Seakeeping Computations in Following Waves *

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A numerical solution to the transient, forward-speed, seakeeping problem is presented for the case of waves incident from abaft the beam. The solution technique implements a low-order panel method to compute the potential flow due to a freely floating ship traveling through waves (at steady forward speed U) in a semi-infinite fluid with a free surface. The ship's unsteady motions are assumed to be small excursions from its mean position, and the exact initial-boundary-value problem is linearized about an undisturbed free stream.

Based on the above assumptions the total velocity potential may be written $\Phi(\vec{x},t) = -Ux+\bar{\phi}(\vec{x})+\sum_{k=1}^{6}\phi_k(\vec{x},t)+\phi_I(\vec{x},t)+\phi_S(\vec{x},t)$, where the first-order disturbance of the free stream is decomposed into: a steady potential $\bar{\phi}$, six unsteady radiation potentials ϕ_k, and a scattering potential ϕ_S; while the incident wave is represented by ϕ_I. Each of these unknown perturbation potentials is computed by solving an integral equation on the submerged ship surface, with impulsive forcing. (A more complete description of the formulation and numerical techniques involved can be found in [1]).

Associated with each perturbation potential is a first-order pressure $p = -\rho(\phi_t - U\,\phi_x)$. Integrating these pressures over the ship gives a set of impulse-response functions which can be used to compute the motions of the ship while traveling through a specified (but otherwise arbitrary) wave field. Assuming the ship to be a stable linear system, the equations of motion may be written

$$\sum_{k=1}^{6}(M_{jk}+a_{jk})\ddot{x}_k + b_{jk}\dot{x}_k + (C_{jk}+c_{jk})x_k + \int_{-\infty}^{t} d\tau\, K_{jk}(t-\tau)\dot{x}_k(\tau) = \int_{-\infty}^{\infty} d\tau\, K_{jD}(t-\tau,\beta)\zeta(\tau) \quad (1)$$

where $j = 1,...,6$, and an overdot indicates differentiation with respect to time. The ship's inertia matrix is M_{jk}, and the first-order hydrostatic restoring-force coefficients are given by C_{jk}. Following [5], the exciting-force components appearing on the right hand side of (1) are expressed by means of convolution integrals with the incident wave elevation $\zeta(t)$ defined at a prescribed reference point in a coordinate system fixed to the mean position of the ship. ('Force' is understood in the generalized sense to include the moments, for $j = 4,5,6$.) The kernel $K_{jD}(t,\beta)$, is the ship's diffraction impulse-response function at a heading angle of β, which is measured from the positive x-axis ($\beta = 0°$ for stern seas, and $\beta = 180°$ for head seas.) The hydrodynamic coefficients a_{jk}, b_{jk}, and c_{jk}, and the kernel of the convolution K_{jk} comprise the radiation impulse-response functions.

If the unsteady ship motions and the incident wave elevation are assumed to be time harmonic at the frequency of encounter ω (*i.e.* $x_k(t) = \Re\{\xi_k(\omega,\beta)\,e^{i\omega t}\}$, $\zeta(t) = \mathcal{A}\,\Re\{e^{i\omega t}\}$), then as $t \to \infty$ the equations of motion become

$$\sum_{k=1}^{6}\left\{-\omega^2[M_{jk}+A_{jk}(\omega)] + i\omega\, B_{jk}(\omega) + C_{jk} + c_{jk}\right\}\frac{\xi_k(\omega,\beta)}{\mathcal{A}} = X_j(\omega,\beta). \quad (2)$$

The frequency-dependent coefficients in (2) are related to the impulse-response functions through the Fourier transforms

$$A_{jk}(\omega) = a_{jk} + \Re\left\{\frac{-i}{\omega}\int_0^{\infty} dt\, K_{jk}(t)e^{-i\omega t}\right\},\quad B_{jk}(\omega) = b_{jk} + \Im\left\{i\int_0^{\infty} dt\, K_{jk}(t)e^{-i\omega t}\right\},\quad (3)$$
$$X_j(\omega,\beta) = \int_{-\infty}^{\infty} dt\, K_{jD}(t,\beta)e^{-i\omega t},$$

