A New Algorithm (SIMAC) for the Solution of Free Surface Unsteady High Reynolds Flows

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Introduction

Many problems in ship hydrodynamics are unsteady, influenced by the viscosity of the water and characterized by the presence of the free surface. Large amplitude sloshing of liquids in baffled tanks or the hydrodynamic interaction between water and oscillating bodies below or at the free surface are typical problems in this area.

In the past several techniques have been developed for treating such kinds of problems. The MAC methods family [1-4] solves the unsteady Navier-Stokes equations, written in primitive variables,using an explicit second order centered discretization of the convective and diffusive terms on a uniform staggered grid. Moreover the domain of integration of the governing equations is unchanged during the time evolution of the problem, and the liquid portion is individuated by the flagging of the cells, depending on their own position with respect to the instantaneous position of the markers.

The explicit updating of the velocities field doesn't allow the boundary layers to be solved accurately, even if not uniform grid are employed, mainly because of a not acceptable restriction on the time step necessary to preserve the stability of the diffusive term.

At the moment an alternative may be given by writing the equations in generalized coordinates within a time dependent domain of integration [5]. This allows the use of implicit algorithms and therefore the use of fine grids where needed, and more efficient pressure solvers than SOR. Neverthless this approach is not actractive for unsteady problems, mainly because large memory is requested for storage of the metric quantities and CPU time needed to recalculate them at each time step. Finally when large distorsions of the free surface are expected (breaking waves) these methods usually fail.

In this abstract a new way, that should embrace the advantages of the two approaches described above , has been developed.

The principal features of such a method, called SIMAC (Semi-Implicit Marker and Cell method) , are the following:

The NSE are solved on a non uniform staggered grid, employing a fractional step approximate factorization tecnique. In particular the convective term is calculated by means of a second order Adams-Bashforth discretization in time and a third order upwinding scheme (QUICK) in space, while the diffusion is treated implicitely by means of the Crank-Nicholson scheme using the approximate factorization tecnique as described in [6]. The advantages of such a semi implicit treatment of the momentum equation permits to use high order schemes for the discretization of the convective terms and, at the same time, to avoid the diffusive restriction on the time step;

The Poisson equation for the pressure is solved by a Line Gauss Seidel method in conjunction with the Additive Correction strategy [7] to speed up the convergence rate;

The domain of integration of the equations is divided into two parts by massless particles that individuate the instantaneous position of the free surface as in a standard MAC method [3] and the velocities and the pressure points are flagged 1 or 0 depending on their own position with respect to the free surface.

At the moment the method has been applied to the analysis of large amplitude sloshing of liquids in rectangular containers.

A FORTRAN 77 computer code, working as follows, has been developed:

- The geometry of the tank, the liquid level and the initial position of the free surface are read together with the number of cells, the kinematic viscosity of the liquid, the Courant number, the number of iterations in time and the characteristics of the external excitation.

- The pressure and velocities points in the computational domain are flagged depending on their own position with respect to the free surface.

- The irregular stars [8] requested for the calculation of the pressure at the free surface are calculated

- An extrapolation of the velocities above the free surface, according to [3] is performed for two reasons: the evaluation of the convective and diffusive terms in the points immediately below the free surface, and the calculation of the velocities of the markers.

- The convective terms and the external forces acting on the body are calculated.

- The coefficients of the tridiagonal matrices to be inverted for the evaluation of the diffusive terms are calculated

- An intermediate velocities field satisfying the momentum equation and the corresponding source terms of the Poisson equation are calculated.

- The coefficients of the tridiagonal matrices to be inverted for the evaluation of the pressure are calculated and the LGS solver is applied firstly along the x-direction and successively along the y-direction.Then the following test $Res^{k+1} \leq 0.5\ Res^{k}$ is performed, being Res^{k+1} and Res^{k} respectively the residuals after the k+1 and k iterations. If the previous relation is not satisfied the Additive Correction [7] is invoked.

-The new velocities field is evaluated and the position of the markers is updated.

-The cells are re-flagged and a new time step according to the CFL condition is calculated, finally the cycle can start again.

