(Abstract for the 12th International Workshop on Water Waves and Floating Bodies, March 1997)

A study on wave-drift damping by fully nonlinear simulation

Katsuji Tanizawa † and Shigeru Naito ‡

† Ship Research Institute, Tokyo, Japan
‡ Osaka University, Osaka, Japan

1 Introduction

The low-frequency motion of floating marine structures such as moored ships and oil platforms are one of the main concern in ocean engineering. The low-frequency motions are excited by slowly oscillating second-order wave forces and the resonance causes large amplitude horizontal motions. The accurate estimation of the amplitude is very important for the design of mooring system. The conventional damping force due to viscous effect and wave radiation are small and wave-drift damping, resulting from the interaction between an incident wave and low-frequency oscillatory motion of a floating body, is dominant. Therefore the accurate estimation of wave-drift damping is indispensable.

Wave-drift damping has been studied experimentally and theoretically by Wicheres and Slujis [1)], Saito, Takagi, Ohkubo and Hirashima [2)], Faltinsen [3)], Hearn and Tong [4)], Nossen, Grue and Palm [5)], Zhao and Faltinsen [6)], Eatock Taylor and Teng [7)], Sunahara [9)] and others. In their approach, wave-drift damping is analyzed in a quasi-steady manner, based on the rate of change of the added resistance in waves, with respect to small steady forward velocity. Newman [8)] outlines a procedure for the more direct derivation of wave-drift damping from a perturbation analysis and extracted it from second order radiation force at low-frequency.

In this study, a numerical approach is taken for the fully nonlinear analysis of the interaction between an incident wave and low-frequency oscillatory motion of a floating body. Using the fully nonlinear simulation method [12)], three motions of a moored two-dimensional body are simulated in presence of a regular wave field and the hydrodynamic force due to the interaction is extracted from horizontal hydrodynamic force act to the body. Based on this numerical study, added mass and damping coefficient due to the interaction are analyzed and a rational explanation of wave-drift damping is proposed. The relation between this explanation and the conventional explanation based on quasi-steady analysis is discussed.

2 Target of the numerical simulation

Motions of a moored two dimensional floating body in a regular wave is considered. Fig.1 shows the target of the simulation. The ideal fluid is bounded by free surfaces, a piston wave maker, a flat bottom, a vertical wall and a floating body. The fluid motion is described by velocity potential and acceleration potential. Motion of the floating body coupled with the fluid motion is solved in the acceleration filed using the implicit body surface boundary condition. All three degrees of freedom are simulated.

Following a preceding work of Cointe et al. [10)], artificial damping zones are applied to prevent wave reflection from both ends of wave basin. Inside of the damping zones, damping terms are added to both dynamic and kinematic free surface boundary conditions to give damping effect to free surface. These damping zones effectively work as a wave breaker and an absorbing wave maker. The motions of floating body can be simulated for long time without affected much by reflection waves.

The detail of this simulation method with damping zone is presented in reference paper [12)].

3 The interaction between incident wave and low-frequency body motion

To study the interaction between incident wave and low-frequency body motion, the hydrodynamic force purely due to the interaction is extracted from following four nonlinear simulations presented in §3.1, §3.2, §3.3 and §3.4.

3.1 Simulation of moored body motions in still water

To obtained the basic characteristic of the mooring system in still water, free oscillatory motion is simulated. Fig.2 shows the simulated velocity of the body and the horizontal hydrodynamic force acts to the body. Fourier analysis of the simulated results gives following information.

- The natural frequency of the moored body motion in still water is $\tilde{\omega}_o = 0.384\ rad/s$.
- The added mass of the motion is $m_{sd}(\tilde{\omega}_o) = 155.75\ kg$.
- The damping coefficient of the motion is $c_{sd}(\tilde{\omega}_o) = 0.1091\ N/(m/s)$.

Since the radiation wave length correspond to $\tilde{\omega}_o$ is about $390m$ and more than 520 times wider than body breadth, these hydrodynamic coefficients are similar equal to the limit values $m_{sd}(0) = 148.56\ kg,\ \ c_{sd}(0) = 0$ when $\omega_o \to 0$.

