On the generation of wave free oscillatory bodies and of trapped modes

E. Fontaine and M.P. Tulin

University of California, Ocean Engineering Laboratory, Santa Barbara CA 93106-1080.

1 Introduction

The motion of bodies piercing a free surface in the presence of gravity usually leads to the formation of waves on the surface, which propagate away from the body. In this paper, we explore within the framework of first order potential flow theory, the existence of wave-free two dimensional flows past surface piercing oscillating bodies. The shape of the body is found by constructing a wave free potential which decays to zero at infinity and interpreting some of its streamlines as body boundaries. Two different general techniques can be used to construct such a potential. First, through phase cancellation of the wave fields due to discrete singularities with appropriate spacings. For example, McIver (1997) considers two sources separated by half a critical wavelength. There exists then a relation between the characteristic length of the body shape and the critical wave frequency. The use of this method is thus limited to higher frequencies. Another technique, introduced by Tulin (1976, 1982) in connection with ship waves, is based on the use of wave-free compound singularities. It has been successfully applied in three-dimensions for the minimization of ship wave resistance by Tulin & Oshri (1994). A portion of Tulin's results were re-discovered and applied by Tuck (1992). In this paper, we apply this technique to the seakeeping problem in order to find shapes of bodies that do not generate waves while oscillating at a given frequency. It is also shown how body shapes that generate the so called "trapped modes" can be derived using this theory. Simple examples are considered here using a single wave-free compound singularity, but results for singularity distributions, which can be interpreted in terms of body volume and vertical force distributions, can also be derived using the same basic ideas. These results can also be extended in three-dimensions as carried out by Tulin in the case of bodies in uniform streams.

2 Wave-free compound singularities

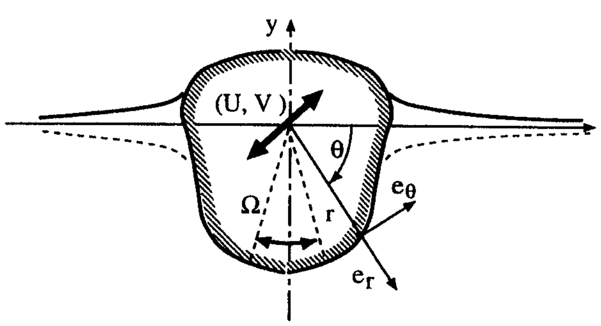

Figure 1: Geometric definitions.

We consider the two-dimensional case of a surface piercing body oscillating with pulsation ω in heave, sway and roll (see. fig.1). The variables are non-dimensionalized using $1/\omega$ as the time scale and g/ω^2 as the length scale, where g is the acceleration due to gravity

$$\tilde{x} = \frac{\omega^2 x}{g}, \qquad \tilde{y} = \frac{\omega^2 y}{g}, \quad \tilde{t} = \omega t,$$
$$\tilde{\eta} = \frac{\omega^2 \eta}{g}, \qquad \tilde{\phi} = \frac{\omega^3 \phi}{g^2} \tag{1}$$

The complex potential

$$\Psi = [\tilde{\phi} + i\,\tilde{\psi}\,]\,e^{i\tilde{t}} = [\Psi_h(\tilde{z}) + \Psi_s(\tilde{z})\,]\,e^{i\tilde{t}} \tag{2}$$

is considered as the sum of a *hard* singularity system, Ψ_h, generating only horizontal velocities on the free surface, and a *soft* system Ψ_s

which generates only vertical velocities on the free surface. Ψ_h and Ψ_s are defined so that[1] $\Im([\Psi_h']_{y=0}) = \Im([\Psi_h]_{y=0}) = 0$ and $\Re([\Psi_s']_{y=0}) = \Re([\Psi_s]_{y=0}) = 0$. It follows that the complex potential Ψ will satisfy the linearized free surface boundary condition

$$\Re\Big([\Psi - i\,\Psi']_{y=0}\Big) = 0 \qquad (3)$$

provided that $\Re\Big([\Psi_h - i\,\Psi_s']_{y=0}\Big) = 0$. This last relation is verified by simply choosing

$$\Psi_h = i\,\Psi_s' \qquad (4)$$

Finally, the compound singularity defined by $\Psi = \big(\Psi_s + i\,\Psi_s'\big)\,e^{i\tilde{t}}$ satisfies the free surface boundary condition and is wave free. Simple wave-free compound systems can be constructed directly using soft systems based on standard singularities such as sources and vortices, centered about a fixed point and its image (see Tulin, 1994). On the other hand, soft systems can be obtained from the hard ones, and vice et versa, by using (4). It can also be checked that if Ψ_s is a soft system, then its derivatives also represent soft systems. There is therefore a wide range of possibilities for the choice of the wave-free potential. Given the potential generated by a real oscillating body, the body shape must be determined.

