Removing the Irregular Frequencies in Wave-Body Interactions

By

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Introduction

The solutions of the boundary integral equations, with the free surface Green function, suffer from the effect of 'irregular frequencies' when a body intersects the free surface. The irregular frequencies are a set of infinite discrete resonance frequencies of the nonphysical interior flow where the formulation breaks down. In a discretized problem, the effect appears as a numerical error over the substantial frequency band around the irregular frequencies resulting from the bad conditioning of the linear system.

Despite this defect of the formulation, the 'panel method' has been widely used for the analysis of three-dimensional wave-body interactions without much attention to the correction of the irregular frequencies effect. In general, the irregular frequencies are higher than the frequency range of practical interest of typical sea spectra. Interpolation of the numerical results may be possible when the solution is smooth in frequency, since the frequency band of the 'polluted' solution can be reduced arbitrarily with increasing number of panels. However there are several applications where it is essential to remove the effect of the irregular frequencies. For vessels with a large waterplane area, such as barges, the irregular frequencies may be within the range of practical interest. In multiple-body interaction, it is necessary to distinguish the physical resonance frequencies from the irregular frequencies. The effect of the irregular frequencies is particularly detrimental to second order solution for structures such as tension leg platforms, due to the density of the irregular frequencies over the sum-frequency range.

Various methods have been used to suppress the effect of the irregular frequencies. We apply three of these methods and compare the robustness and efficiency of the methods in connection with the low-order panel method.

Modified integral-equation method

Lee and Sclavounos [1] showed that if Green's integral equation and its normal derivative are combined in the form

$$2\pi\varphi(\mathbf{x}) + \iint_{S_b} \varphi(\boldsymbol{\xi})\frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi}d\xi + \alpha\frac{\partial}{\partial n_x}\iint_{S_b} \varphi(\boldsymbol{\xi})\frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} =$$

$$\iint_{S_b} \frac{\partial \varphi(\boldsymbol{\xi})}{\partial n_\xi}G(\mathbf{x};\boldsymbol{\xi})d\xi - 2\pi\alpha\frac{\partial\varphi(\mathbf{x})}{\partial n} + \alpha\frac{\partial}{\partial n_x}\iint_{S_b} \frac{\partial \varphi(\boldsymbol{\xi})}{\partial n_\xi}G(\mathbf{x};\boldsymbol{\xi})d\xi \qquad \mathbf{x} \in S_b \tag{1}$$

there are no irregular frequencies when the complex constant α has a nonzero imaginary part.

The accuracy of the numerical solution of (1) depends on the value of the imaginary part of α. If this value is too small, the irregular frequency effects are not completely removed. On the other hand, if the value of the imaginary part of α is too large, numerical errors are introduced due to the fact that the additional integral equation associated with the normal derivative is of the first kind. One can find numerically the optimal value to minimize the latter error but a substantially larger number of panels is required to achieve the same accuracy as in the unmodified integral equation.

Modified Green function method

When the field point is on the body boundary S_b or on the interior free surface S_i inside the body, Green's integral equation has the following forms

$$2\pi\varphi(\mathbf{x}) + \iint_{S_b} \varphi(\boldsymbol{\xi}) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} d\xi = \iint_{S_b} \frac{\partial \varphi(\boldsymbol{\xi})}{\partial n_\xi} G(\mathbf{x};\boldsymbol{\xi}) d\xi \qquad \mathbf{x} \in S_b \qquad (2a)$$

$$\iint_{S_b} \varphi(\boldsymbol{\xi}) \frac{\partial G(\mathbf{s};\boldsymbol{\xi})}{\partial n_\xi} d\xi = \iint_{S_b} \frac{\partial \varphi(\boldsymbol{\xi})}{\partial n_\xi} G(\mathbf{s};\boldsymbol{\xi}) d\xi \qquad \mathbf{s} \in S_i \qquad (2b)$$

Green's integral equation (2a) does not have irregular frequencies when it is combined with equation (2b) or its spatial derivative for one or more points $\mathbf{s}$ in the interior domain. In equation (2b), $\mathbf{s}$ should not be on the nodal points of the eigenmodes. In the usual discretized problem, (2a) and (2b) are an over-determined system, and the rectangular matrix of the linear system is less efficient to solve than a square matrix.

