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A New Slender-Ship Theory with Arbitrary Forward Speed and Oscillation Frequency

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1. Introduction

In the context of forward-speed slender-ship theories, there are at hand two distinctive theories of practical utility: the unified theory and the high-speed slender-body theory.

The unified theory was proposed by Newman[1], and can be applied irrespective of the order of oscillation frequency. However, since the free-surface condition in the inner problem is of zero speed, the order of forward-speed must be less than $O(1)$.

On the other hand, the high-speed slender-body theory initiated by Chapman[2] satisfies the speed-dependent free-surface condition in the inner problem. The resulting solution includes only the downstream influences and thus the frequency must be high, but the forward speed can be arbitrary.

The theory developed in this paper embraces both of the unified theory and the high-speed slender-body theory, thus being consistent for all the frequencies and forward speeds. The near-field velocity potential includes a particular solution which is the same as that of the high-speed slender-body theory, plus a homogeneous component which is constructed by the superposition of the real-flow potential and the reverse-flow reverse-time potential. The unknown coefficient of the homogeneous component is determined from the matching with the outer solution, which makes the present theory different from Yeung & Kim's[3], developed with the same objective as the present paper.

2. Formulation

Consider a ship advancing with constant speed U and undergoing small-amplitude motions of circular frequency ω in deep water. For simplicity, only the radiation problem of heave and pitch are considered here. The x-axis of the adopted coordinate system is positive in the direction of the forward motion and the z-axis positive downward.

Assuming the flow inviscid with irrotational motion, we introduce the velocity potential which is expressed as

$$\Phi = U\Big[-x + \varphi_S(x,y,z)\Big] + \mathrm{Re}\Big[i\omega \sum_{j=3,5} X_j\,\phi_j(x,y,z)\,e^{i\omega t}\Big] \tag{1}$$

Here φ_S denotes the steady-state disturbance potential, ϕ_j the unsteady potential of the j-th mode with unit velocity, and X_j is the complex amplitude of oscillation.

The unsteady potentials, ϕ_j ($j = 3,5$), are governed by the 3-D Laplace equation and subject to the linearized free-surface boundary condition:

$$\nabla^2_{3D}\,\phi_j = 0 \tag{2}$$

$$\Big(i\omega - U\frac{\partial}{\partial x}\Big)^2\phi_j - g\frac{\partial \phi_j}{\partial z} + \mu\Big(i\omega - U\frac{\partial}{\partial x}\Big)\phi_j = 0 \quad \text{on } z = 0 \tag{3}$$

Here μ in (3) is Rayleigh's artificial viscosity coefficient ensuring that the radiation condition is satisfied far from the ship.

When a ship is viewed from the far field, the ship looks like a line along the x-axis. Therefore the body boundary condition can not be imposed and the flow field is governed by (2) and (3).

On the other hand, in the near field close to the ship hull, the body boundary condition must be satisfied. Instead, however, the radiation condition need not be imposed, since the inner observer is not concerned about the far-field asymptotic behavior of the flow.

Therefore the inner problem is defined as follows:

$$\nabla^2_{2D}\,\phi_j = 0 \tag{4}$$

$$\left(i\omega - U\frac{\partial}{\partial x}\right)^2 \phi_j - g\frac{\partial \phi_j}{\partial z} = 0 \quad \text{on } z = 0 \tag{5}$$

$$\frac{\partial \phi_j}{\partial N} = N_j + \frac{U}{i\omega} M_j \quad \text{on } S_H \tag{6}$$

where N is the 2-D unit normal on the contour of transverse sections (positive when pointing out of the ship hull), N_j is its component, and M_j is the slender-body approximation of the so-called m-term introduced by Timman & Newman[4].

3. 2-D Green function

In order to construct the inner solution, let us consider the Green function associated with the inner problem (4)-(6), satisfying the followings:

$$\nabla^2_{2D}\, G^{\pm}(x-\xi; y, z; \eta, \zeta) = \delta(x-\xi)\delta(y-\eta)\delta(z-\zeta) \tag{7}$$

$$\left(i\omega - U\frac{\partial}{\partial x}\right)^2 G^{\pm} - g\frac{\partial G^{\pm}}{\partial z} \pm \mu\left(i\omega - U\frac{\partial}{\partial x}\right) G^{\pm} = 0 \quad \text{on } z = 0 \tag{8}$$

where δ is Dirac's delta function, and the superscripts '$\pm$' are taken according to the sign in front of μ in (8).

