Multiple-body Simulations using a Higher-Order Panel Code

Donald Danmeier [1]

Formulation

Potential theory is used to formulate an initial-boundary value problem (IBVP) directly in the time domain. The free-surface is linearized about its calm position, but the exact body boundary condition is imposed on its instantaneous position. Thus, we allow large-amplitude motions, but the waves generated by the body must be small. Assuming an ideal fluid and irrotational flow, the these boundary condition and are:

$$\Phi_{tt} + g\Phi_z = 0 \quad on \quad Z = 0 \tag{1}$$

$$\Phi_n = (\vec{U} - \nabla\varphi)\cdot\vec{n} \quad on \quad \mathcal{B}(t). \tag{2}$$

Here, $\mathcal{B}(t)$ is the instantaneous body surface, with velocity $\vec{U}$. The incident potential is φ, and Φ represents the disturbance due to the presence of the body. The $Z = 0$ plane lies at the mean sea level.

Global forces on the body are found from direct pressure integration.

$$\vec{F} = -\rho \iint_{\mathcal{B}(t)} \left(\Psi_t + \frac{1}{2}\nabla\Psi\cdot\nabla\Psi + gZ\right)\vec{n}\,dS + \frac{\rho}{2}\int_{\Gamma(t)} \Psi_t^2\vec{n}\,dl \tag{3}$$

where, $\Gamma(t)$ represents the waterline, and the potential of the total flow is $\Psi = \Phi + \varphi$. The quadratic terms from Bernoulli's expression and the waterline contribution may be loosely thought of 'second-order' effects due to a 'first-order' solution, and are the leading-order contributors to the steady drift force.

In order to facilitate a solution, Green's theorem is used to recast the above IBVP into an integral equation.

$$2\pi\Phi + \iint_{\mathcal{B}(t)} (\Phi G_n^o - G^o\Phi_n)\,dS = \int_o^t d\tau \iint_{\mathcal{B}(\tau)} (\Phi H_{\tau n} - H_\tau\Phi_n)\,dS \tag{4}$$
$$+\frac{1}{g}\int_o^t d\tau \int_{\Gamma(\tau)} (\Phi H_{\tau\tau} - H_\tau\Phi_\tau)\,\vec{U}_{2d}\cdot\vec{n}_{2d}\,dl$$

where, G^o and H are, respectively, the Rankine and wave parts of the transient free-surface Green function. $\vec{U}_{2d}$ and $\vec{n}_{2d}$ are projections of the body's velocity and normal vector onto the XY plane.

Numerical Solution

We use higher-order panel methods to efficiently solve the integral equation. The body surface and velocity potential are mapped to a square parametric plane via the B-spline basis. A Galerkin procedure gives a linear system of equations. The success of this B-spline based panel method in the frequency domain has been demonstrated by Maniar [1] [2]. A major advantage of the method is the ability to analytically differentiate the solution, and we exploit this by including the $(\nabla\Phi\cdot\nabla\Phi)$ term in the Bernoulli pressure.

The results presented in this abstract only include horizontal modes, however, if our finite-amplitude simulations include vertical motion, the body needs to be remeshed at every time step. In these cases, an automated marching algorithm detects the body/free-surface intersection in the parametric plane. We then produce new B-spline coefficients for the portion of the body below the mean free-surface by a least-square fit.

[1] Department of Ocean Engineering, MIT, USA (danmeier@mit.edu)

N-body Simulation

Hydrodynamic interactions may greatly change wave loads when multiple bodies operate in close proximity. Ohkusu [3] examined the motions of a ship in the neighborhood of an offshore structure. His ship-structure system included a large moored cylindrical structure with a horizontal axis. The floating ship lies parallel to the structure's longitudinal axis and is subject to monochromatic beam waves. Ohkusu looked at how the fixed structure influenced the wave-frequency motions, as well as the drift force on the freely floating ship. At certain wavelength/separation conditions, his calculations predicted the upwave drift of the ship observed during model testing. We will use the finite-amplitude IBVP formulation to study the hydrodynamic interactions of two truncated circular cylinders.