The modified Green function of Ogilvie and Shin [2] can be obtained by adding (2b) or its spatial derivatives with the multiplication factor $cG(\mathbf{x},\mathbf{s})$ to (2a) and setting the point $\mathbf{s}$ at the midpoint of S_i. We have tested this modified Green function method for bodies with two planes of symmetry, with $\mathbf{s}$ at the midpoint of S_i. Our numerical results [3] indicate that the irregular frequencies can be removed, but the value of c depends on the mode of motion. Also the range of c is very restricted if an iterative solver is used due to the poor conditioning of the linear system. For arbitrary bodies the optimum location of the point $\mathbf{s}$ is not obvious.

Extended boundary condition method

Following Kleinman [4], one may extend the boundary to include S_i, where the normal direction points downward:

$$2\pi\varphi(\mathbf{x}) + \iint_{S_b} \varphi(\xi) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} d\xi + \iint_{S_i} \varphi(\xi) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} d\xi = \iint_{S_b} \frac{\partial \varphi(\xi)}{\partial n_\xi} G(\mathbf{x};\boldsymbol{\xi}) d\xi \qquad \mathbf{x} \in S_b \quad (3a)$$

$$-4\pi\varphi(\mathbf{x}) + \iint_{S_b} \varphi(\xi) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} d\xi + \iint_{S_i} \varphi(\xi) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} d\xi = \iint_{S_b} \frac{\partial \varphi(\xi)}{\partial n_\xi} G(\mathbf{x};\boldsymbol{\xi}) d\xi \qquad \mathbf{x} \in S_i \quad (3b)$$

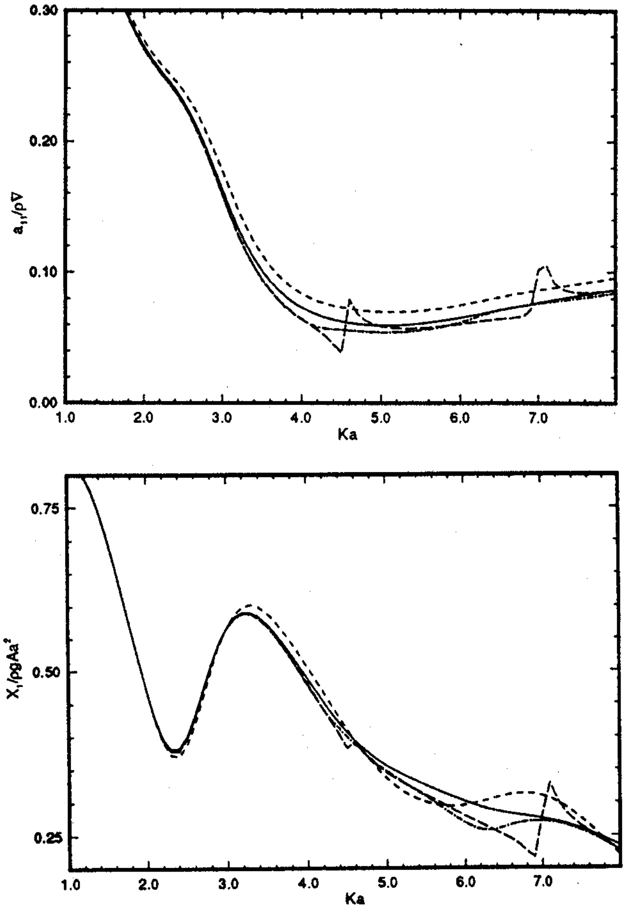
The complete derivation of the above equations and proof of the uniqueness are provided in [4], however the first term of (3b) is affected by an erroneous jump condition on S_i in [4]. This is corrected in (3b) following the derivation outlined in the Appendix, and in more detail in [3]. Corresponding equations for the source formulation are also derived in [3].

This method requires additional panels on S_i, so the dimension of the linear system increases. Numerical tests indicate that the number of panels on S_i can be much smaller than on the body boundary. The required number of panels on S_i increases with the order of the irregular frequencies. Using too many panels on S_i causes convergence problems with the iterative solver.

The Green function $G(\mathbf{x};\boldsymbol{\xi})$ includes (in addition to the two Rankine singularities) a weaker logarithmic singularity which must be accounted for analytically in the second integral of (3b) when the points $\mathbf{x}$ and ξ coincide. An appropriate numerical technique which integrates the logarithmic singularity analytically over each panel is derived in [5], and is used for the results shown below.

Numerical results

Numerical results based on these methods are presented in Figures 1 and 2 for a truncated cylinder and a barge.