3 Generation of wave-free oscillating bodies and of trapped modes

We now look for body shapes that would generate, while oscillating, the wave-free flow described by a given complex potential Ψ. The oscillations of the body are given by $V = g\,\tilde{V}_0/\omega\,e^{i\tilde{t}}$ for heave, $U = g\,\tilde{U}_0/\omega\,e^{i\tilde{t}}$ for sway and $\Omega = \omega\tilde{\Omega}_0\,e^{i\tilde{t}}$ for roll. Using $\tilde{r} = f(\theta)$ as a parameterization for the shape of the body, the body boundary condition is written as

$$\frac{\partial\phi}{\partial n} = (U\,\vec{e}_x + V\,\vec{e}_y + \Omega\,\vec{e}_z \times \vec{r}) \cdot \vec{n} \qquad (5)$$

where $\vec{n} = -\vec{e}_r + 1/f \cdot df/d\theta\,\vec{e}_\theta$ is the normal to the body. This leads to the following differential equation for f

$$\frac{1}{f}\frac{df}{d\theta} = \frac{N(f,\theta)}{D(f,\theta)} \qquad (6)$$

where

$$N(\tilde{r},\theta) = \cos\theta\,(\tilde{\phi}_{\tilde{x}} - \tilde{U}_0 + \tilde{r}\,\tilde{\Omega}_0\,sin\theta) + \sin\theta\,(\tilde{\phi}_{\tilde{y}} - \tilde{V}_0 - \tilde{r}\,\tilde{\Omega}_0\cos\theta), \qquad (7)$$

and

$$D(\tilde{r},\theta) = -\sin\theta\,(\tilde{\phi}_{\tilde{x}} - \tilde{U}_0 + \tilde{r}\,\tilde{\Omega}_0\sin\theta) + \cos\theta\,(\tilde{\phi}_{\tilde{y}} - \tilde{V}_0 - \tilde{r}\,\tilde{\Omega}_0\cos\theta). \qquad (8)$$

Families of body shapes that do not generate waves at a given frequency while oscillating with velocities $\tilde{U}_0, \tilde{V}_0, \tilde{\Omega}_0$ can therefore be constructed by numerical integration of equation (6), starting from different initial conditions. Of course in our linear approximation, the amplitude of body motion must remain small when compared to the characteristic length describing the body shape.

On the other hand, bodies which would not oscillate in the presence of an oscillating free surface perturbation must obey the same differential equation (6) with $\tilde{U}_0 = \tilde{V}_0 = \tilde{\Omega}_0 = 0$, which may also be recognized as the equation of stream lines in polar coordinates. These bodies therefore generate the so called "trapped modes". It is therefore expected that resonance phenomena occur while oscillating these bodies. This leads to non-uniqueness of the solution and infinite added mass, McIver (1997).

For the examples presented below, the integration has performed using a standard fourth order Runge Kutta algorithm with adaptation of the incremental step to the solution.

4 Simple examples

As an illustration of the previous ideas, we now look for symmetric bodies that do not generate waves while oscillating at a given frequency in heave. The velocity field has to satisfy the symmetry condition $\tilde{\phi}_{\tilde{x}}(\tilde{r}, \pi - \theta) = -\tilde{\phi}_{\tilde{x}}(\tilde{r},\theta)$ and $\tilde{\phi}_{\tilde{y}}(\tilde{r}, \pi - \theta) = \tilde{\phi}_{\tilde{y}}(\tilde{r},\theta)$ so that the shape of the body obtained by integrating (6) remains symmetric. A simple wave-free compound singularity centered at $z = 0$ which satisfies this condition is given by

$$\Psi = -\alpha\,(\frac{i}{\tilde{z}} + \frac{1}{\tilde{z}^2})\,\Re\Big(e^{i\,\tilde{t}}\Big) \qquad (9)$$

[1] the notation $\Re$ and $\Im$ stands for real and imaginary parts of a complex number. Ψ' indicate a derivation of analytic function Ψ with respect to the complex variable $\tilde{z} = \tilde{x} + i\,\tilde{y} = \tilde{r}\,e^{i\theta}$.

