3rd Order Wave Loads Using a Higher Order Panel Method

by

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In the past few years, there have been several attempts to predict the phenomenon of 'ringing'. The long wave approximation approach by Faltinsen *et al.* [1] (FNV hereafter), assumes that the wave amplitude A and the cylinder radius a are of the same order, and that both are small compared to the wavelength. Therefore potentials of the order A^2a and A^3 are comparable in magnitude and retained in the analysis. In this regime, an analytical solution is derived for an infinitely deep cylinder. Using the conventional perturbation expansion, Malenica and Molin [2] (M. & M. hereafter) calculate the third order sum frequency solution for a bottom mounted cylinder. Expressing the free surface Green function and the potential as a Fourier series, the third order solution is obtained. They find that their third order solution agrees with the third harmonic solution by FNV only when the wavenumber K is extremely small, in the range $Ka < 0.05$.

To investigate the disagreement between the solutions of M. & M. and FNV, an independent study of the third order problem is necessary. We present a preliminary investigation of the third order diffraction effects based on the conventional perturbation expansion. For the integral equation solution of the third order problem, we extend a B–spline based panel code (Maniar [4]). The method represents the potential on the body as B–spline expansions, and in our extension of the method we also approximate the potential on the free surface by B–spline expansions. The use of differentiable B–spline approximants for the potential on the free surface permit the evaluation of the first and second derivatives of the potential on the free surface in a straightforward manner. Further, an adaptive scheme is employed to ensure the accurate evaluation of the 'nearly singular' integrals of the Rankine singularity encountered at the free surface region close to the waterline. As a result, the panel method not only provides accurate pointwise values of the potential and its derivatives on the body surface, but also at the waterline and in its vicinity.

Since we consider a range where the fundamental wavenumber is relatively small, the far field free surface integration contribution to the integral equations forcing and the second order potential contribution to the third order forcing are negligible (see FNV). Hence, in our preliminary study we neglect both: the far field free surface integration contribution to the integral equations forcing, and the second order potential contribution to the forcing of the third order potential. Some results for regular waves are presented in this abstract. These results will be compared with the analytic solution of FNV and the numerical solution by M. & M..

The Formulation

We consider the nonlinear diffraction of waves incident on a fixed body in a fluid of infinite depth. The problem is formulated using the conventional perturbation expansion method, and in the following we only list some relevant details. The inhomogeneous free surface condition for the second order potential, $\phi^{(2)}$, is

$$-4K\phi^{(2)} + \phi_z^{(2)} = f^{(2)} = -\frac{i\omega}{g}\nabla\phi^{(1)}\cdot\nabla\phi^{(1)} + \frac{i\omega}{2g}\phi^{(1)}(-K\phi_z^{(1)} + \phi_{zz}^{(1)}), \tag{1}$$

where $\phi^{(1)}$ is the first order diffraction potential, ω the wave frequency ($K = \omega^2/g$), g the acceleration due to gravity, and $\phi_I^{(1)}$ the incident wave potential given by $\phi_I^{(1)} = \Re(gA/\omega)e^{(Kz-iKx+i\omega t)}$. As mentioned above since the second order potential is negligible, the free surface condition for the third order potential, $\phi^{(3)}$, is

$$\begin{aligned}-9K\phi^{(3)} + \phi_z^{(3)} = f^{(3)} = & -\frac{K}{g}\phi^{(1)}\nabla\phi^{(1)}\cdot\nabla\phi_z^{(1)} - \frac{1}{8g}\nabla\phi^{(1)}\cdot\nabla(\nabla\phi^{(1)}\cdot\nabla\phi^{(1)}) \\ & +\frac{1}{4g}(K\phi^{(1)}\phi_z^{(1)} + \frac{1}{2}\nabla\phi^{(1)}\cdot\nabla\phi^{(1)})(-K\phi_z^{(1)} + \phi_{zz}^{(1)}).\end{aligned} \tag{2}$$