The calculation of the coefficients of the matrices requests some manipulations at and above the free surface, in order to mantain the diagonal dominance. This latter point is described in detail in [9].

Numerical Results

Numerical tests have been performed in order to evaluate the consistence of the algorithm. As previously discussed the sloshing of a liquid in a rectangular container 1 meter long and liquid depth equal to 0.21 m has been analysed. The domain of integration of the equation is 1 meter long and 0.4 meter height, both uniform and stretched grids have been used for different Reynolds numbers ($Re= (b^3g)^{1/2}/\nu$ being b the tank breadth [5, 10]) and several cases of external excitation have been studied.

At first the case of a tank subjected to a constant sway acceleration $A_x = 1\ m/s^2$ has been analysed using three different uniform grids. The maximum allowed time step in these simulations was chosen equal to 5.0×10^{-3} and Re = 320. In (Fig. 1, 2) the wave elevation at the left wall and the percentage of mass variation versus time are presented for three grids (20x20,40x40,80x80). It is clearly showed the consistence of the algorithm.

Successively a re-gridding test has been performed using the same grids and Reynolds number as the previous case, considering an harmonic large amplitude sway excitation (Sway Ampl.= 0.1 m, Sway Period = 2.5 s). Also in this simulations the results (Fig. 3) are very satisfactory.

Furthermore the algorithm has been tested on stretched grids for the same harmonic sway excitation as in the previous case and Re = 940 (the stretching is computed using the TANH function, see [9]). Looking at (Fig. 4), it is showed that some differences appear on the coarse grid, neverthless the results relatives to the finer grids are practically the same.

The wave amplitudes at the left wall for several Reynolds numbers using a 50x50 stretched grid are showed in (Fig. 5). It appears that the higher is the Reynolds number the more the oscillations are characterized by the overlapping of several wave modes. In particular for Re= 313 only the mode corresponding to the period of the external motion is excited, neverthless for Re = 3130 and Re = 31300 the presence of high frequency modes, resulting in the appearence of short waves, can be observed. Furthermore a phase lag in the wave propagation, due to the effect of viscosity on the liquid motion, can be observed The results of the numerical simulations agree with the physical observations [10].

Furthermore, in order to analyse the characteristics of the boundary layer at the vertical walls for several Reynolds number, the vertical component of the velocities field at y= 0.086 m for the previous simulations at t = 2.5 s are plotted in (Fig. 6).

Finally the velocities field for Re = 31300 at t=2.5 is presented in (Fig.7).

In conclusion the following considerations can be made:

1) The application of the Additive Correction strategy, that can be thought as a two levels multigrid tecnique, demonstrates that a more levels multigrid method can be easily applied to a MAC-type algorithm; this step will be made in the next future.

2) The application of the method to the study of an unsteady external free surface flow (i.e. for the calculation of the viscous damping of oscillating bodies near the free surface) is straightforward.

Finally, at the moment an attempt to simulate turbulent flows by meansof the Subgrid Scale (SGS) model is in progress; Results will be presented at the Workshop.

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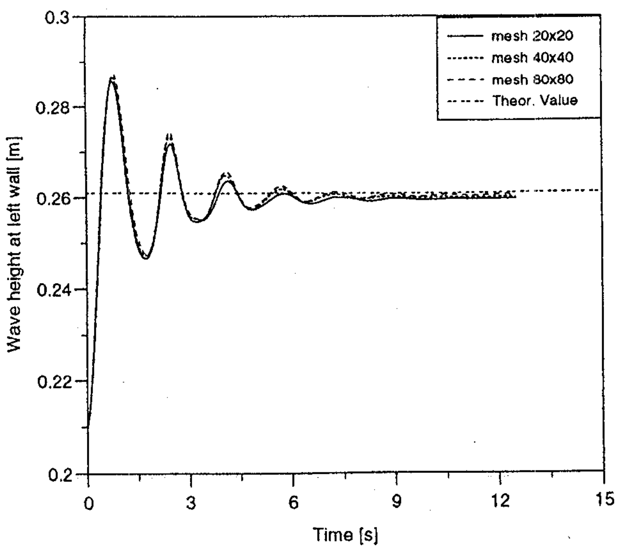