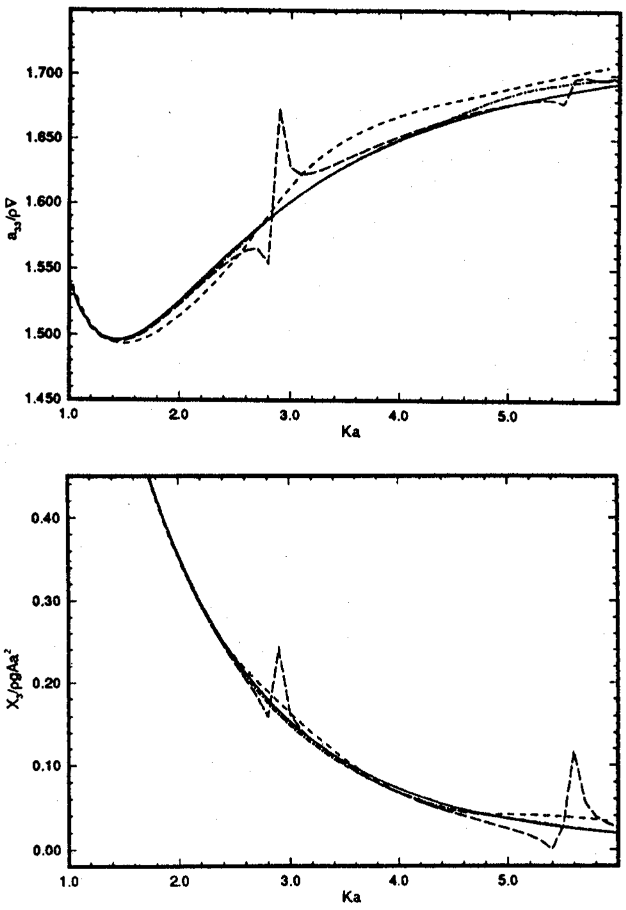


Figure 1 — Surge added-mass coefficients and Haskind exciting force of a barge(L/B=2,B/T=2). The number of panels on the body is $4 * 96$.

Figure 2 — Heave added-mass coefficients and Haskind exciting force of a cylinder (R/T=2). The number of panels on the body is $4 * 80$.

Methods: (– – – –) original Green's integral equation.
(- - - - -) modified integral equation method, $\alpha = 0.2$.
(– · – · –) modified Green function method, $c = 0.02$ in Figure 1, $c = 0.3$ in Figure 2.
(———) extended boundary condition method. The number of panels on S_i is $4 * 8$ in Figure 1, $4 * 16$ in Figure 2.

References

[1] Lee, C. H. and Sclavounos, P. D. 1989 Removing the irregular frequencies from integral equations in wave-body interactions. *J. Fluid Mech.* **207**, 393-418.

[2] Ogilvie, T. F. and Shin, Y. S. 1977 Integral-equation solution for time-dependent free surface problems. *J. Soc. Nav. Arch.* Japan **143**, 86-96.

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[4] Kleinman, R. E. 1982 On the mathematical theory of the motion of floating bodies — an update. DTNSRDC-82/ 074.

[5] Newman, J. N and Sclavounos, P. D. 1988 The computation of wave loads on large offshore structures. Conference on the Behaviour of Offshore Structures (BOSS '88), Trondheim.

Appendix

Equations (3a) and (3b) are derived by adding to (2a) and (2b) the following pair of integral equations:

$$\iint_{S_i} \varphi(\boldsymbol{\xi}) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} d\boldsymbol{\xi} = 0 \qquad \mathbf{x} \in S_b \qquad (4a)$$

$$-4\pi\varphi(\mathbf{x}) + \iint_{S_i} \varphi(\boldsymbol{\xi}) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} d\boldsymbol{\xi} = 0 \qquad \mathbf{x} \in S_i \qquad (4b)$$

Our objective here is to show that there is no nontrival solution of (4a) and (4b).

First it is important to have the correct limiting behaviors of the normal derivatives of the Green function when $\boldsymbol{\xi} \in S_i$ and $\mathbf{x} \to S_i$, and these are

$$\frac{\partial}{\partial n_\xi}\left(\frac{1}{r} + \frac{1}{r'}\right) = -\frac{\partial}{\partial \zeta}\left(\frac{1}{r} + \frac{1}{r'}\right) = 0 \qquad (5)$$

$$\frac{\partial}{\partial n_{\mathbf{x}}}\left(\frac{1}{r} + \frac{1}{r'}\right) = -\frac{\partial}{\partial z}\left(\frac{1}{r} + \frac{1}{r'}\right) = \frac{2z}{r^3} \qquad (6)$$

Next define the potential $\phi_-(\mathbf{x})$ inside the body V_- due to the dipole distribution of density $\varphi(\mathbf{x})$ on S_i:

$$\phi_-(\mathbf{x}) = \iint_{S_i} \varphi(\boldsymbol{\xi}) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} d\boldsymbol{\xi} \qquad \mathbf{x} \in V_- \qquad (7)$$

