Impulsive diffraction by an array of three cylinders

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It has recently been discovered that trapped waves are present in an array of cylinders. Frequency-domain work reported by Maniar and Newman [3] and others, includes both analytical solutions and computational results. One of the important questions this phenomenon raises is how long it will take to build up a trapped wave or nearly trapped wave, and how important this will be in the generation of time series. It is also of interest to understand when the interaction between the cylinders occurs.

Impulsive-diffraction analysis by an array consisting of three cylinders has been performed to answer these questions. Results have been reported in the frequency-domain on arrays of the size of 100 cylinders using a B-spline methodology, but the computational expense using a planar, constant strength panel method in the time-domain has so far limited this study to three cylinders. However, the phenomenon found in the frequency-domain are recovered.

During work with arbitrary generalized modes in the time-domain, it was found that a wide variety of problems could be addressed [2]. Generalized modes were therefore used to study the diffraction by the three cylinders. The total potential Φ describing the flow satisfies Laplace's equation. The free surface condition is linearized and the body boundary condition is implied on the mean wetted surface of the global structure. The total potential is decomposed by

$$\Phi = \phi_I + \phi_S + \sum_{j=1}^{J} \phi_j \tag{1}$$

where the incident potential is ϕ_I, the scattered potential is ϕ_S and for all rigid body modes and deformation modes there is an associated radiation potential ϕ_j. If the number of bodies is N then J=6N.

J normal vectors are also defined, in an N body problem such that n_1 is zero on all other bodies except the first. The same is true for n_2 to n_6. The normal vectors n_7 to n_{12} are nonzero on the second body and so on. The diffraction force can then easily be obtained by

$$F_j = \int_{\bar{S}} (\phi_I + \phi_S) n_j \mathrm{dS} \tag{2}$$

where $\bar{S}$ is the mean wetted surface of the global body. The problem is solved using an integral formulation and a free-surface Green function as explained by Bingham et al. [1]. Giving the body an impulsive velocity in a mode, impulse-response functions for the influence on all modes are obtained.

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The global problem includes three truncated cylinders in infinite depth of radius and draft a. The separation distance between cylinder centroids is $2d = 4a$. The wave heading is parallel to the array. Each cylinder consists of 240 panels total.

Figure 1 presents the exciting force coefficient vs. nondimensional wavenumber for each of the three cylinders. The exciting force coefficients are found by Fourier transform of the diffraction impulse-response functions. The existence of trapped modes is evident. The results from the time-domain are compared to quantities produced by the frequency-domain code WAMIT, and the comparison confirms the method used. The peak occurring close to $Kd/\pi = 1/2$ is the Neumann trapped wave whereas the peak at $Kd/\pi \approx 1$ is the Dirichlet trapped wave. These names correspond to the boundary conditions for the trapped waves, for further explanation see [3]. Peaks for higher wavenumbers are present as well, all peaks will become sharper in a larger array and with larger draft for each cylinder.

The diffraction impulse-response functions for each of the three cylinders is presented in Figure 2. The response function is compared with the impulsive diffraction of a single cylinder at the same spatial location. To interpret the results it is important to understand that the impulsive wave is a delta function in time at $x = 0$, the same spatial location as the center of the second cylinder. As one would expect, it is found that no interaction is present before the wave-packet is close to the second cylinder. From the time when the majority of this wave-packet is coming close to the second cylinder the interaction is evident from Figure 2-a, where trapped waves are present for $t/(L/g)^{1/2} > 0$. As scattered waves of the second cylinder are becoming important, there is a rapid build-up of a nearly trapped wave-force acting on the first cylinder. The trapped wave has a slow decay rate and when the computation was stopped interaction effects could still be found. Figure 2-a indicates that it takes 4-5 wave periods to dissipate the energy associated with the trapped wave for this geometry.

Figure 2-b indicates that the interaction effects on the second cylinder take place earlier, as can be expected. The sheltering effect is easily seen for $t/(L/g)^{1/2} < 0$, whereas a trapped wave is seen for larger time. The magnitude of the wave is initially about the same as for cylinder one, but the decay rate is faster, of the order of 2-3 wave periods. This might be due to the interaction with the third cylinder, but results reported in the frequency-domain indicate that the separation distance is important as well. The third cylinder experiences a strong sheltering effect, and the trapped wave is not so clearly defined in Figure 2-c. The Fourier transform of the impulse-response function confirms this.

