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Runup on a Vertical Cylinder in Long Waves

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1. Introduction

The maximum elevation of the free surface at the intersection of a body, generally known as 'runup', is important from the standpoint of predicting the occurrence of wave impact on offshore platforms. In extreme conditions the runup can be affected significantly by nonlinear effects, and predictions based on linear theory generally underestimate the runup. Within the framework of the conventional perturbation expansion, results for second-order runup have been presented by several authors.

In extreme wave conditions, where the wave period and wavelength are relatively large, a different asymptotic approach can be followed along the lines of Faltinsen, Newman and Vinje (1994), referenced hereafter as 'FNV'. The velocity potential consists of the usual linear solution plus a nonlinear component ψ which is derived from a novel perturbation expansion with the fundamental assumptions that the wave amplitude A and the cylinder radius a are of the same order, and both are small compared to the incident wavelength. The nonlinear potential is forced by an inhomogeneous Neumann boundary condition on the oscillatory horizontal plane which moves up and down with the incident wave. The nonlinear potential ψ includes third order components proportional to A^2a and A^3. The forcing is significant only within the *inner* region where the distance from the cylinder and free surface is $O(a)$. The linear solution applies in the complementary *outer* region, where these distances are comparable to or large compared to the wavelength.

The work of FNV was motivated by observations of 'ringing' on platforms in extreme waves. In FNV the incident waves are regular, the cylinder is fixed, and it extends vertically throughout the fluid of infinite depth. The main emphasis is on the evaluation of the horizontal wave load and force, particularly the components which are of third order in the wave amplitude and oscillatory at the third harmonic of the fundamental frequency. This analysis has been extended to the case of irregular waves by Newman (1994), referenced hereafter as 'N94', and Faltinsen (1995) has derived the more general extension applicable to slender cylinders of varying radius. In the present work the wave runup is evaluated to third order, using the approach described above. This requires the computation of additional Fourier components of ψ which do not contribute to the horizontal force. In addition to this third-order extension a comparison is given below of the second-order runup with computations based on the conventional diffraction analysis.

2. The velocity potential

Following FNV, the velocity potential is derived in the form $\phi = \phi_D + \psi$, where $\phi_D = \phi_I + \phi_S$ is the linearized solution of the diffraction problem, including the incident and scattered components, and ψ is the nonlinear correction. Cartesian coordinates (x, y, z) are defined with $z = 0$ the plane of the undisturbed free surface, $z = \zeta$ the exact free surface, and $z < \zeta$ the fluid domain. Long-crested incident waves propagate in the $+x$-direction. In regular waves the corresponding linear potential is

$$\phi_I(x, z, t) = \mathrm{Re}\Big\{(gA/\omega)\exp(Kz - iKx + i\omega t)\Big\}. \tag{1}$$

Here A is the wave amplitude, g is the acceleration of gravity, ω is the radian frequency and $K = \omega^2/g$ is the wave number. Cylindrical coordinates (r, θ) are defined such that $x + iy = r^{i\theta}$. A fixed circular cylinder with radius $r = a$ is considered, and the boundary condition $\phi_r = 0$ is imposed on $r = a$. It is assumed that $KA = \epsilon << 1$, and that $A/a = O(1)$. Thus $Ka = O(\epsilon)$.

The solution can be extended to include long irregular waves following N94, if the incident wave is described more generally in terms of the potential φ, horizontal velocity u, and velocity gradient u_x, all evaluated on the axis $r = 0$. With these definitions the linearized solution for the diffraction potential can be expressed in the inner region $r = O(a)$ in the form

$$\begin{aligned}\phi_D =&\varphi + u(r + a^2/r)\cos\theta - \tfrac{1}{2}a^2 u_x(\log\tfrac{1}{2}Ka + \gamma) - \frac{\pi}{4}a^2 u_z \\ &+ \tfrac{1}{4}u_x\Big[r^2 + \cos 2\theta(r^2 + a^4/r^2) - 2a^2\log(r/a)\Big] + o(\epsilon^3).\end{aligned} \tag{2}$$

The principal boundary conditions for the nonlinear potential ψ are

$$\psi_r = 0, \quad \text{on}\, r = a, \tag{3}$$

and

$$\psi_{tt} + g\psi_z = -2\nabla\phi\cdot\nabla\phi_t - \tfrac{1}{2}\nabla\phi\cdot\nabla(\nabla\phi)^2, \quad \text{on}\, z = \zeta. \tag{4}$$