Applying the Green theorem to the potential at each order, the boundary integral equations for the higher order potentials are

$$2\pi\phi^{(2)} + \iint_{S_b} \phi^{(2)} \frac{\partial G(\mathbf{x},\xi)}{\partial z} d\xi = \iint_{S_f} f^{(2)} G(\mathbf{x},\xi) d\xi, \tag{3}$$

$$2\pi\phi^{(3)} + \iint_{S_b} \phi^{(3)} \frac{\partial G(\mathbf{x},\xi)}{\partial z} d\xi = \iint_{S_f} f^{(3)} G(\mathbf{x},\xi) d\xi, \tag{4}$$

where the free surface Green function G satisfies the homogeneous form of the second and third order free surface condition respectively. On solving for the nonlinear potentials, the nonlinear forces are obtained by

$$F^{(2)} = -\rho \iint_{S_b} 2i\omega\phi^{(2)} \mathbf{n} dS, \tag{5}$$

$$F^{(3)} = -\rho \iint_{S_b} (3i\omega\phi^{(3)} + \nabla\phi^{(1)} \cdot \nabla\phi^{(2)}) \mathbf{n} dS. \tag{6}$$

Above, the contributions from the waterline integral have been neglected.

The Numerical Method

In the higher order panel method, the geometry and the unknown potential are approximated by tensor product B-spline expansions in the parametric space of the body surface. A semi–discrete Galerkin scheme is employed to discretize the integral equation, and set-up a linear system of equations for the coefficients of the potential B–spline representation. The use of B–splines allows nearly exact geometry description and lends the potential several degrees of continuity. The salient features of the panel method are: its ability to obtain accurate pointwise values of the potential and its derivatives, its computational efficiency and that it is less influenced by the effects of irregular frequencies. Further details of the basic numerical scheme can be found in Maniar [4].

With the use of the free surface Green function, the panel method only solves for the first order potential ($\phi^{(1)}$) on the body surface. This solution on the body surface in turn is used for the pointwise evaluation of $\phi^{(1)}$ on the free surface. On the free surface, besides $\phi^{(1)}$, equations (1)–(2) require the evaluation of its first and second derivatives. To evaluate these the following procedure is used: (a) The free surface within a circle (of arbitrary radius) surrounding the body waterline is divided into several regions ('patches'). (b) Over each patch, the pointwise values of the first order potential are 'fit' by a tensor product B–spline expansion using a linear least squares procedure. Similarly, the geometry of the region is also approximated by a B–spline expansion. (c) Using the free surface condition, Laplace's equation, and the parametric representations for $\phi^{(1)}$ and the patch geometry, the Cartesian first and second derivatives of $\phi^{(1)}$ can be obtained (see Bingham and Maniar [5]).

For a given distribution of $\phi^{(1)}$ on the free surface patch, numerical experimentation indicates that the accuracy of this procedure is mainly dependent on the accuracy of the pointwise evaluation of $\phi^{(1)}$ and the choice of the B–spline characteristics (the total number of splines used, their degree and their distribution over the region). While a loss of accuracy is inherent in obtaining derivatives by analytic differentiation of the polynomial representation of $\phi^{(1)}$, it can be reduced with a suitable refinement of the B–spline characteristics. Finally, since the above procedure results (locally) in a polynomial form for the free surface forcing, $f^{(i)}, i = 2, 3$, we write

$$\iint_{\Delta} f^{(i)} G dS = \sum_{m=0}^{\infty} \sum_{n=0}^{m} f^{(i)}_{mn} \iint u^m v^n G dS, \tag{7}$$

thereby permitting the use of adaptive schemes for efficient evaluation of the integrals of G, independent of the behaviour of $f^{(i)}$.

The Results

We present results for the diffraction forces on a truncated cylinder (radius $a = 1$; draft $T = 6$). In Table 1 we compare the second order horizontal force as computed by the higher order panel method (HIPAN3D) with those from WAMIT, a low order panel code. A cosine spaced discretization is used for the cylinder, and a uniform discretization is used for the free surface. The free surface is truncated at the specified radius r. In Table 2 we compare our truncated cylinder computations for the third order horizontal force due to the third order potential with the analytic result for an infinitely deep cylinder by FNV. For these computations, some long wave approximations have been invoked: only the second term in the right hand side of the free surface forcing (Equation (2)) is retained (the same as the third harmonic forcing term in FNV, the second term of the right hand side of Equation (3.3)), and the Green function used is, $G = 1/r + 1/r'$. The agreement with the analytic results for an inifnitely deep cylinder by FNV are within the expected numerical accuracy. Next, retaining all the terms in the free surface forcing (Equation (2)) and with the use of a complete free surface Green function, we list in Table 3 our computations for the third order horizontal force. Though the order of the magnitude is about the same for the solutions in table 2 and 3, the phase of the force is changed.

