Computing the double-body m-terms using a B-spline based panel method *

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This abstract describes a technique for computing the double-body m-terms over a body using a panel method in which both the geometry and the potential are represented by B-splines of arbitrary order. The m-terms arise in linearizations of the exact potential-flow seakeeping problem for a body which is traveling at steady forward speed U through waves. We expect a linearized theory to be appropriate for the analysis of displacement ships at sea, or offshore platforms which, although bluff, tend to operate in low speed currents. In a co-ordinate system attached to the body, the total velocity potential may be written as

$$\Phi(\vec{x},t) = \bar{\Phi}(\vec{x}) + \phi(\vec{x},t), \tag{1}$$

where it is assumed that there is a large steady "base" flow characterized by $\bar{\Phi}(\vec{x})$, and an unsteady perturbation to this flow, denoted by $\phi(\vec{x},t)$. The perturbation flow describes the combination of diffracted incident waves and radiated waves due to the motions of the body. If the exact boundary-value problem for $\Phi(\vec{x},t)$ is linearized about the base flow potential (*e.g.* as in [5]), then the body boundary condition for a canonical impulsive radiation problem can be written

$$\vec{n}\cdot\nabla\phi_k = n_k\,\delta(t) + m_k\,h(t). \tag{2}$$

In Equation (2) $\delta(t)$ is the Dirac function, $h(t)$ the Heaviside step function, while

$$n_k = \left\{ \begin{array}{ll} \vec{n} & k=1,2,3 \\ \vec{x}\times\vec{n} & k=4,5,6 \end{array} \right\} \quad \text{and} \quad m_k = \left\{ \begin{array}{ll} -(\vec{n}\cdot\nabla)\vec{W} & k=1,2,3 \\ -(\vec{n}\cdot\nabla)(\vec{x}\times\vec{W}) & k=4,5,6 \end{array} \right\} \tag{3}$$

where $\vec{n}$ the unit normal vector to the body surface, and $\vec{W} = \nabla\bar{\Phi}$ are the components of fluid velocity due to the steady base flow. The simplest choice of base flow is an undisturbed stream, $\bar{\Phi} = -Ux$, and results in the Neumann-Kelvin linearization. The m-terms in this case reduce to $m_k = (0,0,0,0,Un_3,-Un_2)$.

Another possible choice of base flow is generally referred to as the "double-body" flow: the result of the submerged portion of the body, plus its reflection about the $z=0$ plane, traveling with speed U in an infinite fluid. To compute this potential we let $\bar{\Phi} = -Ux + \phi^{db}$ where $\phi^{db}\to 0$ at spatial infinity, $\phi^{db}_z = 0$ on the free-surface ($z=0$), and $\vec{n}\cdot\nabla\phi^{db} = Un_1$ on the submerged body surface $\bar{S}_b$. A Green function for this problem is

$$G^{(\infty)}(\vec{x},\vec{\xi}) = \frac{1}{r} + \frac{1}{r'}, \qquad \left.\begin{array}{c} r \\ r' \end{array}\right\} = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z\mp\zeta)^2}, \tag{4}$$

and by applying Green's theorem to ϕ^{db} and $G^{(\infty)}$ an integral equation for this potential can be written as

$$2\pi\phi^{db}(\vec{x}) + \int\!\!\int_{\bar{S}_b} d\vec{\xi}\;\phi^{db}(\vec{\xi})\,G^{(\infty)}_n(\vec{x},\vec{\xi}) = \int\!\!\int_{\bar{S}_b} d\vec{\xi}\;\phi^{db}_n(\vec{\xi})\,G^{(\infty)}(\vec{x},\vec{\xi}). \tag{5}$$

Equation (5) is solved using a B–spline based panel method as described in [4]. This method allows the body geometry to be modeled in a patch-wise fashion, where each patch is a parametric representation of the form

$$\vec{x}(u,v) = \sum_{m,n} \vec{x}_{mn}\;\tilde{U}_m(u)\;\tilde{V}_n(v). \tag{6}$$