As is clear from (3), the real-flow solution satisfying the proper radiation condition should be the '+' case. However in the inner problem the '$-$' case is also allowed owing to the absence of the radiation condition. Physically, this '$-$' case corresponds to a reverse-flow and reverse-time problem.

The present Green function may easily be obtained by means of the Fourier transform. The final result can be expressed in the form[3]

$$G^{\pm}(x-\xi; y, z; \eta, \zeta) = \frac{\delta(x-\xi)}{2\pi}\ln\frac{R}{R_1} + G_p^{\pm}(x-\xi; |y-\eta|, z+\zeta) \tag{9}$$

$$\begin{aligned} G_p^{\pm *}(k; y, z) &= \int_{-\infty}^{\infty} G_p^{\pm}(x, y, z)\, e^{ikx}\, dx \\ &= -\frac{1}{\pi}\mathrm{Re}\Big[e^{-\nu(z+i|y|)} E_1\{-\nu(z+i|y|)\} \Big] \pm i\,\epsilon_k\, e^{-\nu(z \pm i\epsilon_k |y|)} \end{aligned} \tag{10}$$

where

$$R,\, R_1 = \sqrt{(y-\eta)^2 + (z\mp\zeta)^2}\,, \quad \nu = (\omega + kU)^2/g\,, \quad \epsilon_k = \mathrm{sgn}(\omega + kU) \tag{11}$$

$E_1(u)$ in (10) is the exponential integral with complex argument.

By using known expansions of the exponential integral, the asymptotic properties of the Green function can be readily obtained for large and small values of νR, with the result

$$G_p^{\pm *}(k; y, z) = \frac{\cos\theta}{\pi\nu R} \pm i\,\epsilon_k\, e^{-\nu(z \pm i\epsilon_k |y|)} + O((\nu R)^{-2}) \qquad \text{for } \nu R \gg 1 \tag{12}$$

$$\begin{aligned} G_p^{\pm *}(k; y, z) = \frac{1}{\pi}(1-\nu z)\Big(\ln \nu R + \gamma \pm \pi i \epsilon_k\Big) + \frac{1}{\pi}\nu R(\cos\theta + \theta\sin\theta) \\ + O(\nu^2 R^2) \qquad \text{for } \nu R \ll 1 \end{aligned} \tag{13}$$

where $z = R\cos\theta$, $|y| = R\sin\theta$, and γ is Euler's constant.

We can see from (10) that the following relation holds:

$$G_p^{-*}(k; y, z) = G_p^{+*}(k; y, z) - 2i\,\epsilon_k\, e^{-\nu z}\cos(\nu y) \tag{14}$$

4. 3-D Green function

The outer solution will be constructed in terms of the 3-D Green function satisfying the free-surface condition (3). This Green function has been well studied in the past slender-body theories. By referring to the analysis of Newman[1] or Yeung & Kim[3], the Fourier transform of the Green function can be expanded for large and small values of νR as follows:

$$G^*_{3D}(k;y,z) = \frac{\cos\theta}{\pi\nu R} + i\,\epsilon_k\, e^{-\nu(z+i\epsilon_k|y|)} + O(\,(\nu R)^{-2},\ k^2/\nu^2,\ k^2 y/\nu\,) \quad \text{for } \nu R \gg 1 \tag{15}$$

$$G^*_{3D}(k;y,z) = \frac{1}{\pi}(1-\nu z)\Big(\ln\frac{|k|R}{2}+\gamma\Big) + \frac{1}{\pi}\nu R(\cos\theta+\theta\sin\theta) + \frac{1}{\pi}(1-\nu z)\left[\begin{array}{c}\dfrac{1}{\sqrt{1-k^2/\nu^2}}\Big\{\pi i\epsilon_k + \cosh^{-1}\Big(\dfrac{\nu}{|k|}\Big)\Big\} \\ \dfrac{1}{\sqrt{k^2/\nu^2-1}}\Big\{-\pi+\cos^{-1}\Big(\dfrac{\nu}{|k|}\Big)\Big\}\end{array}\right] + O(\nu^2R^2,\, k^2R^2) \quad \text{for } \nu R \ll 1 \tag{16}$$

Here the top and bottom expression in brackets corresponds to $\nu > |k|$ and $\nu < |k|$ respectively.