Figure 1 shows two identical truncated circular cylinders, with planar waves incident in the $-x$ direction. The upwave cylinder is free to move in surge, but the downwave body is fixed. The initial separation between centers is $2d = 5$, and the cylinders have unit radius and draft. From Ohkusu's experience, we may expect the fixed cylinder to repel the floating body at some critical spacing. In order to confirm this prediction, the 3D frequency domain panel code `WAMIT` is used to calculate the mean force on the upwave cylinder for a variety of wave frequencies. Results from three different separation distances are given in Figure 2. From this preliminary computation, we see that negative drift forces arise for a range of wavelength/spacing values.

Results from the finite-amplitude simulation are shown in Figure 3. The large initial separation produces weak interaction effects, but the hydrodynamic coupling grows as the free cylinder drifts downwave. After several wave periods, the upwave cylinder experiences a negative drift force and is repelled from the fixed cylinder. Note that the body's acceleration and velocity are $O(\epsilon)$, but its motions are $O(1)$ in magnitude. This requires an exact treatment of the body boundary condition. The steady flow induced by the slow drift is of secondary interest, since the diffraction field produces the strong interactions effects. For this reason, a linear free-surface boundary condition captures the hydrodynamics to the desired accuracy.

References

[1] Hiren Maniar A B-spline based higher-order method in 3D. In *Tenth International Workshop on Water Waves and Floating Bodies*, Oxford, April 1995.

[2] Hiren Maniar. *A three dimensional higher order panel method based on B-splines.* PhD dissertation, Massachusetts Institute of Technology, Department of Ocean Engineering, September 1995.

[3] Makoto Ohkusu. Ship motions in vicinity of a structure. *Proc. Behaviour of offshore structures*, Trondheim, August 1976.

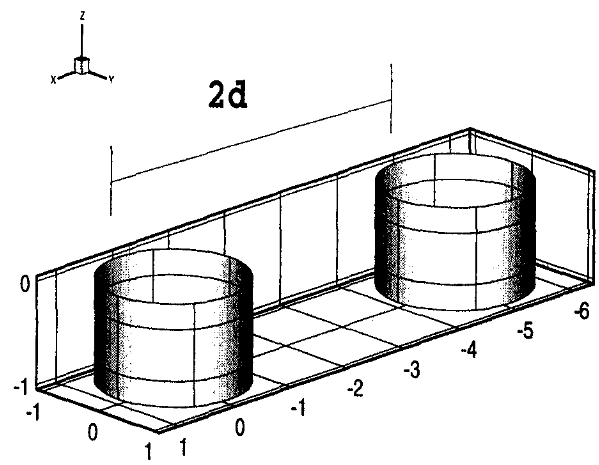

Figure 1: Two interacting cylinders with unit radii and draft. Distance between centers is 5. Waves in $-x$ direction. Upwave cylinder is free in surge, downwave body is fixed.

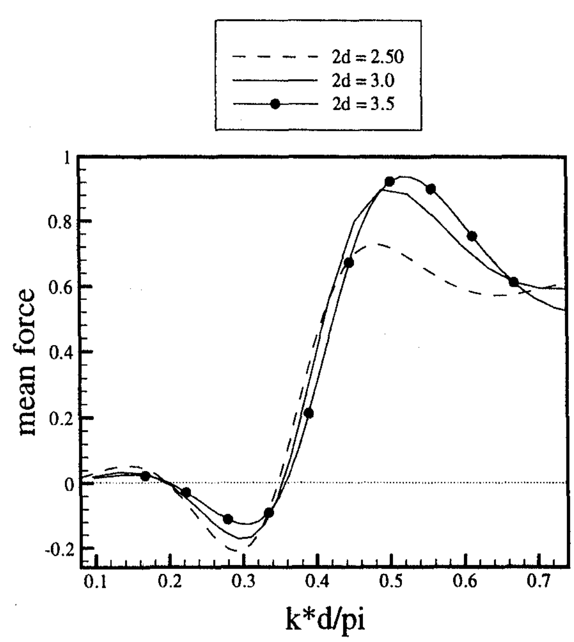

Figure 2: Mean drift force on upwave cylinder calculated from WAMIT. Non-dimensional force is $\bar{F} = \frac{F}{\rho g A^2 R}$, where A is incident wave amplitude and R is radius of cylinder.

