Optimization of Ship Wave Resistance in Shallow Water

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1 Introduction

With increasing speed of inland and coastal ships, ship hydrodynamics in shallow water is now an important practical subject. As described in many papers, e.g., Graff et al. (1964), the wave resistance of a ship increases steeply with the increase of speed in the subcritical range and then reaches a maximum value at a near-critical speed but still in subcritical range. Meanwhile, the "squat" phenomenon takes place so that the ship may touch ground. Beyond that point the wave resistance first decreases and then increases again slowly. In the transcritical and supercritical range the problem is significantly nonlinear. Besides this, it has been confirmed theoretically in the last decade that the problem is essentially unsteady when ship moves at a near-critical constant speed in a channel. The linearized theory or even nonlinear theory only for steady flow fails to predict the resistance in this transcritical range.

Recently we have developed a new method (Chen & Sharma 1992) with matched asymptotic expansions based on nonlinear shallow water wave theory in far-field and slender body theory in near-field for the problem of ship in shallow water at near-critical speeds. It has been shown that the calculated results agree well with model experiments of Graff et al. (1964), Ertekin et al. (1984) and Millward & Bevan (1986) respectively for three hull forms: Taylor Standard Series $C_P = 0.64$, Series 60 $C_B = 0.8$ and Wigley Model. The corresponding calculation method is also numerically efficient so that we were able to carry out numerical experiments on optimization of ship hull using a Workstation HP9000/730.

In this paper we apply it to optimize a ship hull form for minimum wave resistance in shallow water. This attempt is not only for finding the best ship for a high subcritical speed, but also it has practical relevance to the transition problem of enabling a high-speed ship to reach more easily the favorable supercritical region through the critical Froude number. In the last section we discuss how large is the speed range to which our theoretical model can be applied. We give a new KP-kind of equation which includes high transverse dispersion effect and can be applied to a wider speed range without numerical handicap.

2 Formulation

We consider a slender ship of length l^*, beam b_o^* and draft d^* moving along x-direction in a shallow channel of depth h^* and width w^* at a near-critical speed U^* and basically assume the flow to be irrotational and incompressible and the ship free only to heave and pitch. A Cartesian coordinate system $Oxyz$ moving at same speed as the ship is used with origin O located in the midship section, plane Oxy on the quiet free surface, plane Oxz on ship centre-line plane, z-direction positive upward and x positive forward.

We analyze the problem with the technique of matched asymptotic expansions. We divide the flow region into two parts, i.e., near-field and far-field, which mean the fields near and far away from the ship respectively. In the near-field the typical length scale for transverse directions y and z is r_o^* and for the longitudinal direction x it is l^*. On the other hand, in the far-field the typical length scale for horizontal directions x and y is L^* and for the vertical direction z it is h^*. Based on these scales, using nonlinear shallow water theory in far-field and slender body theory in near-field, we separately formulate equations with multiple-scale expansions for each field and then match them with each other asymptotically. For details please to see Chen & Sharma (1992). In the following, the main formulae are given, in which the KP equation has been modified slightly here for a wider speed range about depth Froude number equal to unity.

Upon normalization

$$\zeta = \frac{\zeta^*}{\varepsilon h^*}, \quad \varphi = \frac{\varphi^*}{\varepsilon^{\frac{1}{2}} h^* \sqrt{g^* h^*}}, \quad x = \varepsilon^{\frac{1}{2}} \frac{x^*}{h^*}, \quad Y = \varepsilon \frac{y^*}{h^*}, \quad z = \frac{z^*}{h^*},$$

$$\tau = \frac{\varepsilon^{\frac{3}{2}} t^* \sqrt{g^* h^*}}{h^*}, \quad U = \frac{U^*}{\sqrt{g^* h^*}}, \quad L^* = h^*/\varepsilon^{\frac{1}{2}}, \quad p = \frac{p^*}{\rho^* g^* h^*}, \tag{1}$$

basic equation is modified KP equation

$$\varphi_{x\tau} + \frac{1-U^2}{2U\varepsilon}\varphi_{xx} + \frac{1}{2U}\varphi_{YY} + \frac{3}{2}\varphi_x\varphi_{xx} + \frac{U}{6}\varphi_{xxxx} = O(\varepsilon), \tag{2}$$

with boundary condition at the cut of ship location,

$$\varepsilon\frac{\partial\varphi}{\partial Y}|_{Y\to 0\pm} = \mp\frac{1}{2}\frac{S_o^*}{\varepsilon^{\frac{1}{2}} l^* h^*(1+\varepsilon\zeta_o)}\frac{d}{d\hat{x}}[\hat{S}(U-\varepsilon\varphi_x)|_{Y\to 0}] + \varepsilon V, \tag{3}$$

where φ is the depth-averaged potential, ζ is wave elevation, U is depth Froude number, V is velocity of cross flow, ζ_o is wave elevation along the ship sidewall,

$$\hat{S}(\hat{x},\tau) = S_o(\hat{x}) + \frac{b_o^* h^*}{S_o^*} b(\hat{x})[\frac{l^*}{h^*}(s-\hat{x}\theta) + \varepsilon\zeta_o(\hat{x},\tau)], \quad -0.5 < \hat{x} = \frac{x^*}{l^*} < 0.5, \quad \varepsilon^{\frac{3}{2}} = \frac{S_o^*}{l^* h^*}, \tag{4}$$

and all dimensional variables are marked by asterisk '*'. Wave elevation and the pressure acting on the ship hull can be expressed approximately as,

$$\zeta = U\varphi_x - \varepsilon\varphi_\tau - \frac{1}{2}\varepsilon\varphi_x^2 - \frac{\varepsilon U}{3}\varphi_{xxx} + O(\varepsilon^2), \quad p = \varepsilon\zeta|_{Y\to 0} - z + O(\varepsilon^2). \tag{5}$$