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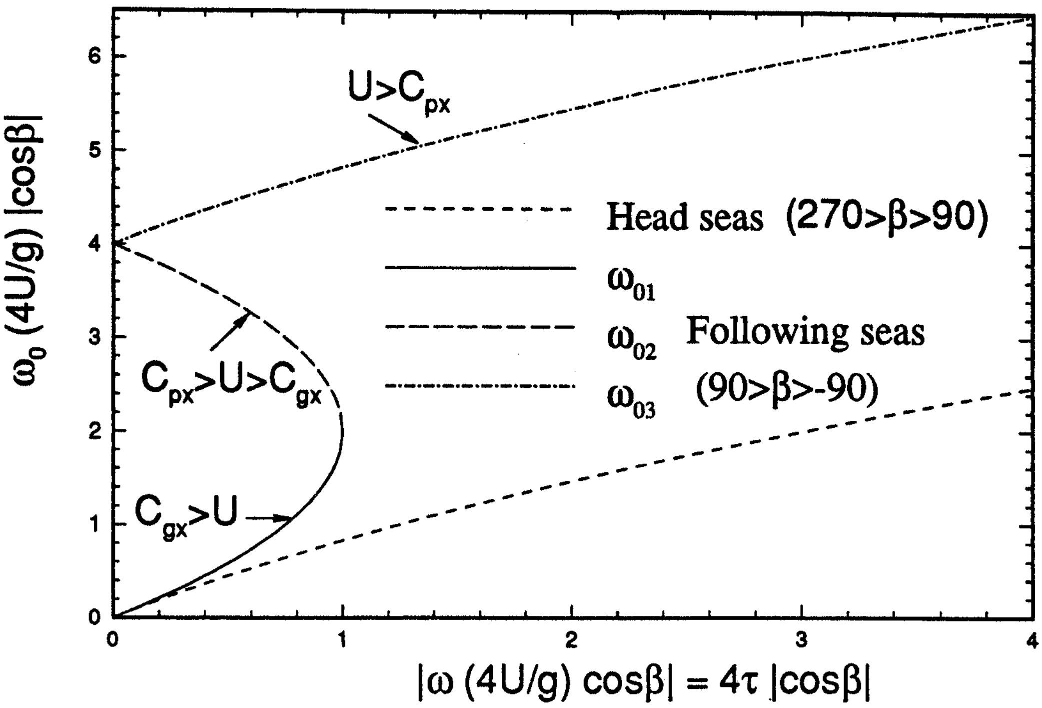

Figure 1: A plot of absolute frequency vs. encounter frequency for all $U > 0$ and $\beta \neq \pm 90°$.

and $\xi_k/\mathcal{A}$ is the response-amplitude operator (RAO).

In order to extend this technique to include waves which are incident from abaft the beam, consider the relationship between the frequency of encounter, ω, and the absolute wave frequency, ω_0: $\omega = \omega_0 - \omega_0^2 \frac{U \cos\beta}{g}$ or

$$\omega_0 = \frac{g}{2U\cos\beta}\left(1 \pm \sqrt{1 - \omega\frac{4U\cos\beta}{g}}\right) \qquad \text{for } \beta \neq \pm 90°, \tag{4}$$

($\omega = \omega_0$ when $\beta = \pm 90°$). Figure 1 shows a plot of equation (4) normalized so that all ship speeds and incident wave heading angles appear on the same two curves. For waves incident from ahead of the beam ($90° < \beta < 270°$), $\cos\beta < 0$ and $\omega \geq 0$, so there is only one root to (4) giving a unique relationship between the two frequencies. For waves incident from abaft the beam ($-90° < \beta < 90°$), $\cos\beta > 0$ and there can be as many as three waves producing the same frequency of encounter.

For following waves, the absolute frequency spectrum may be split into three domains so that overlapping values of encounter frequency may be distinguished. The first domain covers $0 \leq \omega_{01} \leq g/2U\cos\beta$, and corresponds to long waves with $C_{gx} > U$, where C_{gx} is the component of wave group velocity in the x-direction. The second domain covers $g/2U\cos\beta \leq \omega_{02} \leq g/U\cos\beta$, and corresponds to intermediate waves with $C_{px} > U > C_{gx}$, where C_{px} is the component of wave phase velocity in the x-direction. The last domain covers $g/U\cos\beta \leq \omega_{03} < \infty$, corresponding to short waves with $U > C_{px}$. (This last domain actually produces a negative frequency of encounter which simply results in a change in the phase by 180°.) Since the problem is solved in the ship-fixed reference frame, the relevant frequency is the encounter frequency, and these three domains of absolute frequencies must be considered separately.