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Here $\tilde{U}, \tilde{V}$ are B–splines of order k_g in parameters u, v respectively, and $\vec{x}_{mn}$ are known coefficients or vertices. Over the parametric space of each patch, the potential is approximated by

$$\phi(u,v) = \sum_{m,n} \phi_{mn}\; U_m(u)\; V_n(v) \quad , \tag{7}$$

where U, V are B–splines of order k_p (not necessarily the same as k_g) and ϕ_{mn} are the unknown coefficients to be solved for through Equation (5). (The superscript on ϕ^{db} has been dropped for brevity since only the double-body potential will be discussed in the following.) Here we note that with a suitable choice of the order of the splines (k_g, k_p), Equations (6)–(7) are continuous and differentiable with respect to the parameters (u, v) over each patch.

To obtain the Cartesian derivatives of the potential on the body, it is convenient to consider the gradient operator as the combination of a surface gradient and the derivative in the direction normal to the surface. Further, through a relation from differential geometry [2], the surface gradient can be expressed directly in terms of derivatives with respect to the (non-orthogonal) parameters u and v. Thus

$$\nabla\phi = \nabla_s\phi + \vec{n}\,\frac{\partial\phi}{\partial n}, \text{ where } \quad \nabla_s \equiv \frac{1}{H^2}\left[\vec{x}_u\left(G\frac{\partial}{\partial u} - F\frac{\partial}{\partial v}\right) + \vec{x}_v\left(E\frac{\partial}{\partial v} - F\frac{\partial}{\partial u}\right)\right] \quad , \tag{8}$$

and the subscripts indicate partial differentiation with respect to the parametric variables. In Equation (8), $H = \sqrt{EG - F^2}$, $\vec{n} = (\vec{x}_u \times \vec{x}_v)/H$, and E, F, G are the coefficients of the first fundamental form of the surface given by

$$E = \vec{x}_u \cdot \vec{x}_u, \quad F = \vec{x}_u \cdot \vec{x}_v, \quad G = \vec{x}_v \cdot \vec{x}_v \quad . \tag{9}$$

Operating on $\nabla\phi$ with the gradient operator in Equation (8) the second gradient matrix can similarly be written as

$$\nabla\nabla\phi \equiv \begin{bmatrix} \phi_{xx} & \phi_{xy} & \phi_{xz} \\ \phi_{yx} & \phi_{yy} & \phi_{yz} \\ \phi_{zx} & \phi_{zy} & \phi_{zz} \end{bmatrix} = \nabla_s\nabla\phi + \vec{n}\,\frac{\partial\nabla\phi}{\partial n},$$

or

$$\nabla\nabla\phi - \vec{n}\,(\vec{n}\cdot\nabla\nabla\phi) = \nabla_s\nabla\phi. \tag{10}$$

Equation (10) defines nine equations for six unknowns, once the symmetry of the matrix is exploited (*i.e.* $(\nabla\nabla\phi)_{ij} = (\nabla\nabla\phi)_{ji}$). The Laplace equation may be used to further reduce the number of unknowns by one, and by choosing the appropriate five from these equations, a solvable system can always be constructed (*i.e.* a linear system whose matrix has a non-zero determinant.) The right hand side of Equation (10) involves derivatives of $\phi(u, v)$ and the geometric quantities defined in Equation (9) with respect to the parametric variables only. Once the second derivatives of the potential have been computed, the m-terms readily follow.

A more elegant way of computing the m-terms however is to use the tensor identity (see [2])

$$m_k = -\frac{\partial}{\partial n}\nabla\bar{\Phi} = -\frac{\partial}{\partial n}\nabla\phi = -[\vec{n}\cdot^2\nabla_s\nabla\phi - \vec{n}\,\nabla_s\cdot\nabla\phi], \quad k = 1,2,3; \tag{11}$$

where the operation $\vec{n}\cdot^2\nabla_s\nabla\phi \equiv \sum_{i=1}^{3} n_i\nabla_s\nabla\phi_i$ (with n_i and $\nabla\phi_i$ the three components of $\vec{n}$ and $\nabla\phi$ respectively), and the surface divergence operator

$$\nabla_s\cdot \;\equiv \frac{1}{H^2}\left[\vec{x}_u\cdot\left(G\frac{\partial}{\partial u} - F\frac{\partial}{\partial v}\right) + \vec{x}_v\cdot\left(E\frac{\partial}{\partial v} - F\frac{\partial}{\partial u}\right)\right].$$