3.2 Simulation of the moored body motions in a regular wave

Next, free motion is simulated in the presence of a regular incident wave (wave length $\lambda = 2.7\ m$, wave amp. $\zeta_a = 5\ cm$, wave period $T_W = 1.316\ s$). Fig.3(a) shows the simulated sway motion and swaying force act to the body. For the analysis of the slow motion, low-frequency components are extracted by FFT from swaying velocity and swaying force. These are plotted in Fig.3(b) as U and F_X . Two differences exist between Fig.2 and Fig.3(b).

- The natural frequency of slow oscillatory motion in the regular wave is $\tilde{\omega} = 0.420\ rad/s$ and different from that of in still water.
- The significant damping is observed in the regular wave meanwhile damping is weak in still water.

F_X in Fig.3(b) is composed of steady wave-drift force $\bar{F}_X$ and slowly varying component $\tilde{F}_X$, which is considered to be the main cause of the differences.

3.3 Simulation of forced oscillation of the body in still water

To obtain the hydrodynamic force purely due to the interaction between incident wave and slow oscillatory motion, we have to remove the conventional hydrodynamic force F_{sd} due to low-frequency oscillation from F_X . F_{sd} can be obtained from the simulation of forced oscillated body motion in still water. The bottom raw of Fig.4 shows the simulated F_{sd} . The added mass and damping coefficient for $\tilde{\omega}$ can be obtained from Fourier analysis of this force. Then F_{sd} for arbitrary motions is given as

$$F_{sd} = m_{sd}(\tilde{\omega})\dot{U} + c_{sd}(\tilde{\omega})U \ . \qquad (1)$$

Theoretically, F_{sd} can be removed from F_X in Fig.3(b). But for quantitative study, the simulation of transient motion is not adequate and periodically steady state of low-frequency oscillation should be simulated.

3.4 Simulation of the moored body motions oscillated by a low-frequency external force in the regular wave

An external low-frequency force G_X , which is synchronized to $\tilde{\omega}$, is added to the body to oscillate the periodically steady low-frequency motion in the presence of the regular wave. The results are plotted in Fig.5. The amplitude of G_X is set to $|G_X| = 1.8\ N \approx 0.2\bar{F}_X$ in this simulation.

The top row of Fig.5 shows simulated sway motion from $t = 0$ to $250\ T_W$, the 2nd to the 4th row show sway motion, swaying velocity and horizontal hydrodynamic force magnified in time from $t = 150\ T_W$ to $200\ T_W$ and the 5th to the 6th row show low-frequency components of them extracted by FFT. Since the amplitude and the phase of forced oscillating motion shown in Fig.4 are equally set to those of low-frequency motion in Fig.5, the hydrodynamic force due to the interaction is simply given as $F_{wd} = F_X - F_{sd}$. Here, we call F_{wd} as wave-drift force, which is composed of steady wave drift force $\bar{F}_{wd} = \bar{F}_X$ and unsteady force $\tilde{F}_{wd}$. The bottom row of Fig.5 shows F_{wd}

3.5 Interaction between incident wave and slow oscillatory motion

Using F_{wd} and U presented in Fig.5, the interaction can be quantitatively studied. Fourier analysis of F_{wd} and U gives following information.

- The amplitude of F_{wd} is $|F_{wd}| = 1.814\ N$
- The amplitude of U is $|U| = 0.0551\ m/s$
- The phase between F_{wd} and U is $\theta = 2.712\ rad.$

When the simulation converges to the periodically steady state, the damping force balances with the exciting force. Above results, obtained from independent simulation shown in Fig.4 and Fig.5, well satisfy this condition $|F_{wd}| = |G_X|$ and that demonstrate the accuracy of these simulations.