which leads for the free surface elevation $\tilde{\eta} = -\alpha \sin(\tilde{t})/\tilde{x}^2$. The oscillations of the body therefore generate an evanescent free surface deformation.
Considering α as a parameter, a family of flat bottom bodies can be found (see fig. 2) starting from initial conditions $\theta = -\pi/2$, $f = -\tilde{y}_0$ where $\tilde{y}_0$ is the only real root of equation $-(\tilde{V}_0/\alpha)\,\tilde{y}^3 + \tilde{y} + 2 = 0$ $(\alpha > 0)$. Let us denote $\tilde{x}_0$ the half width of the flat body at the waterline. For each value of α, two additional one parameter families can be obtained, starting from initial conditions $\theta = 0$, $f = f_0$. For $f_0 > \tilde{x}_0$, body shapes are wine glass like and extend down to $-\infty$ (see fig. 3) while for $f_0 < \tilde{x}_0$, twin hull closed bodies are obtained. For the resonant problem, a family of twin hull bodies is obtained (fig. 4).

We now consider the case of a forced roll motion, i.e. $\tilde{U}_0 = \tilde{V}_0 = 0$. In order to respect the symmetry condition, the velocity field has to satisfy $\tilde{\phi}_{\tilde{x}}(\tilde{r}, \pi - \theta) = \tilde{\phi}_{\tilde{x}}(\tilde{r}, \theta)$ and $\tilde{\phi}_{\tilde{y}}(\tilde{r}, \pi - \theta) = -\tilde{\phi}_{\tilde{y}}(\tilde{r}, \theta)$. A simple wave-free compound singularity centered at $z = 0$ satisfying this condition is given by

$$\Psi = \alpha\,(\frac{1}{\tilde{z}} - \frac{i}{\tilde{z}^2})\,\Re\Big(e^{i\tilde{t}}\Big) \qquad (10)$$

Starting the integration of eq. (6) from $\theta = 0$, $f = f_0$ leads to the definition of two families of bodies (see fig. 5 and 6). For large values of f_0, bodies are very close to circular since eq. (6) leads to $df/d\theta = 0$ as r goes to infinity. For small values of f_0 a family of twin hull bodies is found. The resonant problem also leads to a family of twin hull bodies (fig. 7).

5 Summary & conclusions

A method is presented using compound wave-free singularities for the determination of families of two dimensional body shapes that do not produce waves while oscillating at a given frequency in heave, sway or roll. Body shapes that generate trapped modes are also derived. Examples are given showing that a wide range of shapes can be generated even with a simple singularity system.

In view of the Haskind formula relating radiation damping and wave exitation, Newman (1977), bodies which are wave free when oscillated will be force free in the same mode when placed in an incident wave field at the same frequency. The latter can be chosen, for example, as the natural resonant frequency of the body, suggesting an application for this theory.

Using distributed compound singularities, a wide variety of wave-free realistic bodies can be developed, and the method can be extended to the axially symmetric case.

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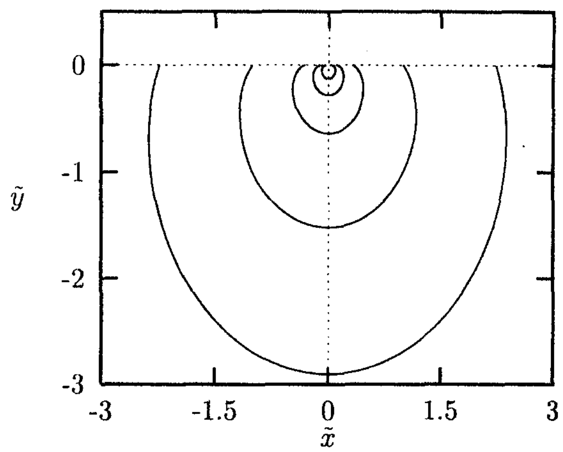

Figure 2: Heave motion. One parameter family of wave free flat bottom bodies for $\alpha/\tilde{V}_0$ varying from 10^{-3} (smallest body) to 5.

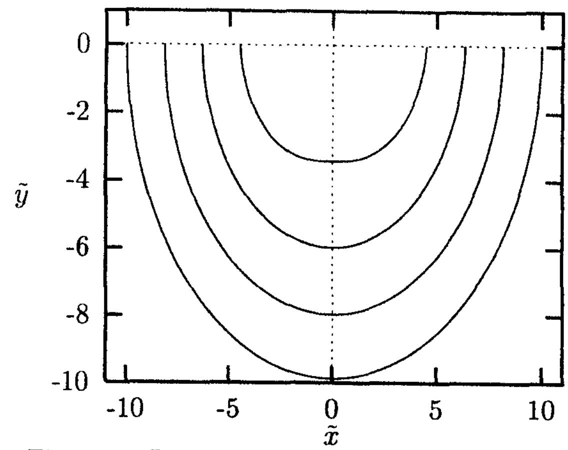

Figure 5: Roll motion. One parameter family of wave free flat bottom bodies for $\alpha/\tilde{\Omega}_0 = 10$.