This potential is defined throughout the domain inside the body. Since it is continuous as the field point approaches S_b, we obtain

$$\phi_-(\mathbf{x}) = \iint_{S_i} \varphi(\boldsymbol{\xi}) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} d\boldsymbol{\xi} \qquad \mathbf{x} \in S_b \qquad (8)$$

Now consider the limiting behavior of $\phi_-(\mathbf{x})$ as $\mathbf{x} \to S_i$. From (5) it follows that

$$\phi_-(\mathbf{x}) = \iint_{S_i} \varphi(\boldsymbol{\xi}) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} d\boldsymbol{\xi} \qquad \mathbf{x} \in S_i \qquad (9)$$

Before evaluating the normal derivative of (7) as $\mathbf{x} \to S_i$, we can use the free surface boundary condition to rewrite (7) in the form of a source distribution. Differentiating this expression and using (6), we get:

$$\frac{1}{K}\frac{\partial \phi_-(\mathbf{x})}{\partial n_{\mathbf{x}}} = -4\pi\varphi(\mathbf{x}) + \iint_{S_i} \varphi(\boldsymbol{\xi}) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_{\mathbf{x}}} d\boldsymbol{\xi} \qquad \mathbf{x} \in S_i \qquad (10)$$

where the integral should be a principal value. Since both $z = 0$ and $\zeta = 0$ in (10), and since the integral in (10) excludes the point where (6) is singular, the free surface condition $\partial G(\mathbf{x};\boldsymbol{\xi})/\partial n_\xi = \partial G(\mathbf{x};\boldsymbol{\xi})/\partial n_{\mathbf{x}}$ can be used now to replace the above normal derivative as follows:

$$\frac{1}{K}\frac{\partial \phi_-(\mathbf{x})}{\partial n_{\mathbf{x}}} = -4\pi\varphi(\mathbf{x}) + \iint_{S_i} \varphi(\boldsymbol{\xi}) \frac{\partial G(\mathbf{x};\boldsymbol{\xi})}{\partial n_\xi} d\boldsymbol{\xi} \qquad \mathbf{x} \in S_i \qquad (11)$$

It is now easy to show that there are no nontrivial solutions of (4a) and (4b). From (4a) and (8), $\phi_-(\mathbf{x})$ satisfies a homogeneous Dirichlet condition on S_b, and from (4b) and (11), $\phi_-(\mathbf{x})$ satisfies a homogeneous Neumann condition on S_i. Since (7) is a harmonic function in the interior of the body, with these two homogeneous boundary conditions, it must be zero everywhere inside the body, and thus the integral in (9) is zero. Since the same integral is in (11), it follows that $\varphi(\mathbf{x}) = 0$.

DISCUSSION

Kuznetsov N.: Is there any relation between φ on S_i and the solution φ of the original problem in your third method?

Lee C.H.: No.

Clement A.: I have experienced a method similar to your second method in 2D seakeeping computations. It consists in forcing ϕ to be null at one or more points located on the inner free surface S_i. If you choose a point at a short distance of the body surface S_b, the first irregular frequency is pushed away in a high frequency range, where we never have to solve the problem, in least at first order. The method is cheap and works well, in 2D.

Lee C.H.: Thanks for your comment. We are trying to remove the effect in a higher frequency range in sum-frequency problem.

Ando S.: Based on your experience, how would you rank the three methods (M1, M2, and M3) in terms of the ease of numerical implementation?

Lee C.H.: It is difficult to rank the easiness. I would rather point out the difficulties which may be encountered in numerical implementation. They are evaluation of the double normal derivative of the Green function on the body surface (M1), double spacial derivative of the normal dipole on the free surface (M2), robust treatment of the log singularity of the Green function on the free surface (M3), respectively.

Bingham H.: The fact that the condition number becomes uniformly large in your method #1 seems to explain the departure of these calculations from the unmodified computations away from the irregular frequencies. I was surprised then to see the same behavior of the condition numbers in methods #2 & #3, where the calculations are very close to the unmodified ones away from the irregular frequencies. Can you explain this?

Lee C.H.: The condition number, including the sensitivity of the linear system, is the upper bound of the amplification factor of the input error which arises from various sources. Both the magnitude and direction of the input error affect the behavior of the solution and the full account of it may require detailed analysis of the linear system. However we may roughly say that the input error in method 1 (the modified integral equation) is larger, due to the direct evaluation of the hypersingular integral, than those in methods 2 and 3 and thus method 1 shows largest departure of the solution from the unmodified original solution over the frequency range.