Next, here we write

$$U = |U| \sin \tilde{\omega}t \qquad (2)$$

and decompose F_{wd} into sin and cos components

$$F_{wd} = |F_{wd}| \cos\theta \sin\tilde{\omega}t + |F_{wd}| \sin\theta \cos\tilde{\omega}t \ . \qquad (3)$$

Then added mass and damping coefficient due to the interaction are written as

$$A_X = -\frac{|F_{wd}| \sin\theta}{\tilde{\omega}|U|} \qquad (4)$$

$$B_X = -\frac{|F_{wd}| \cos\theta}{|U|} \ . \qquad (5)$$

Substituting the values of $|F_{wd}|, |U|$ and θ in these equations, we have $A_X = -32.65\ kg,\ B_X = 29.93\ N/(m/s)$. We should not confuse A_X, B_X with the conventional hydrodynamic coefficient m_{sd}, c_{sd} due to the oscillatory motion. A_X and B_X do not exist without incident wave.

Using A_X , the difference between $\tilde{\omega}_o$ and $\tilde{\omega}$ can be explained. The spring constant of the mooring $51.07 N/m$, the body mass $191.79 kg$ and the added mass in still water $155.75 kg$ gives the estimation of the natural frequency in still water $0.383 rad/s \approx \tilde{\omega}_o$. Taking the added mass reduction A_X into account, the natural frequency in the wave field becomes $0.403 rad/s \approx \tilde{\omega}$.

3.6 Wave-drift damping

B_X is considered to be wave-drift damping for frequency $\tilde{\omega}$. On the other hand, in the theoretical studies, wave-drift damping is defined in quasi-steady manner as

$$\bar{B}_X = -\left.\frac{\partial \bar{F}_X}{\partial U}\right|_{U=0} . \quad (6)$$

The relation between B_X and $\bar{B}_X$ can be clearly shown as follows.

Using eq.(2), eq.(3) can be written as

$$F_{wd} = \frac{|F_{wd}|\cos\theta}{|U|}U + \frac{|F_{wd}|\sin\theta}{\tilde{\omega}|U|}\dot{U} . \quad (7)$$

When the slow drift motion is in periodically steady state, $|F_{wd}|$ and $|U|$ are constant. Therefore, partial derivative of F_{wd} with respect to U is given as

$$\frac{\partial F_{wd}}{\partial U} = \frac{|F_{wd}|\cos\theta}{|U|} + \frac{|F_{wd}|\sin\theta}{\tilde{\omega}|U|}\frac{\partial \dot{U}}{\partial U} . \quad (8)$$

Taking the relation

$$\frac{\partial \dot{U}}{\partial U} = -\tilde{\omega}^2 \frac{U}{\dot{U}} \quad (9)$$

into account, $\partial \dot{U}/\partial U$ becomes zero at $U = 0$ and we have formula

$$\left.\frac{\partial F_{wd}}{\partial U}\right|_{U=0} = \frac{|F_{wd}|\cos\theta}{|U|} . \quad (10)$$

Therefore, Eq.(5) is finally written as

$$B_X = \left.\frac{\partial F_{wd}}{\partial U}\right|_{U=0} . \quad (11)$$

This definition of wave-drift damping is valid for $\tilde{\omega} \geq 0$. When $\tilde{\omega}$ tends to zero, $F_{sd} \to 0$ and $F_{wd} \to \bar{F}_X$ can be substituted to eq.(11) to have the conventional definition eq.(6).

4 Conclusion

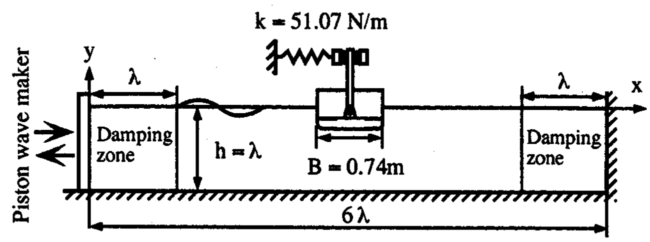
The hydrodynamic force purely due to the interaction between incident wave and low-frequency body motion is extracted by following nonlinear simulations

1. Free motions of the moored body in still water is simulated to obtain the natural frequency $\tilde{\omega}_o$ in still water.
2. Free motions of the moored body in a regular wave is simulated to obtain the natural frequency $\tilde{\omega}$ in the regular wave field.
3. Forced oscillatory body motions in still water in frequency $\tilde{\omega}$ is simulated to obtain the conventional hydrodynamic force F_{sd} due to the low-frequency oscillation.
4. The moored body motions oscillated by a low-frequency external force in frequency $\tilde{\omega}$ in the regular wave field is simulated to obtain the hydrodynamic force $F_{wd} = F_X - F_{sd}$ purely due to the interaction,

and the rational explanation of added mass A_X and the damping coefficient B_X due to the interaction is given. This explanation reflects the dynamics of the interaction. Series of simulations with different $\tilde{\omega}$ will teach us the frequency dependency of A_X and B_X .