As in [5], the impulsive incident wave potential is written as an integral over all absolute wave frequencies. For waves incident from abaft the beam, this potential is split into three parts, each of which contains frequencies from only one of the three domains discussed above.

$$\begin{aligned}\phi_I(\vec{x},t) &= \textstyle\sum_{m=1}^{3}\phi_{Im} \\ &= \sum_{m=1}^{3}\Re\left\{\frac{ig}{\pi}\int d\omega_{0m}\,\frac{1}{\omega_{0m}}\left(1-2\omega_{0m}\frac{U\cos\beta}{g}\right)e^{\frac{\omega_{0m}^2}{g}\{z-i[(x+Ut)\cos\beta+y\sin\beta]\}}\,e^{i\omega_{0m}t}\right\}.\end{aligned} \tag{5}$$

The first spatial and temporal derivatives of these potentials can be related to the complex error function [4], and computed using algorithms found in [3]. Figure 2 illustrates the incident-wave elevations for each of the three parts of the impulsive incident wave for the case of $Fn = U/\sqrt{gL} = 0.3$ and $\beta = 0°$. In this figure the ship's waterline runs roughly from -1 to 1 along the x-axis. These three incident waves are used to solve three diffraction problems, resulting in three psuedo-impulse-response functions for a single wave heading. Consequently the expression for the exciting forces appearing in (1) becomes the sum of three separate convolutions

$$F_j(t) = \sum_{m=1}^{3}\int_{-\infty}^{\infty} K_{jDm}(t-\tau)\zeta_m(\tau)d\tau. \tag{6}$$

Here the actual incident wave elevation record must also be decomposed into three incident wave records, each of which contains frequencies from only one of the three domains. Given an arbitrary time history of incident wave elevation at some point in an earth-fixed coordinate system, $\zeta_0(t)$, the necessary decomposition into $\zeta_m(t)$ can be made using

$$\zeta_m(t) = \Re\left\{\frac{1}{\pi}\int d\omega_{0m}\,\tilde{\zeta}_0(\omega_{0m})\,e^{i\omega_{0m}\left(1-\frac{\omega_{0m}U\cos\beta}{g}\right)t}\right\}, \quad m = 1,2,3; \tag{7}$$

where $\tilde{\zeta}_0(\omega_0)$ is the Fourier transform of $\zeta_0(t)$ with respect to absolute wave frequency. Equation (7) is equivalent to the transformation presented in [5], but is computationally more convenient since it contains no singularities.

Figure 3 shows computed results for the magnitudes of the heave and pitch RAO's for an SL-7 hull at Froude number of 0.3 encountering waves from directly astern. The calculations are compared to model test experiments presented in [2].

References

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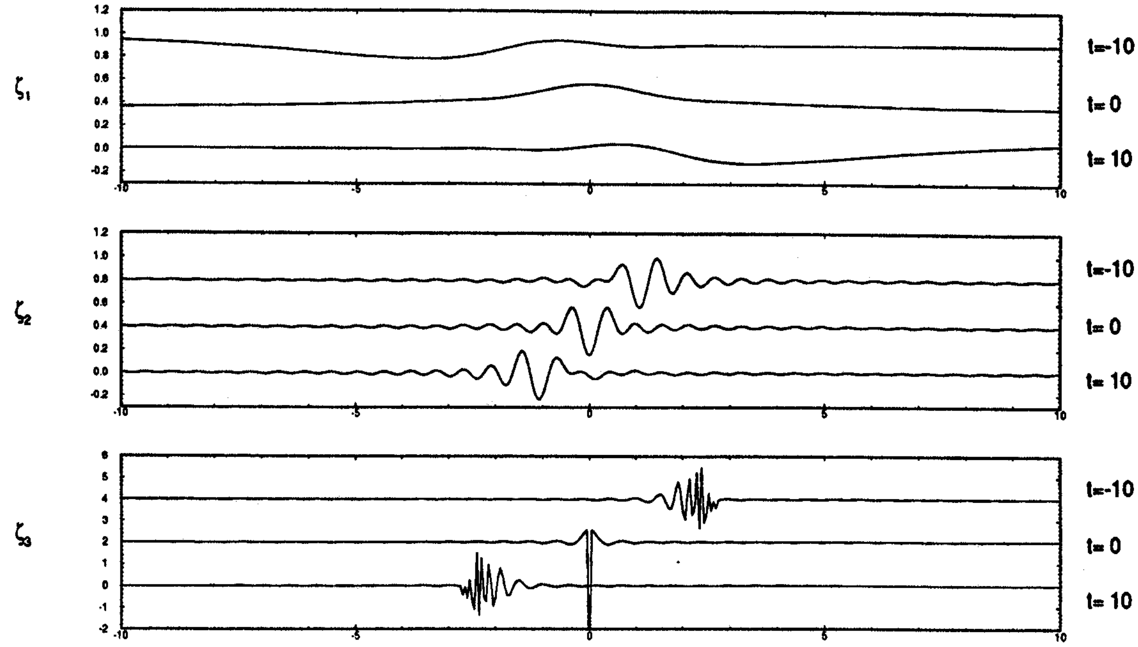