Fig. 1 Liquid elevation at the left wall versus time
(Re=320, Constant sway acc. = 1 m/s^2)
(Uniform grids)

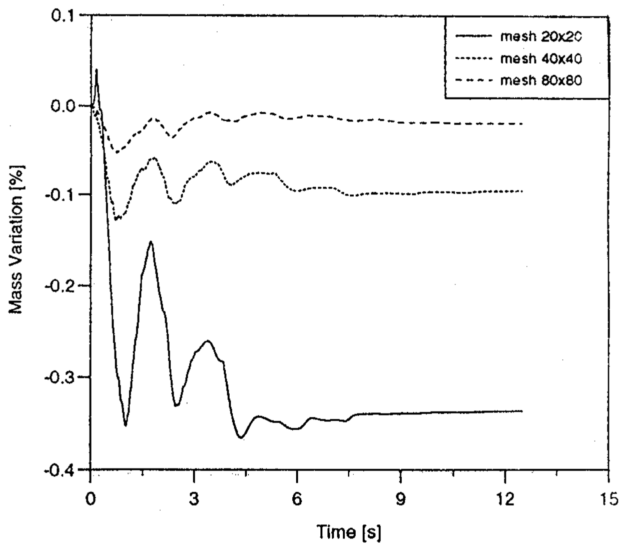

Fig. 2 Percentage of mass variation versus time
(Re=320, Constant sway acc. = 1 m/s^2)
(Uniform grids)

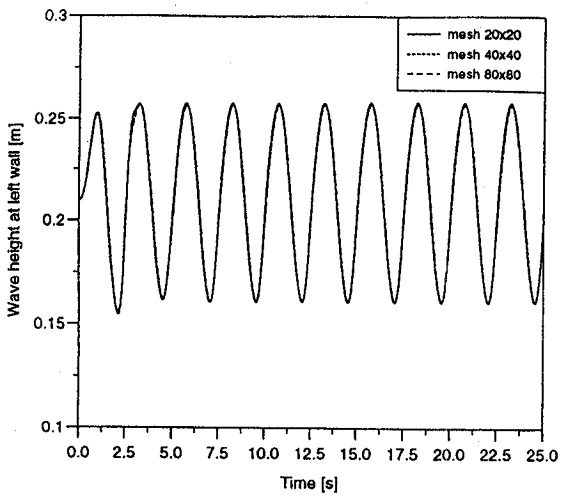

Fig. 3 Liquid elevation at the left wall versus time
(Re=320, Sway Ampl. =0.10 m, Period = 2.5 s)
(Uniform grids)

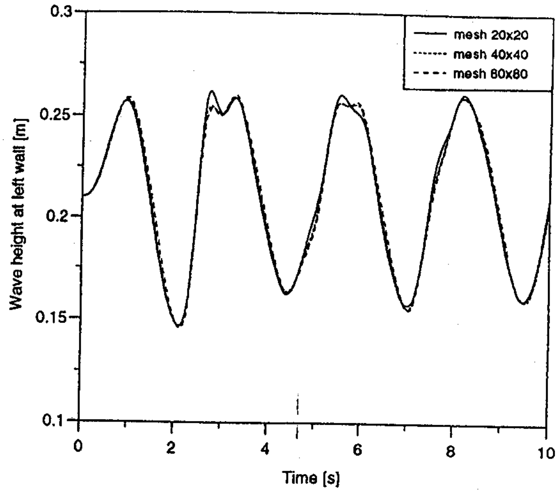

Fig. 4 Liquid elevation at the left wall versus time
(Re=940, Sway Ampl. =0.10 m, Period = 2.5 s)
(Stretched grids)