References

1) Wicheres,J.E.W. and Slujis,M.F.: The influence of wave on the low frequency hydrodynamic coefficients of moored vessels, *Proc. of Offshore Tech. Conf. No.3625*,pp2313-2324, (1979)
2) Saito,K., Takagi,M., Ohkubo,H. and Hirashima,M. : On the low-frequency damping forces acting on a moored body in waves, *J. Kansai Soc. Narval Arch. Japan, Vol.195*, pp51-59, (1984)
3) Faltinsen,O.M. : Slow-drift damping and responses of moored ship in irregular waves, *5th OMAE Symp.*, Tokyo, (1986)
4) Hearn,G.E. and Tong,K.C. : A comparative study of experimentally measured and theoretically predicted wave drift damping coefficients, *Proc. OTC, No.6136*, (1989)
5) Nossen,J., Grue,J and Palm,E. : Wave forces on three-dimensional floating bodies with small forward speed, *JFM, Vol.227*, (1991)
6) Zhao,R. and Faltinsen,O.M. : Interaction between current, waves and marine structures, *Proc. 5th Int. Conf. on Num. Ship Hydro.*, Hiroshima, (1989)
7) Eatock Taylor,R. and Teng,B. : The effect of corners on diffraction/radiation forces and wave drift damping, *Proc. OTC, No.7187*, (1993)
8) Newman,J.N. : Wave-drift damping of floating bodies, *JFM, Vol.249*, (1993)
9) Sunahara : A study on wave drift damping acting on multiple floating cylinders, *Ph.D Thesis, The Univ. of Tokyo*, (1994)
10) Cointe,R., Geyer,P., King,B., Molin,B. and Tramoni,M. : Nonlinear and linear motions of a rectangular barge in a perfect fluid, *Proc. 18th Symp. Naval Hydro.*, (1990)
11) Tanizawa,K. : A Nonlinear Simulation Method of 3-D Body Motions in Waves, *J. Soc. Naval Arch. Japan, Vol.178*, (1995)
12) Tanizawa,K. : Long time fully nonlinear simulation of floating body motions with artificial damping zone, *J. Soc. Narval Arch. Japan, Vol.180*, (1996)