Comparison of (15) and (12) tells us that, for $\nu R \gg 1$, the 2-D Green function for the real flow, G_p^{+*}, is compatible with the 3-D Green function, G^*_{3D}. Another comparison between (16) and (13) gives the relation

$$G^*_{3D}(k;y,z) = G_p^{+*}(k;y,z) - \frac{1}{\pi}(1-\nu z)\,g^*(k) + O(\nu^2R^2,\, k^2R^2) \tag{17}$$

where

$$g^*(k) = \ln\frac{2\nu}{|k|} + \pi i\,\epsilon_k - \left[\begin{array}{c}\dfrac{1}{\sqrt{1-k^2/\nu^2}}\Big\{\pi i\epsilon_k + \cosh^{-1}\Big(\dfrac{\nu}{|k|}\Big)\Big\} \\ \dfrac{1}{\sqrt{k^2/\nu^2-1}}\Big\{-\pi+\cos^{-1}\Big(\dfrac{\nu}{|k|}\Big)\Big\}\end{array}\right] \tag{18}$$

It should be noted in (18) that $g^*(k)$ tends to zero for $\nu \gg |k|$, meaning that (17) is a general expression for arbitrary values of νR. Since it is known that the 2-D Green function G_p^+ represents only the divergent wave system, $g^*(k)$ is a correction term associated with the transverse wave system in the genuine 3-D wave field.

5. Solutions and matching

According to the concept of Newman's unified theory, the inner solution can be of the form of a particular solution plus a homogeneous component. In the present case, the particular solution is the same as the high-speed slender-body-theory solution which satisfies the extraneous radiation condition.

Since the appropriate Green function for the real flow, G^+, is given by (9), the particular solution may be easily obtained by the boundary integral-equation method. (This solution will be denoted by ϕ_j^+ to mean the 'real-flow' potential.)

Turning to the homogeneous solution, we introduce the reverse-flow reverse-time potential, ϕ_j^-, where the signs of U and ω are reversed in the formulation. In view of (4)–(6), we can take the following as a homogeneous component

$$\phi_H(x;y,z) = \phi_3^+(x;y,z) - \phi_3^-(x;y,z) \tag{19}$$

With this component, the general inner solution can be written in the form

$$\phi_j^{(i)}(x;y,z) = \phi_j^+(x;y,z) + \int_{-\infty}^{\infty} C_j(\xi)\,\phi_H(x-\xi;y,z)\,d\xi \tag{20}$$

where $C_j(\xi)$ is the unknown coefficient, representing physically 3-D interactions among transverse sections along the ship's length.

This unknown can be determined using the matched asymptotic expansion method. For that purpose, we consider the outer expansion of (20) in the Fourier domain.

Applying the Fourier transform to the particular solution ϕ_j^+ and then considering its asymptotic expansion for large y and z, we can write

$$\phi_j^{+*}(k;y,z) \sim H_j^+(k)\,G_p^{+*}(k;y,z) \tag{21}$$

where

$$H_j^+(k) = \int_{-L/2}^{L/2} d\xi\, e^{ik\xi} \int_{B(\xi)} \Big\{\frac{\partial\phi_j^+(\xi;Q)}{\partial n} - \phi_j^+(\xi;Q)\frac{\partial}{\partial n}\Big\} e^{-\nu\zeta}\cos(\nu\eta)\, d\ell(\eta,\zeta) \tag{22}$$

Here (22) is the Kochin function, the definition of which must include the contribution from the so-called line-integral term, but here that contribution is omitted for simplicity of the expression.

Similarly the outer expansion of ϕ_j^- can be written in the form

$$\phi_j^{-*}(k;y,z) \sim H_j^-(k)\, G_p^{-*}(k;y,z)$$
$$= H_j^-(k)\Big\{G_p^{+*}(k;y,z) - 2\,i\,\epsilon_k\, e^{-\nu z}\cos(\nu y)\Big\} \tag{23}$$

Here the relation (14) has been substituted, and the Kochin function $H_j^-(k)$ may be defined in a form analogous to (22).