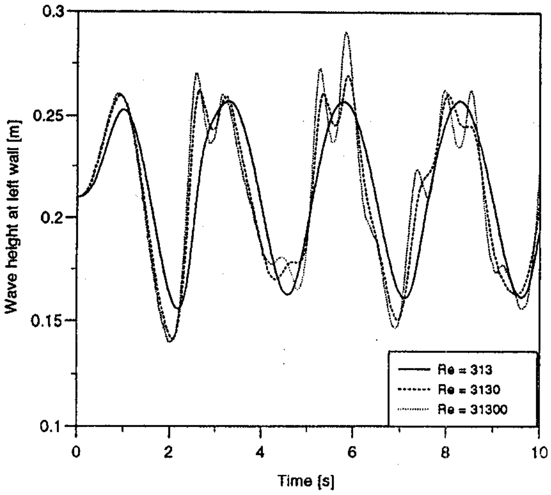

Fig. 5 Liquid elevation at the left wall versus time
for three Reynolds numbers.
(Sway Ampl. =0.10 m, Period = 2.5 s)
(50x50 stretched grid)

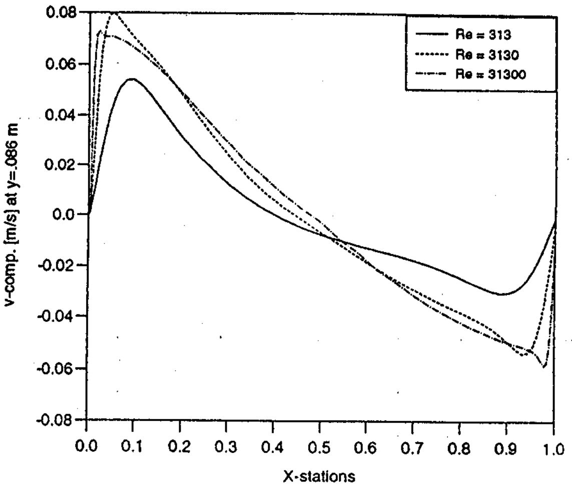

Fig. 6 Vertical velocity at y = 0.086 m for three Reynolds
numbers at t=2.5 s.
(Sway Ampl. =0.10 m, Period = 2.5 s)
(50x50 stretched grid)

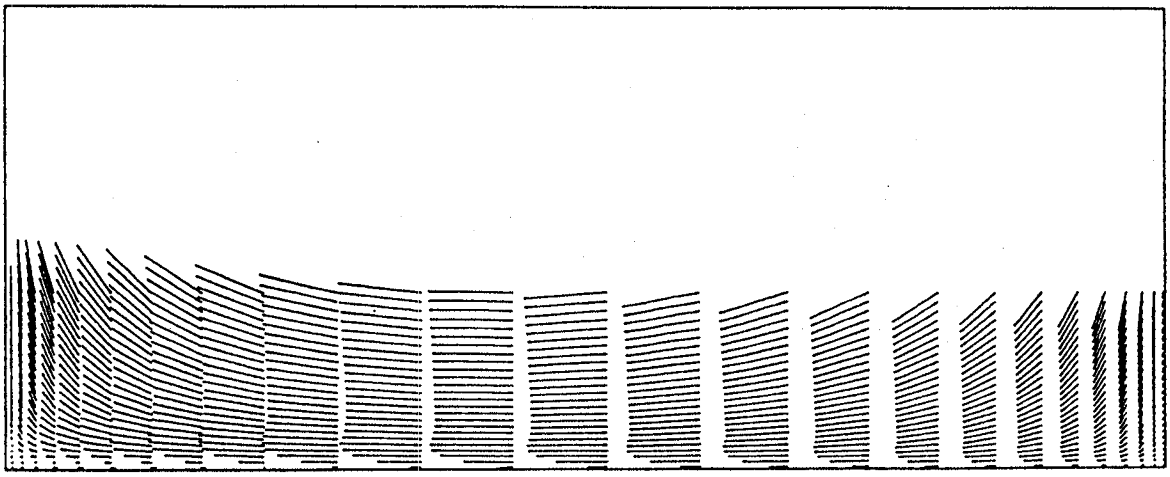

Fig. 7 Computed velocity vector field for Re = 31300 at t = 2.5 s (Sway Ampl. = 0.10 m, Period = 2.5 s)