Equation (11) expresses the translational m-terms directly in terms of parametric derivatives of the double-body velocities $\nabla\phi$ and the geometry. Some manipulation of Equation (3) further allows the rotational m-terms to be written in terms of the steady velocities and the translational m-terms,

$$m_k = \vec{n} \times \vec{W} + \vec{x} \times \vec{m}, \quad k = 4, 5, 6; \tag{12}$$

where $\vec{m} = (m_1, m_2, m_3)$. Note that once the linear m-terms have been computed, Equation (10) may be written

$$\frac{\partial^2 \phi}{\partial x_i \partial x_j} = \vec{e}_i \cdot \nabla(\vec{e}_j \cdot \nabla\phi) = \vec{e}_i \cdot \nabla_s(\vec{e}_j \cdot \nabla\phi) + n_i m_j; \quad i, j = 1, 2, 3 \tag{13}$$

and used explicitly to obtain the Cartesian second derivatives. (Here $[\vec{e}_1, \vec{e}_2, \vec{e}_3]$ are the unit vectors directed along the Cartesian $[x, y, z]$ axes.)

As an example problem to investigate the accuracy of the method we consider the double-body flow around a floating hemisphere. Equations (11) and (12) have been used to obtain the results presented below. The geometric B–spline representation used in the computations has $k_g = 6$ with 36 panels, resulting in 64, 81, 100, 121 unknowns on one octant of the sphere as the order of the potential solution is increased from $k_p = 3$ to $k_p = 6$. The Gaussian integration scheme employed 5x5 nodes per panel, and the calculations were made using double-precision arithmetic. The chosen geometric representation of the sphere is accurate to six digits, with maximum errors on the order of 10^{-7} in the geometry $[\vec{x}(u, v)]$, the surface area and the volume. Table 1 shows the maximum and the average absolute errors in the double-body velocities for a sample of 144 points over the sphere, as the order of the potential solution is increased. Table 2 shows the corresponding errors in the m-terms. It should be noted that the rotational double-body m-terms on a sphere are identically zero, which may explain the behavior of the errors for these quantities. Maximum errors tend to occur

	$k_p = 3$		$k_p = 4$		$k_p = 5$		$k_p = 6$	
	max.	ave.	max.	ave.	max.	ave.	max.	ave.
W_1	.0009	.0003	.00006	.00003	5×10^{-6}	2×10^{-6}	5×10^{-6}	2×10^{-6}
W_2	.0003	.0001	.00007	.00003	8×10^{-6}	2×10^{-6}	7×10^{-6}	1×10^{-6}
W_3	.0003	.00008	.00007	.00002	6×10^{-6}	2×10^{-6}	5×10^{-6}	2×10^{-6}

Table 1: Absolute errors in the double-body velocities on a sphere for a fixed geometric representation as the order of the potential solution is increased.

	$k_p = 3$		$k_p = 4$		$k_p = 5$		$k_p = 6$	
	max.	ave.	max.	ave.	max.	ave.	max.	ave.
m_1	.05	.02	.002	.0005	.0005	.00007	.0006	.00007
m_2	.08	.03	.001	.0004	.0002	.00008	.0002	.00002
m_3	1.1	.1	.01	.001	.002	.0003	.008	.0001
m_4	.00007	6×10^{-6}	.00007	6×10^{-6}	.00007	6×10^{-6}	.00007	6×10^{-6}
m_5	.0002	.00001	.0002	.00001	.0002	.00001	.0002	.00001
m_6	.00003	3×10^{-6}	.00003	3×10^{-6}	.00003	3×10^{-6}	.00003	3×10^{-6}

Table 2: Absolute errors in the double-body m-terms on a sphere for a fixed geometric representation as the order of the potential solution is increased.

near the pole (on the z-axis for this discretization) where the parameterization is singular, and are most significant in the heave m-term m_3,although even these results are reasonably accurate with $k_p > 4$.