Using the convolution theorem to transform (24) and then substituting (25) and (27), the desired result of the outer expansion can be expressed in the form

$$\phi_j^{(i)*}(k;y,z) \sim \Big[H_j^+(k) + C_j^*(k)\Big\{H_3^+(k) - H_3^-(k)\Big\}\Big]\, G_p^{+*}(k;y,z)$$
$$+2\,i\,\epsilon_k\, e^{-\nu z}\cos(\nu y)\, C_j^*(k)\, H_3^-(k) \tag{24}$$

Let us turn to the outer solution; which is the same as that having been studied in the past slender-ship theories and can be written as

$$\phi_j^{(o)}(x,y,z) = \int_{-\infty}^{\infty} Q_j(\xi)\, G_{3D}(x-\xi,y,z)\, d\xi \tag{25}$$

Here $G_{3D}(x,y,z)$ is the 3-D Green function, and $Q_j(x)$ is its unknown strength along the ship's length.

To determine $Q_j(x)$, the inner expansion of (25) must be sought. It can be readily done by using the convolution theorem and substituting the relation (17) for the Fourier transform of $G_{3D}(x,y,z)$, with the result

$$\phi_j^{(o)*}(k;y,z) = Q_j^*(k)\, G_{3D}^*(k;y,z)$$
$$\sim Q_j^*(k)\Big\{\, G_p^{+*}(k;y,z) - \frac{1}{\pi}(1-\nu z)\, g^*(k)\,\Big\} \tag{26}$$

Now we are ready to consider the matching. Requiring (24) and (26) to be compatible in an overlap region gives the following two equations:

$$Q_j^*(k) = H_j^+(k) + C_j^*(k)\Big\{H_3^+(k) - H_3^-(k)\Big\} \tag{27}$$

$$2\,i\,\epsilon_k C_j^*(k) H_3^-(k) = -\frac{1}{\pi} Q_j^*(k)\, g^*(k) \tag{28}$$

Eliminating $Q_j^*(k)$ from these two equations, one can obtain

$$C_j^*(k) = \frac{-g^*(k)\, H_j^+(k)}{2\pi\, i\,\epsilon_k\, H_3^-(k) + g^*(k)\,\{H_3^+(k) - H_3^-(k)\}} \tag{29}$$

With numerical results of $C_j^*(k)$, the outer source strength, $Q_j^*(k)$, can be readily computed from (27).

As described before, the correction function $g^*(k)$ reduces to zero for $\nu \gg |k|$, implying that $C_j^*(k) = 0$ and $Q_j^*(k) = H_j^+(k)$ in this limiting case. This may be a proof that the present theory is an extension of the high-speed slender-body theory.

On the other hand, as in the unified theory or the strip theory, if the particular solution is constructed in just the transverse y–z plane, the homogeneous solution can be provided independent of x, i.e.

$$\phi_H(x;y,z) = \delta(x)\Big\{\phi_3(y,z) - \overline{\phi_3(y,z)}\Big\} \tag{30}$$

where the overbar denotes the complex conjugate, meaning equivalently the reverse-time velocity potential. In this case, (20) becomes identical to the inner solution of the unified theory.

At present, the computer program based on the present theory is being developed.

References

1) Newman, J. N.: Adv. Appl. Mech., Vol.18 (1978)

2) Chapman, R. B.: Proc. 1st Int. Conf. on Numer. Ship Hydrodyn. (1975)

3) Yeung, R. W. and Kim, S. H.: Proc. 15th Symp. on Naval Hydrodyn., Hamburg (1985)

4) Timman, R. and Newman, J. N.: J. S. R., Vol.5, No.4 (1962)

DISCUSSION

Yeung, R. W.: I would like to applaud your fine efforts in further exploring this interesting subject. In this development, you have exploited the reverse-time reverse-flow solutions of Yeung & Kim (1985) extensively. The outer asymptotic behaviour of these solutions in Fourier space follows from our work directly. The proposed "new" theory has a much simpler interaction term than Yeung & Kim's, which is definitely an improvement. However, unlike our proposed solution, your particular solution for the body boundary condition in the inner region, i.e. ϕ_j^+, would require a perfectly calm water initial condition at the bow. This solution is far from obvious when $\tau = U\omega/g < 1/4$, including the case of $\omega = 0$. The bow actually cuts into an ambient but unknown wave field. Because of this observation, the proposed solution will be restricted to the "high-speed" regime. There is no such restriction in Yeung and Kim's work. It was uniformly valid in ω and U. Perhaps the author can comment on this point.