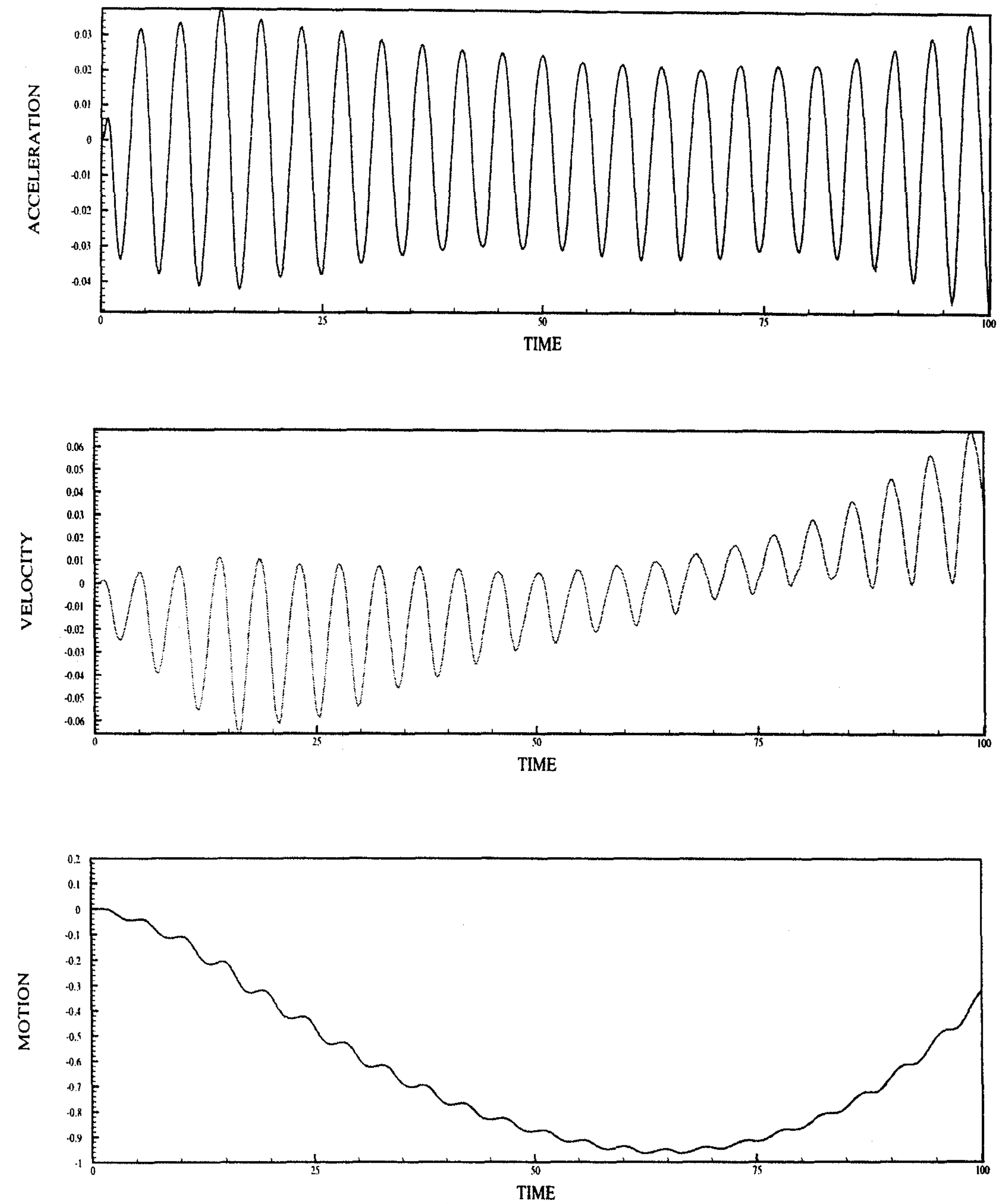

Figure 3: Time histories of upwave cylinder from finite-amplitude simulation. Incident waves are monochromatic, with wavenumber $K = 2$ and amplitude $A = 0.1$. Non-dimensional length and time are: $\bar{x} = \frac{x}{R}, \bar{t} = t(R/g)^{(1/2)}$