Incident wave λ = 2.7 m, $h_w = 2\zeta_a = 10$ cm
Floating body Weight 191.8 kg

Fig.1 Model of the simulation

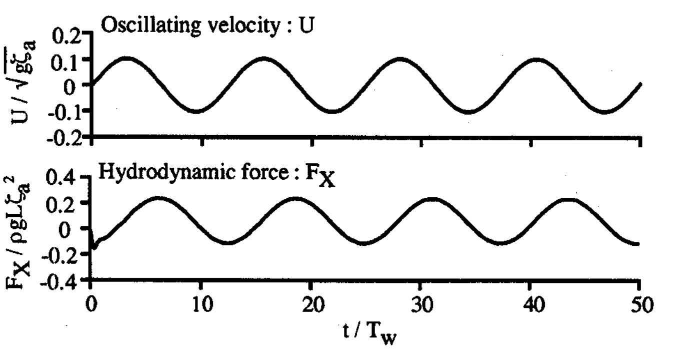
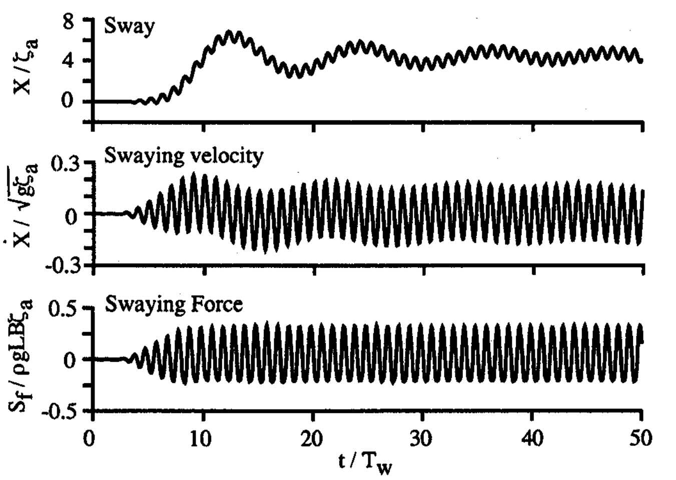
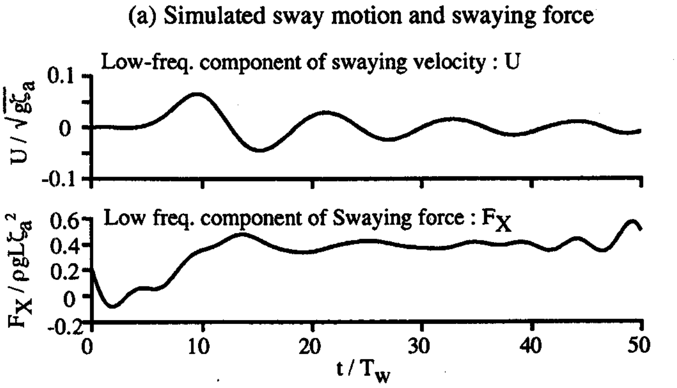


Fig.2 Simulated free oscillatory motion and hydrodynamic force in still water



(a) Simulated sway motion and swaying force



(b) Low-freq. components

Fig.3 Simulated free oscillatory motion and hydrodynamic force in the regular wave