Kashiwagi, M. : A point in your discussion is the treatment of the initial condition at the bow in obtaining the particular solution. I suppose your observation comes from actual 3-D wave fields. However, when considering only the inner particular solution, it is quite obvious from characteristics of the 2-D (pseudo 3-D) Green function that the particular solution has no upstream waves. Therefore, the treatment of zero distance ahead of the bow can be justified at least in obtaining the inner particular solution. Your observation for $\tau < 1/4$ is incorporated in the present theory through the homogenous component; this is the 3-D correction due to the transverse wave system which can exist in the upstream. So, if we look at the final solution (the particular solution plus the homogeneous solution), I'm sure it has the characteristics of your observation.

It seems that Yeung & Kim's theory (1985) tried to obtain the inner solution in a lump, just like a 3-D panel method, using the "generalized" inner Green function with corrections due to the transverse wave system. The way of constructing the solution in the present theory is the same as the unified theory, so understanding the resulting solution from the physical viewpoint can be the same as in the unified theory. When we look at only the strip-theory-like particular solution, it does not have 3-D characteristics at all, but the total solution does have 3-D corrections by virtue of the homogeneous component.

Faltinsen, O. M.: 1) I think it should be clearly stated that your theory does not account for transom stern effects, which are important for high-speed ships.

2) We normally say that a high-speed slender-body theory (2 1/2 D theory) is valid for $Fn > 0.4$ for vertical motions. The reason is that it only accounts for divergent wave systems. It is therefore interesting to see that you predict reasonable values for vertical added mass and damping for $Fn = 0.3$.

Kashiwagi, M. : 1) In computing the particular solution, i.e. the high-speed slender-body theory solution the transom stern effects will be taken into account just by finishing the calculations at the stern point. (This is a slender-body approximation for incorporating the effects of dynamic lift.)

continued/ ...

Probably your concern comes from the computation of the reverse-flow problem for a ship with transom stern. Computationally speaking, however, we can obtain the reverse-flow particular solution without any problem even for a ship with a blunt (wall-like) bow. Perhaps the problem is that the reverse-flow model for a transom-stern ship appears eccentric and unrealistic.

However, I should emphasise that, in the high speed range, the interaction coefficient $C_j(x)$ of the homogeneous solution will reduce automatically to zero irrespective of the form of the reverse-flow solution. Namely in the high-speed range, the solution in the present theory will be the same as that which would be obtained by the theory of Faltinsen & Zhao in 1991.

2) As you stated, in the high-speed slender-body theory, only the divergent waves are taken into account. These waves are outgoing when viewed in each transverse section along the ship. This implies that the high-speed slender-body theory recovers the strip theory as a special case, and actually it is obvious from the boundary conditions for the inner particular solution that the case of $U = 0$ is exactly the same as the 2-D problem used in the strip theory.

Theoretically, the above-mentioned argument is correct, and in fact we can prove it (e.g. please see Adachi's paper in 1980). However, I agree that there might be numerical difficulties in reproducing the strip-theory solution from the numerical scheme for the high-speed slender-body theory adopted here.

Newman, J. N.: You state that $U < 0(1)$ is necessary in the unified theory. This is surprising to me since the original derivation was based on the assumption $U = 0(1)$. Did we overlook something in the inner solution or matching region?

Kashiwagi, M. : My reasoning comes from the fact that the matching in the unified theory requires the difference between the 3-D forward-speed wavenumber ν and the zero-speed wavenumber K to be small. That is, more specifically,

$$(K - \nu)r = kr(2\tau + k/K_o) < 0(1) \text{ where } \tau = U\omega/g,\ K_0 = g/U^2$$

Probably your original derivation is based on the assumption that $kr < 0(1)$ and $\tau = 0(1)$. In this sense you are right and my statement must be corrected.

My experience in the unified theory is that the prediction agrees well with experiments for Froude number less than about 0.3, especially in the heave mode. But this is not necessarily the case in the pitch mode and coupling terms. Actually it is known since the rational strip theory that the forward-speed effects may be pronounced with the order $0(U)$ in the coupling terms and with the order $0(U^2)$ in the pitch mode. Therefore I have been thinking from the practical viewpoint that the unified theory should be applied in the relatively low-speed range.