The leading-order contributions on the right side of (4), due to the first-order potential ϕ, are of order ϵ^2 in the inner domain and vanish to this order in the outer domain. Thus the nonlinear potential is forced only in the inner domain, and the boundary-value problem can be reformulated in terms of the inner coordinates $R = r/a$, $Z = (-z + \zeta_1)/a$, and solved for the inner potential $\Psi(R, \theta, Z) = \psi(r, \theta, z)$. The vertical coordinate is shifted so that the plane $Z = 0$ coincides with the elevation $z = \zeta_1$ of the first-order incident wave at the cylinder axis, and $Z > 0$ is the domain below this plane. The inner boundary conditions are

$$\Psi_R = 0, \quad \text{on}\, R = 1, \tag{5}$$

and, to leading order,

$$\Psi_Z = -2uu_t(a/g)\Big(\frac{2}{R^2}\cos 2\theta - \frac{1}{R^4}\Big) + (u^3/g)\Big[\frac{2}{R^3}\cos 3\theta + \Big(-\frac{4}{R^5} + \frac{2}{R^7}\Big)\cos\theta\Big], \quad \text{on}\, Z = 0. \tag{6}$$

The contribution from ψ_{tt} on the left side of (4) is neglected since it is of higher order. On the right side of (6) the term $-(u^2 + w^2)_t$ is neglected; this term is associated with the nonlinear correction to the incident-wave potential, vanishing in regular waves and oscillating with the difference frequency between spectral components in irregular waves (Newman, 1994).

In the conventional perturbation approach higher-order boundary conditions such as (4) and (6) would be transferred to the plane $z = 0$ using a Taylor expansion of the left-hand side; this procedure is not permissible here since the the potential ψ varies by $O(1)$ over distances of order a. Instead it is appropriate to satisfy (6) on the plane $Z = 0$, which corresponds to the first-order free-surface elevation in the inner region. The boundary-value problem for Ψ is completed by imposing Laplace's equation in the inner domain, and requiring that Ψ tends to zero when $(R^2 + Z^2)^{1/2} \to \infty$.

The right side of (6) suggests writing the solution in the form

$$\Psi(r, z, t) = \sum_{m=0}^{3} c_m(t)\Psi_m(R, Z)\cos m\theta, \tag{7}$$

where $c_0 = c_2 = (2a/g)uu_t$ and $c_1 = c_3 = (u^3/g)$. The functions Ψ_1 and Ψ_2 are evaluated in FNV using separation of variables with Weber transforms of the corresponding terms on the right side of (6) which can be evaluated in terms of Lommel functions. Ψ_0 and Ψ_3 can be evaluated in a similar manner. These solutions have maximum values at the free surface $Z = 0$, and decrease monotonically as Z increases. For the analysis of runup we only require the values of each function on the intersection of the cylinder $R = 1$ and the plane $Z = 0$. These have been computed by numerical integration with the following results:

$$\Psi_0 = -0.5755, \quad \Psi_1 = 0.8004, \quad \Psi_2 = 0.8091, \quad \Psi_3 = -0.4925. \tag{8}$$

3. The free-surface elevation

Next we evaluate the free-surface elevation $z = \zeta$, defined implicitly by the equation

$$\zeta = -(1/g)(\phi_t + \tfrac{1}{2}\nabla\phi \cdot \nabla\phi)_{z=\zeta}. \tag{9}$$

This can be expanded in the form

$$\zeta = \zeta_1 + \zeta_2 + \zeta_3 + \dots \tag{10}$$

where $\zeta_n = O(\epsilon^n)$ in the inner domain. Our intention here is to derive the first three terms on the right side of (10). For this purpose it is convenient to expand the velocity potential in the modified form

$$\phi = \phi_1 + \phi_2 + \phi_3 + \Psi \dots \tag{11},$$

where the first three terms on the right side of (11) correspond to the terms of the same order in the linearized potential (2). Except for the contributions from Ψ, the right side of (9) can be transferred to the plane $z = 0$ in the usual manner, with the following results:

$$\zeta_1 = -(1/g)\phi_{1t}, \tag{12}$$

$$\zeta_2 = -(1/g)(\phi_{2t} + \phi_{1tz}\zeta_1 + \tfrac{1}{2}\phi_{1z}^2 + \tfrac{1}{2}\phi_{2x}^2 + \tfrac{1}{2}\phi_{2y}^2), \tag{13}$$