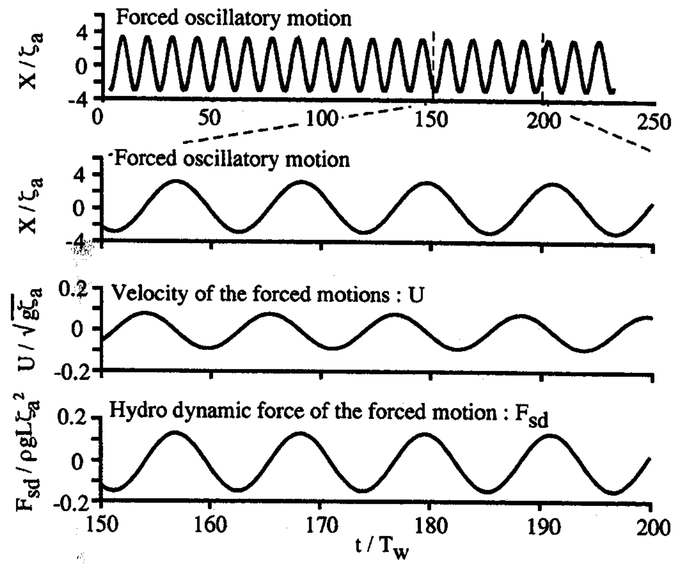


Fig.4 Simulated hydrodynamic force by forced oscillation in still water

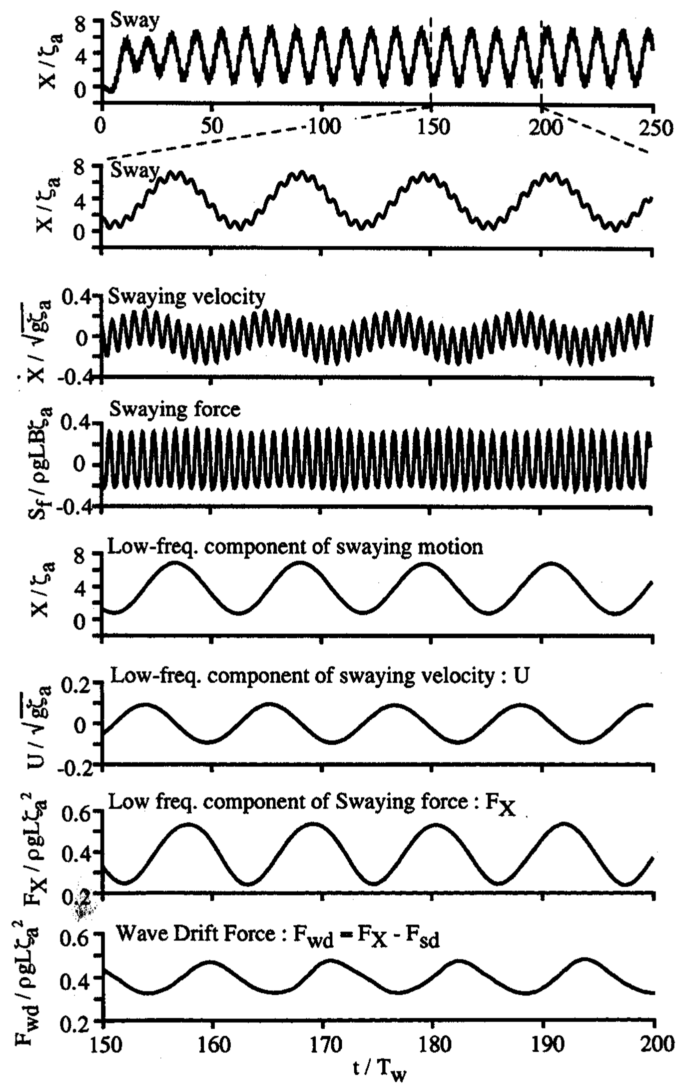


Fig.5 Simulated free oscillatory motion and hydrodynamic force excited by external low-frequency force in the regular wave

DISCUSSION

Molin B.:

1) It seems to me that your procedure to determine wave drift damping is a lot more complicated than what one usually does in a wave tank (i.e. decay tests in still water and regular waves). Can you comment why?

2) Have you made comparisons with published experimental or numerical results?

3) I want to comment that the often observed change in low frequency added mass has more to do with viscous effects than with potential effects.

Tanizawa K. + Naito S.:

1) For the estimation of the wave drift damping, free decay tests are not accurate enough because they are transient phenomena and affected by viscous force. What we usually do is low speed towing test in regular waves based on the conventional definition of the wave drift damping (i.e. the rate of change of the added resistance in waves with respect to small steady forward velocity). But, since this definition is derived from quasi-steady analysis, I proposed dynamic definition of wave drift damping and introduce series of fully nonlinear numerical simulations to determine the wave drift damping based on this dynamic definition.

2) No, not yet.

3) Thank you very much for you comment. Of course viscous effects may affect to the added mass change. What I explain in my talk is pure potential effect to the added mass change.

Newman J.N.: Your approach seems analogous to the rationale in my 1993 paper except that I used a perturbation expansion in powers of the wave amplitude (A) and you use a fully nonlinear simulation. I ignored the $O(A^2)$ added mass since it seemed unimportant compared to $O(1)$ conventional added mass. This suggests that your shift in the natural period of slow drift motions is $O(A^2)$. Do you have any results to confirm this?

Tanizawa K. + Naito S.: I simulated the slow drift motions for various wave heights and checked the dependency of the natural frequency to the wave amplitude. The simulated results in Table A show that the change of natural frequency is almost proportional to $O(A^2)$ and this can be a confirmation of your analysis. So, when wave amplitude is small, the change of frequency is not significant. But for larger amplitude wave, the change is not negligible.

Table A: Natural frequency of slow drift motion in regular waves

A/B	$\tilde{\omega}/\tilde{\omega}_0$
0.0	1.0
0.0169	1.0052
0.0338	0.9948
0.0507	1.0182
0.0676	1.0938
0.0845	1.1901

A : Wave amplitude

B : Body breadth 0.74 *m*

$\tilde{\omega}_0$: The natural frequency in still water 0.384 *rad/s*

λ : Wave lenghth 2.7 *m*