The B-spline solution discussed above is next used to compute the double-body m-terms on a Wigley hull. These are combined with a planar panel description of the geometry and used as input to the constant strength panel method TiMIT [1], in order to compute the linearized hydrodynamic response of the hull. Figure 1 compares the magnitudes of the computed heave and pitch motions of the hull using both Neumann-Kelvin and double-body m-terms. Experimental results of Journée [3] are also shown. Note that in all of these calculations the free-surface boundary condition is the Kelvin linearized condition, $\phi_{tt} - 2U\phi_{tx} + U^2\phi_{xx} + g\phi_z = 0$ [where ϕ is again used to represent the perturbation potential in Equation (1)], so that the double-body results are in fact a mixed linearization of the problem. The next step would be to satisfy the double-body linearized free-surface boundary condition by distributing panels over some portion of the free-surface. We might expect in general that $\phi^{db} \to 0$ rapidly with increasing distance away from the body, and so it is likely that only a small portion of the free-surface will need to be discretized. This step is left as future work.

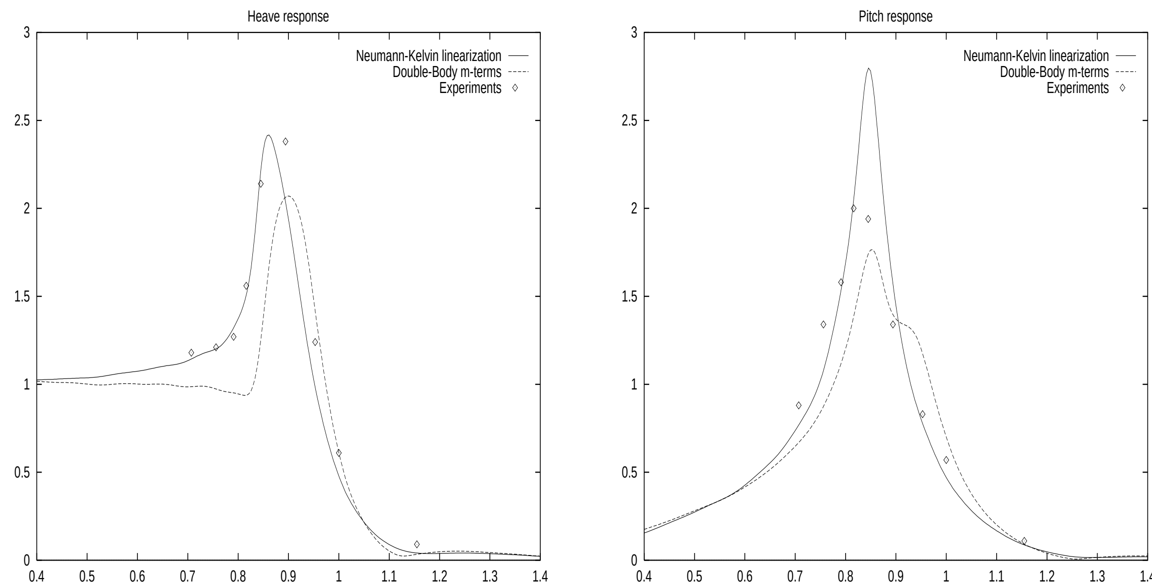

Figure 1: Magnitudes of the non-dimensional heave $(\frac{x_3}{A})$ and pitch $(\frac{x_5}{A})$ responses for a Wigley hull at $Fn = 0.3$, plotted against $(\frac{L}{\lambda})^{\frac{1}{2}}$. A is the wave amplitude, λ the wave length, and L the ship length.

References

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DISCUSSION

Kashiwagi: If we consistently retain the double-body flow effects in the unsteady pressure equation, we can have a term in proportion to the unsteady amplitude of motion, which gives the speed-dependent restoring force and may improve your results of the motion response calculation. Do you include that term in computing the motions?

Bingham & Maniar: No, we did not, but thank you for pointing it out. We will look into it.