$$\begin{aligned}\zeta_3 = -(1/g)(&\phi_{3t} + \phi_{2tz}\zeta_1 + \phi_{1tz}\zeta_2 + \tfrac{1}{2}\phi_{1tzz}\zeta_1^2 + \phi_{1z}\phi_{2z} \\ &+ \phi_{2x}\phi_{2xz}\zeta_1 + \phi_{2y}\phi_{2yz}\zeta_1 + \phi_{2x}\phi_{3x} + \phi_{2y}\phi_{3y} \\ &+ \Psi_t + \phi_{1z}\Psi_z + \phi_{2x}\Psi_x + \phi_{2y}\Psi_y).\end{aligned} \tag{14}$$

The derivatives of the potentials ϕ_j are evaluated on $z = 0$ and the derivatives of Ψ on $Z = 0$.

More explicit results can be derived in regular waves, where the first- and second-order components on the cylinder are

$$\zeta_1 = A\sin\omega t, \tag{15}$$

$$\zeta_2 = -2KAa\cos\theta\cos\omega t - \tfrac{1}{2}KA^2(\tfrac{1}{2} - \cos 2\theta) - \tfrac{1}{2}KA^2(\tfrac{1}{2} + \cos 2\theta)\cos 2\omega t, \tag{16}$$

The corresponding expression for ζ_3 is too long to reproduce here. It includes terms proportional to Aa^2, A^2a, A^3, and A^4/a, with harmonics up to order four. It may seem paradoxical that the third component in this sequence is nonzero when $a \to 0$, and the fourth component is singular in this limit, but the fundamental assumption $A/a = O(1)$ precludes applying these results in that limit. Similarly, the second-order result (16) differs in the last two terms from the second-order elevation of the incident wave, despite the fact that these terms are independent of a.

In Figure 1 we compare the second-order runup given by (16) with computations based on the conventional perturbation expansion using the three dimensional panel code WAMIT with a truncated cylinder of draft/radius=4. A total of 2096 panels are used on the cylinder and 4144 panels on the free surface. The conventional approach applies in the diffraction regime, with $Ka = O(1)$; the runup consists of two components, ζ_q due to quadratic interactions of the linear potential and ζ_p due to the second-order potential. In the long-wavelength approximation ζ_p is one order smaller that the ζ_q. The third-order runup (14) is not included here, but since Ψ and ζ_p are solutions of inhomogeneous free-surface conditions which have some common basis, part of the contribution from ζ_p may be recovered in ζ_3.

Figure 1 shows the second-order runup along the circumference of the cylinder for Ka =0.05, 0.1 and 0.15. The upper figures show the sum-frequency (second harmonic) component and the lower figures show the mean (zero harmonic) component of (16), and the comparison with the computations of ζ_p and ζ_q based on the diffraction analysis. In general the long-wavelength approximation agrees well with ζ_q. The sum-frequency component of ζ_p increases rapidly as Ka increases, and impairs the long-wavelength approximation of the sum-frequency runup for $Ka > 0.1$, particularly near the weather side. Only the real part of the sum-frequency components ζ_p and ζ_q are included for simplicity; the imaginary components are relatively small, with negligible influence on the modulus.