Figure 2: The three pieces of the impulsive incident wave elevation shown in a cut along the x-axis. The waves are incident from $\beta = 0°$, and evaluated at $y = 0$, $z = -.01$ for three different points in time (shifted vertically to show the progression in time). The coordinate system is fixed to the ship which is traveling at $Fn = 0.3$ in the positive x direction (time is made non-dimensional by $(g/L)^{\frac{1}{2}}$ and distance by the ship length L.)

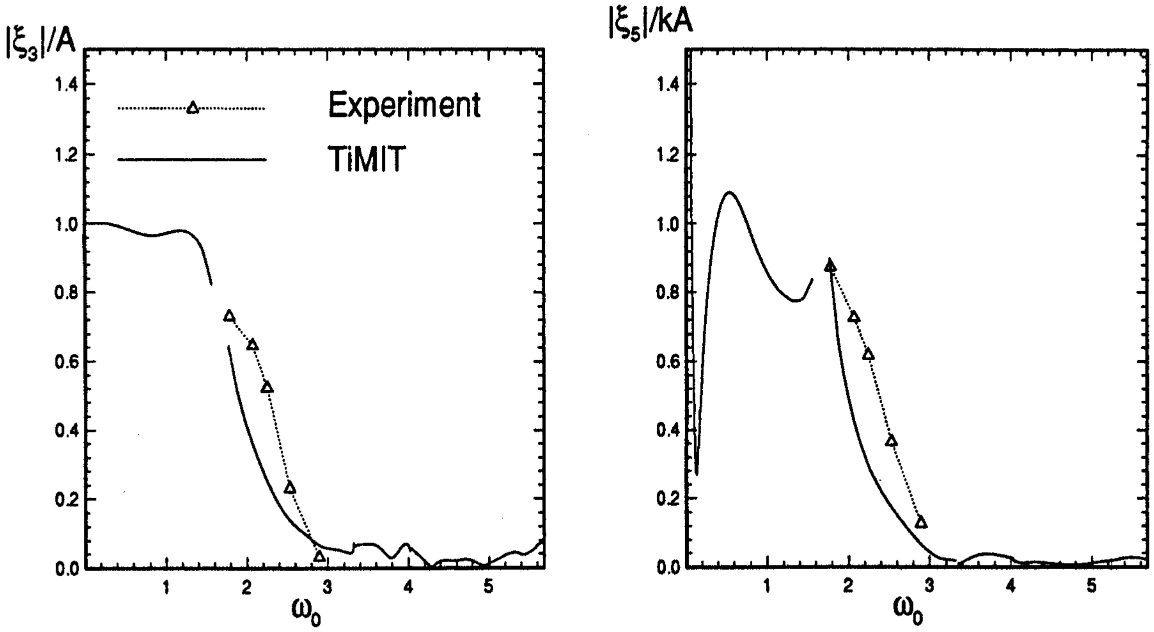

Figure 3: Magnitude of the RAO's for an SL-7 hull at $Fn = 0.3$, $\beta = 0°$.