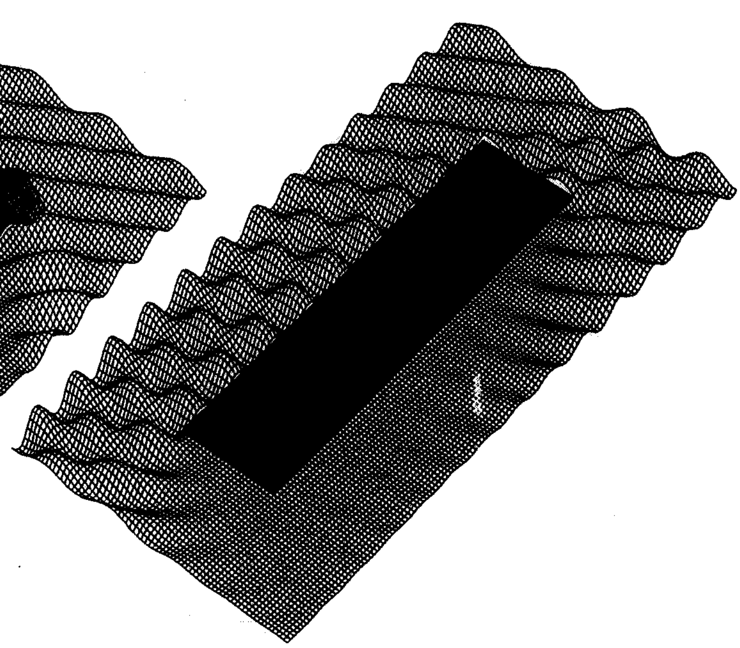

Fig. 2: Wave pattern around a rigid plate. Geometrical dimensions and wave data are the same as Fig. 1.