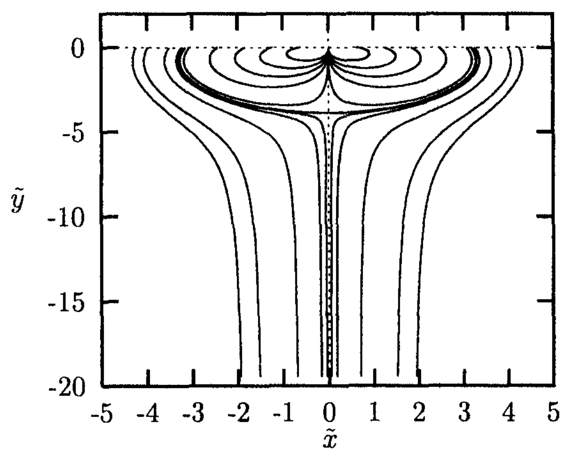

Figure 3: Heave motion. For $\alpha = 1$. inside and outside families of body shapes.

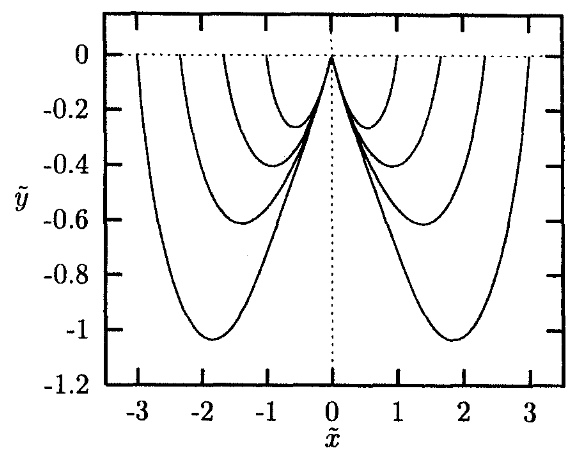

Figure 6: Roll motion. One parameter family of wave free twin hull bodies ($\alpha/\tilde{\Omega}_0 = 10$).

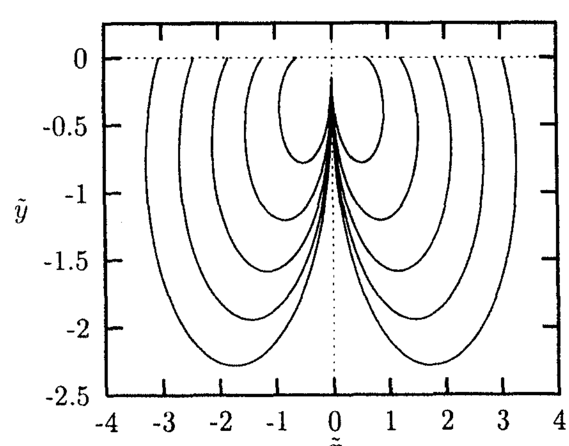

Figure 4: Heave motion, resonant case. One parameter family of body shapes that generate trapped modes.

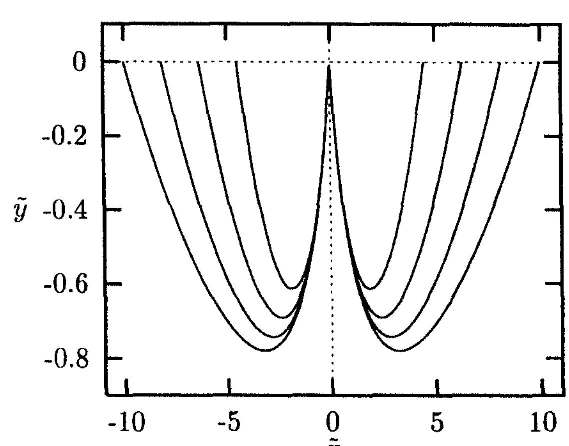

Figure 7: Roll motion, resonant case. One parameter family of body shapes that generate trapped modes ($\alpha/\tilde{\Omega}_0 = 10$).