Applying generalized modes theory, the feasibility of computing impulsive diffraction in the time-domain for an array has been demonstrated, and it is shown that frequency-domain results can be reproduced. This gives confidence in the method. For the particular case studied we find that the trapping effect has a fast build-up, but the decay rate is different for each cylinder. This might be connected with the separation distance between the cylinders, and further studies should therefore include variation of the spatial separation. Further work will also be to study this problem in finite depth with bottom-mounted, rigid, cylinders. Interaction effects have been found to be strongest for this case, and by convolving an arbitrary wave-packet with the diffraction impulse-response function one will be able to study the duration of a nearly trapped wave in a random sea. In the generation of a time series this will be of importance.

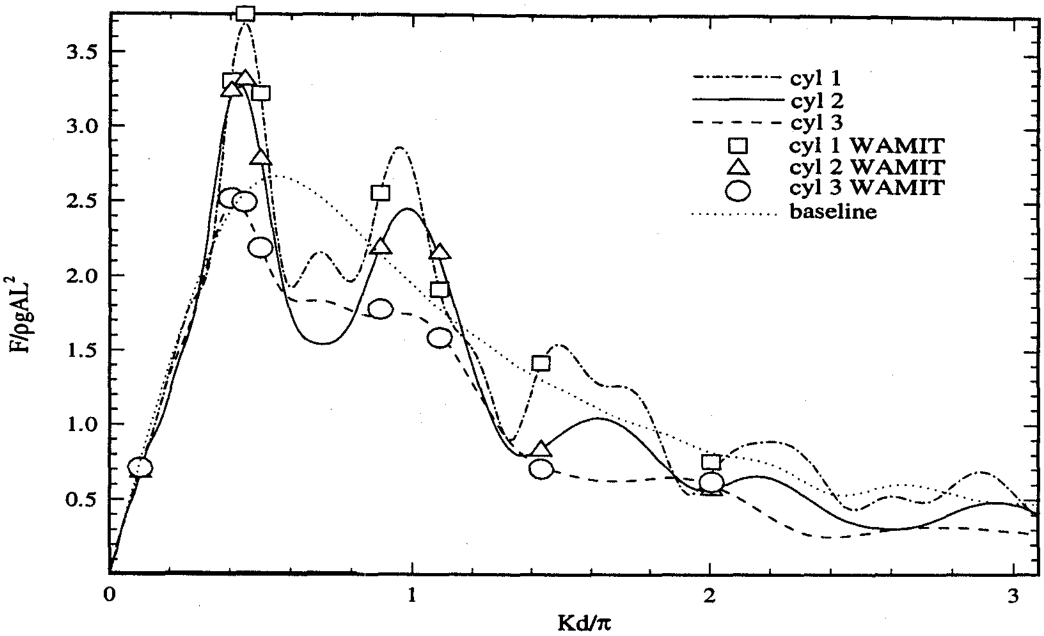

Figure 1: Magnitude of the exciting force coefficient in head seas for an array consisting of 3 truncated cylinders. Cylinder 1 is the first in the row. The separation distance is $2d = 4a$, where a is the radius. The baseline is the force on a single cylinder with no other bodies present.

This work was carried out at the Department of Ocean Engineering, Massachusetts Institute of Technology. Financial support has been received and is acknowledged from the Norway-America Association and the American-Scandinavian Foundation.