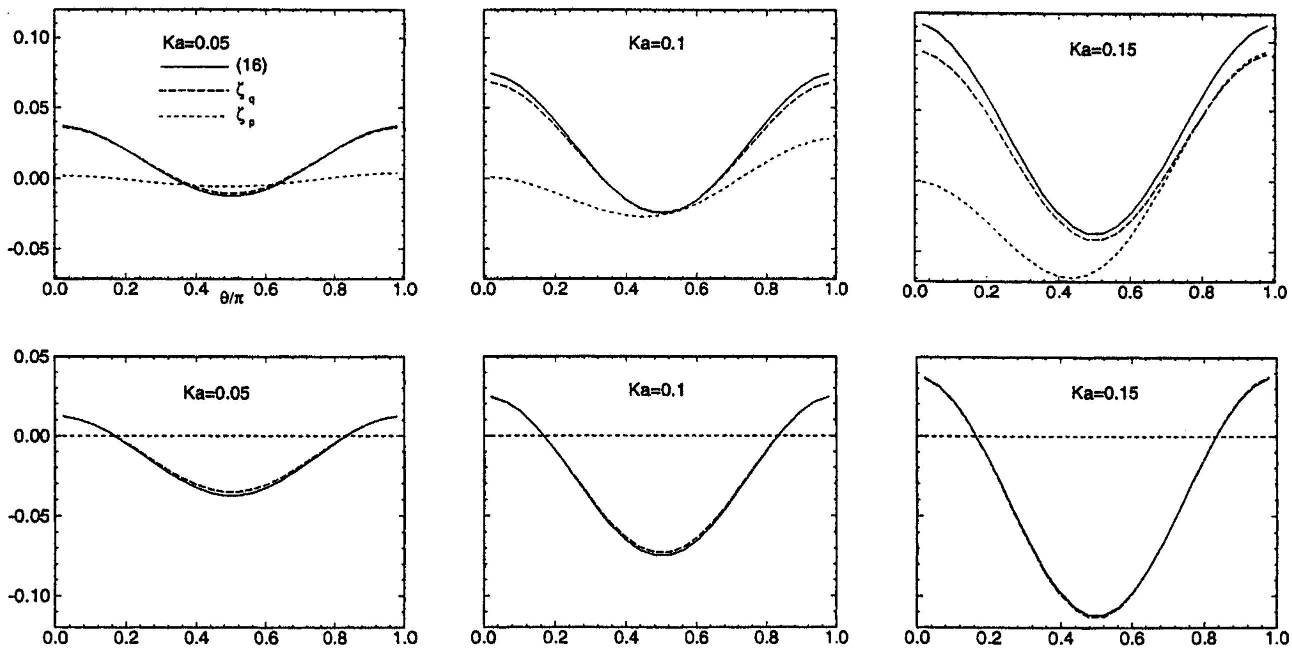

Figure 1 – Comparison of the second-order runup based on the long-wavelength approximation (16) and WAMIT computations. The horizontal axis represents increasing angle from the lee (0.0) to the weather (1.0) side. The upper (lower) figures are for the sum (difference) frequency components.

Acknowledgement

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References

Faltinsen, O.M., 1995 'Ringing loads on gravity based structures,' 10th Workshop on Water Waves and Floating Bodies, Oxford.

Faltinsen, O.M., Newman, J.N., and Vinje, T., 1994 (FNV) 'Nonlinear wave loads on a slender vertical cylinder,' *Jl. of Fluid Mech.*, in press.

Newman, J.N., 1994 (N94) 'Nonlinear scattering of long waves by a vertical cylinder,' Symposium on Waves and Nonlinear Processes in Hydrodynamics, in honour of Professor Enok Palm, Oslo.

DISCUSSION

Rainey, R. C. T.: I believe the position of my "distorted wavy lid" is equal, to second order in Stokes expansion, to your "second order wave elevation in the inner domain". Compare F.N.V. equations (3.12) and (3.13) with equations (2) and (14) in my paper in this workshop. Do you agree?

Newman, J. N. & Lee, C. H.: Strictly speaking, my reply depends on what is meant by "second order in Stokes expansion". The result in your (2) appears to correspond to the second term in our (16) (time average), but not to the third term (second harmonic). And the more complete "Stokes expansion" for the diffraction regime should include the additional contribution from the inhomogeneous free-surface condition, of order $(KA)^2a$.

Grue, J.: Experimental results[1] and strongly non-linear simulations [2], show clear depressions of the surface behind the cylinder on each side of the plane through the cylinder axis in the direction of the wave propagation, while the surface elevation right behind the cylinder along this plane shoots up. These features occur right after the wave crest has left the cylinder, and are pronounced for large amplitude of the incoming wave ($a/d \approx$ 0.5, 0.6) (a = wave amplitude, d = cylinder diameter). Have you obtained such results?

1. Grue, Bjøshol & Strand: *Nonlinear wave exciting loads on a vertical circular cylinder.* Pre-print, Univ. of Oslo, 1993.

2. Mehlum: *UNDA - simulations of non-linear waves.* Report in progress. Oslo 1995.

Newman, J. N. & Lee, C. H.: Our results are restricted to the run up on the cylinder ($r=a$). It should be relatively straightforward to evaluate ζ_2 for $r>a$, but ζ_3 would require numerical solutions for the nonlinear potential Ψ away from the cylinder.