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References

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	HIPAN3D		WAMIT		
	$2*8*(8+3)$	$2*12*(12+3)$	$4*12*(12+4)$	$4*24*(24+4)$	$4*36*(36+4)$
Ka	$2*8*8$	$2*12*12$	$4*12*40$	$4*24*80$	$4*36*120$
0.025	-1.1039853E-02	-1.1024946E-02	-1.2070E-04	-1.4865E-04	-1.6010E-04
0.05	-2.2160457E-02	-2.2162402E-02	-2.9267E-03	-3.3063E-03	-3.4518E-03
0.10	-9.0046681E-02	-9.0047941E-02	-7.1836E-02	-7.5967E-02	-7.7314E-02
0.15	-0.3085249	-0.3085167	-0.2968289	-0.3107565	-0.3147438
0.20	-0.4603241	-0.4603089	-0.4282742	-0.4567858	-0.4647257
0.025	-5.3094198E-05	-5.4267752E-05	1.9245E-03	-2.4011E-04	-9.9521E-04
0.05	-1.5751609E-03	-1.5674086E-03	-1.2173E-03	-6.5922E-03	-8.4147E-03
0.10	-2.7421186E-02	-2.7411198E-02	-3.1477E-02	-4.6015E-02	-5.0610E-02
0.15	4.0208168E-02	4.0218882E-02	5.6256E-02	3.2118E-02	2.4726E-02
0.20	0.3796814	0.3796871	0.4243889	0.3946274	0.3853467

Table 1: The second order horizontal force as computed by a higher order (HIPAN3D) and a constant panel method (WAMIT): The top and bottom tables correspond to the real and imaginary components respectively. The second and third lines from top indicate the number of panels on the body and the free surface respectively. The free surface is truncated at a radius = $6a$ (a = cylinder radius).

Ka	FNV	$r = 3.5a$	$r = 6a$	$r = 9a$	$r = 12a$
0.025	-9.8175E-04	-8.7819E-04	-8.7215E-04	-8.6736E-04	-8.6618E-04
0.05	-3.9270E-03	-3.6001E-03	-3.6102E-03	-3.6113E-03	-3.6160E-03
0.10	-1.5708E-02	-1.5406E-02	-1.5829E-02	-1.6001E-02	-1.6060E-02
0.15	-3.5343E-02	-3.7211E-02	-3.8946E-02	-3.9379E-02	-3.9310E-02
0.20	-6.2832E-02	-6.9664E-02	-7.3216E-02	-7.3107E-02	-7.2047E-02
0.025	0.00	4.0698E-07	4.0176E-07	3.7701E-07	3.4805E-07
0.05	0.00	8.6219E-06	7.6003E-06	6.0725E-06	4.4471E-06
0.10	0.00	1.7865E-04	1.1012E-04	2.0479E-05	-6.2924E-05
0.15	0.00	8.3466E-04	9.5868E-05	-7.6492E-04	-1.4264E-03
0.20	0.00	1.6510E-03	-2.1785E-03	-5.9600E-03	-8.0909E-03

Table 2: The third order horizontal force: Comparsion between the computed and analytic results by FNV. The top and bottom tables correspond to the real and imaginary parts respectively. The computations are shown with the free surface truncated at different radii.

	real part				imaginary part			
Ka	$r = 3.5a$	$r = 6a$	$r = 9a$	$r = 12a$	$r = 3.5a$	$r = 6a$	$r = 9a$	$r = 12a$
0.025	-1.23E-03	-1.18E-03	-1.16E-03	-1.16E-03	1.53E-04	1.15E-04	7.39E-05	6.08E-05
0.05	-6.44E-03	-6.46E-03	-6.31E-03	-6.11E-03	3.54E-03	3.70E-03	3.80E-03	3.66E-03
0.10	-5.65E-03	1.01E-03	-3.14E-03	-3.35E-03	4.02E-02	3.74E-02	3.41E-02	3.90E-02
0.15	4.55E-02	3.60E-02	4.44E-02	3.03E-02	7.52E-02	5.65E-02	7.30E-02	6.28E-02
0.20	0.11E+00	7.45E-02	7.21E-02	9.38E-02	7.48E-02	9.31E-02	6.98E-02	6.64E-02

Table 3: The third order horizontal force: With the use of the complete free surface Green function. The computations are shown with the free surface truncated at different radii.