References

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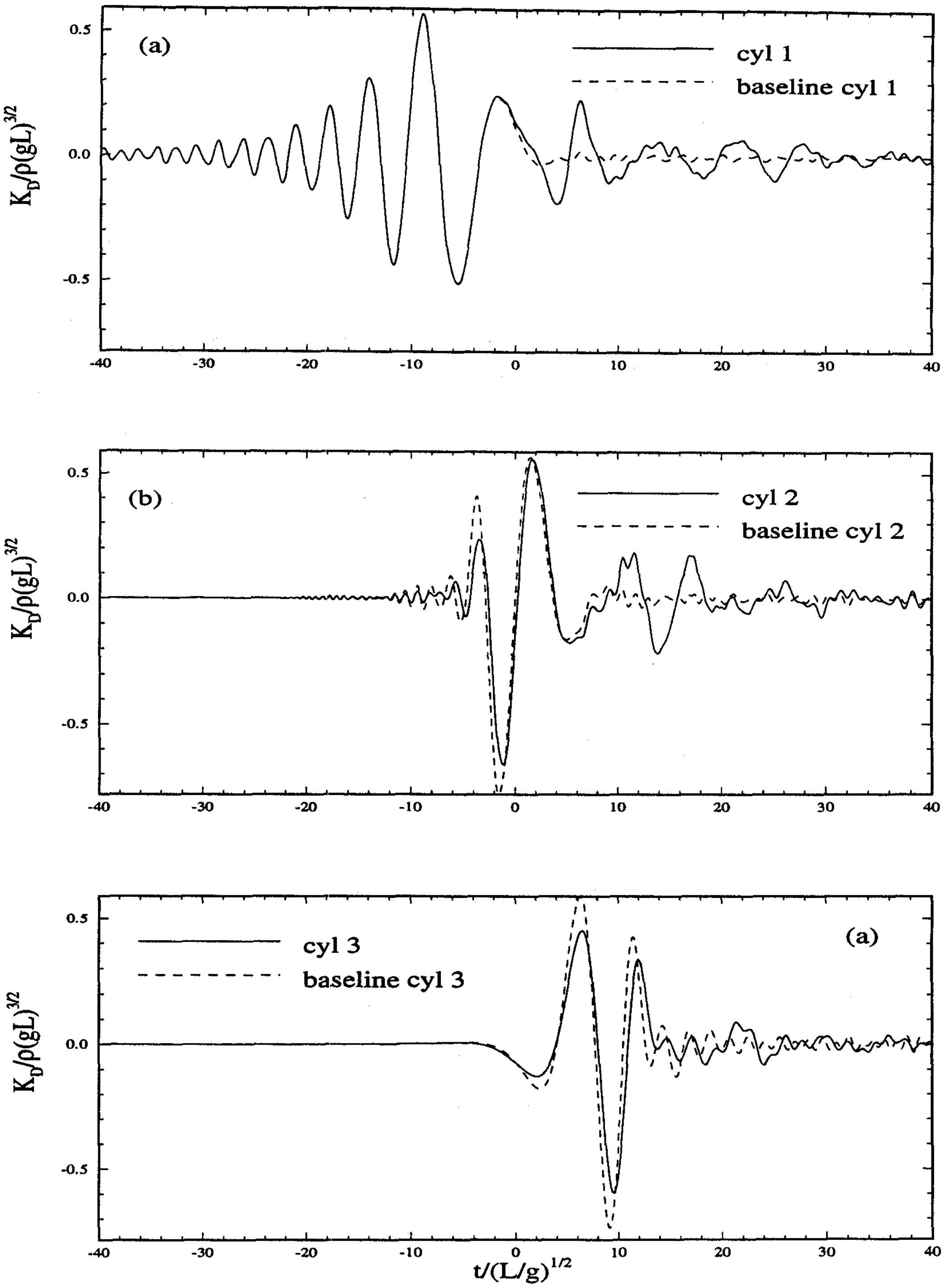

Figure 2: Memory function for the impulsive diffraction problem on an array consisting of three truncated cylinders.

DISCUSSION

Clément A.: Your results are for evenly spaced cylinders; what would happen to the trapped modes in the case of uneven spacing?

Farstad T.H.: I have not studied this problem, but I believe the resonance will disappear if the geometry of the problem is non-symmetric. This can be uneven spacing or cylinders with different diameters, for instance.

Eatock-Taylor R.: Have you encountered any numerical difficulties associated with the high frequency content in the impulsive wave?

Farstad T.H.: The formulation calculating the impulsive wave and performing the water line integral was developed by Bingham, Korsmeyer et. al [1]. The waterline integral is performed at a distance $d/2$ below the free surface, where d is the average height of the panels along the waterline. This attenuates the signal somewhat, and the high frequency problem is avoided.

Molin B.: You seem to hint that one could end up with different design values when using a time domain approach, as compared to the usual frequency domain one. If linearity is assumed, identical values are finally obtained.
On the other hand experiments on TLP like structures show quite different behavior in regular and irregular waves. In regular waves, quasi resonant sloshing motions of the free surface are observed at some frequencies leading to non-linear effects coming into play and ultimately breaking. In irregular waves these resonant sloshing motions get initiated in long wave groups at critical frequencies then disappears. So an aspect of the problem is how many waves it takes for the resonant state to be attained. In this respect your work is quite helpful.