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CALCULATIONS OF SURFACE-WAVE RADIATION AND DIFFRACTION BY REGULARIZED COMPOSITE INTEGRAL EQUATIONS

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The application of boundary integral methods to the problems of surface-wave radiation and diffraction by floating bodies suffers from nonexistence or nonuniqueness problems for a denumerably infinite set of characteristic frequencies each corresponding to an eigenvalue of the integral operator [1]. These characteristic frequencies, or irregular frequencies, depend only on the shape of the wetted surface of a body, and, for an arbitrary-shaped body, their location is not known in advance. Furthermore, the approximating matrix equations become ill-conditioned with respect to the wavenumber k when k lies in the vicinity of an irregular frequency. It is therefore necessary to have a formulation that is well-conditioned at all frequencies.

Lee and Sclavounos [2] made use of the modified integral equations, in which the integral operator itself is modified by linearly combining single- and double-layer potentials. They directly discretized and evaluated the resulting integral equations that include an unbounded operator involving the double normal derivative of the exact Green's function. Their work, while successfully demonstrating the effectiveness of the direct formulation, underscores the importance of choosing an optimal value for the coupling parameter for a particular body geometry in order to minimize the condition number of the approximating matrix equations. Based on their experience, they recommend a pure imaginary number $0.2i$ as the coupling parameter for arbitrarily-shaped bodies. In case this value does not perform well for a given shape, they suggest a search for the optimal value by estimating the position of the irregular frequency followed by determination of the minimum condition number over a range of values of its modulus. This could involve a substantial increase in computational effort; moreover, irregular frequencies for bodies of general geometry such as ship hulls are not usually known, as mentioned above.

This prompts us to examine the regularization of the modified integral equation in order to assess its suitability for practical radiation and diffraction computer programs. Since all the integral operators in the regularized formulation are compact, the numerical evaluation of the integrals is expected to be more amenable to accurate approximation. Thus, the regularized formulation offers the promise of being well-conditioned and of not requiring any special consideration to yield accurate results. Nevertheless, no numerical experience concerning wave radiation and diffraction problems has yet been reported. In operator notations, the regularized formulation used in this paper for the radiation problem can be expressed as ([1], [3])

$$\left\{\left(\frac{1}{2}+M\right)+\alpha\left[L_0(N-N_0)+M_0^2-\frac{1}{4}\right]\right\}\phi_j(p)=\left[L+\alpha L_0\left(-\frac{1}{2}+M^t\right)\right]\frac{\partial\phi_j}{\partial n}(p) \qquad (1)$$

for p on the wetted surface. (A similar expression can be derived for the diffraction problem.) In (1), ϕ_j is the potential for the j-th mode of body motion; L and M are respectively single- and double-layer potential operators based on the exact Green's function for Laplace's equation; $N\phi_j$ is the derivative of $M\phi_j$ in the direction of the outward normal at the field point; and α is a strictly complex coupling parameter. The superscript t indicates the transpose, and the subscript 0 indicates the operators based on the free-space Green's function. Here, L_0 is used as the reducing, or regularizing, operator. The numerical implementation of the regularized formulation and the results of calculations, including comparison with the numerical results by the direct method of Lee and Sclavounos [2], are described in this paper.

Although the regularized formulation (1) is more complicated and requires more computing time than the direct formulation used in [2], this need not be a major drawback. For it can be shown [1] that wavenumbers of irregular frequencies are bounded below by

$$k \geq 1/T \tag{2}$$

where T is the draft of the floating body, so the coupling parameter may be set to zero for $k < 1/T$. Furthermore, a sharper estimate of the lower bound for characteristic wavenumbers than (2) for an arbitrary 3-dimensional body can be obtained by considering a rectangular body of length L, breadth B, and draft T, which encloses the body in question, and for which the irregular frequencies are found by straightforward application of separation of variables:

$$kT = \mu T \coth \mu T, \tag{3}$$

where

$$\mu T = \pi \sqrt{\left(\frac{m}{L/T}\right)^2 + \left(\frac{n}{B/T}\right)^2}, \quad m, n = 1, 2, \ldots$$

and by using the following theorem (the proof is given in the paper):

Theorem. Let S and S' be smooth immersed surfaces, concave upwards, of floating bodies such that $S \supseteq S'$, and let k_1 and $k_1{}'$ be the least wavenumbers for irregular frequencies for S and S', respectively. Then $k_1 < k_1{}'$.

References

1. John, F., "On the motion of floating bodies II", *Comm. on Pure and Applied Math.*, **3**, 1950, pp. 45–101.

2. Lee, C.-H. and P.D. Sclavounos, "Removing the irregular frequencies from integral equations in wave-body interactions," *Jour. Fluid Mechanics*, **207**, 1989, pp. 393–418.

3. Burton, A.J. and G.F. Miller, "The application of integral equation methods to the numerical solution of some exterior boundary-value problems," *Proc. Roy. Soc. Lond.* A. **323**, 1971, pp. 201–210.

DISCUSSION

Clement A.: The robustness of such a method can be characterized by the variation of the condition number with increasing number of panels (refining the discretization). Do you have any numerical experience and/or results to illustrate the robustness of the regularized composite integral equation method?

Ando S.: By a robust method, I mean it gives low condition numbers irrespective of body shopes, mesh sizes as well as wavenumbers. This work is still in an early stage, and I have not yet done any numerical experiment. I'm hoping to show some results at the next Workshop.

Kuznetsov N.: Other integral equations without irregular frequencies were proposed in papers: 1) T.S.Augell, G.C.Hsiao & R.E.Kleinman, J. Fluid Mech. 166 (1986) 161-171
2) N.G.Kuznetsov, Math. Notes. of the Acad. Sci. of the USSR 50 (1991) 1036-1042 (Transl. from Russian)

Ando S.: Thank you for your remarks.

Lee C. H.: Your comment that the bad-conditioning is due to the direct numerical evaluation of the hyper singular kernel is not quite correct. The bad-conditioning of the discretized form of the composite integral equation is due to the property of the additive first-kernel integral. The direct numerical integration, however, produces large discretization error which is amplified by the bad-conditioning of the system. I expect your regularization reduces this discretization error and produces better results than ours.

Ando S.: As you pointed out, integral equations of the first kind suffer computational instability because of the absence of dominant diagonal terms. In the case at hand, this disadvantage is further aggrevated by the strong nonsingularity of the kernel, and this is what I meant.