Then the wave resistance is given as

$$\begin{aligned} R_w &= \frac{R_w^*}{m^* g^*} = \frac{h^*}{l^* C_P}\int_{S_w}(\varepsilon\zeta_o - z)n_x dS \\ &= \frac{h^*}{l^* C_P}[-\int_{-1/2}^{1/2}\varepsilon\zeta_o\frac{b_o^* h^*}{S_o^*}\{\frac{d}{d\hat{x}}[S_o + b(\hat{x})(s-\hat{x}\theta)] + \varepsilon\zeta_o\frac{db}{d\hat{x}}\}d\hat{x} + \int_{-1/2}^{1/2}\frac{\varepsilon^2 h^* b_o^*}{2S_o^*}\zeta_o^2\frac{db}{d\hat{x}}d\hat{x}], \end{aligned} \tag{6}$$

where $C_P = V_o^*/S_o^* l^*$ is called as the longitudinal prismatic coefficient, V_o^* is static displacement volume of the ship, $\zeta_o = \zeta|_{Y\to 0}$, $s = s^*/l^*$ is sinkage, and θ is the trim angle. For a ship free to heave and pitch, s and θ are determined by two linear algebraic equations

$$\int_{-1/2}^{1/2}[\zeta + \frac{l^*}{\varepsilon h^*}(s-\hat{x}\theta)]b(\hat{x})d\hat{x} = 0, \quad \int_{-1/2}^{1/2}[\zeta + \frac{l^*}{\varepsilon h^*}(s-\hat{x}\theta)]b(\hat{x})(-\hat{x})d\hat{x} = 0. \tag{7}$$

The KP equation (2) with the boundary condition (3) is solved numerically by a fractional step scheme of finite difference method.

3 Procedure of Optimization

The wave resistance in our theoretical model geometrically depends on static cross sectional areas $S_o(\hat{x})$ and the waterline beams $b(\hat{x})$, i.e., it is their functional,

$$\bar{R}_w = R[S_o(\hat{x}), b(\hat{x})], \tag{8}$$

where $\bar{R}_w$ is time-averaged wave resistance. For a ship of given principal dimensions: length, beam, midship cross-section area and displacement, we select seven suitable interpolation functions for $\Delta S_o(\hat{x})$, including

$$f_n(\hat{x}) = \hat{x}^2\sin(2\pi n\hat{x}), \quad n = 1,2,3, \tag{9}$$

and four other polynomials of fifth degree which come from $T(x)$ and $N(x)$ in Gertler (1954) representing the tangents to the sectional area curve at the forward and after ends as well as the second derivatives of the curve on both sides of the midship section respectively. So that

$$S_o(\hat{x}) = S_{oo}(\hat{x}) + \sum_{n=1}^{7} a_n f_n(\hat{x}), \tag{10}$$

where S_{oo} is static sectional area of original model. For the present we leave $b(\hat{x})$ unchanged. Then the problem becomes

$$\bar{R}_w = R(a_1,\cdots,a_7) \text{ and } \frac{\partial}{\partial a_n}R = 0, \quad n = 1,\cdots,7. \tag{11}$$

We use direct numerical iteration to find relative minima of this multi-variable function.

4 Results

We select the Taylor Standard Series model of $C_P = 0.64$ with length $l^* = 3$ m, beam $b^* = 0.27845$ m and draft $d^* = 0.0928$ m as the original model, which was used in the systematic shallow water model experiments by Graff et al. (1964) in Duisburg shallow water towing tank.

Fig.1. shows the sectional area curves of modified and original models. The modified model was obtained by optimizing for $h^*/l^* = 0.125$ and depth Froude number $U = 0.9$. Fig.2. shows the calculated wave profiles along the hulls of modified and original models at $U = 0.9$, $h^*/l^* = 0.125$ and $\tau = 2.5$. Averaged wave resistances at several Froude numbers are listed in Table 1. At subcritical speeds $U = 0.8$ and 0.85 the modified model is worse than the original. At optimization point $U = 0.9$, wave resistance of the modified model is about eight percent less than that of the original. At supercritical speeds $U = 1.05, 1.10, 1.15$ and 1.2, the wave resistances decrease by about ten percent.

5 Discussion

It should be noticed that if nonlinear, unsteady and high-dispersion terms are omitted, KP equation (2) becomes identical to (3.9) in Tuck (1966) which was used in the farfield for depth Froude number range 0.5-1.5 except unity in his study. Comparing to Boussinesq equations, KP equation is still believed to be applicable only in a smaller speed range about depth Froude number unity. But the latter is much easier to solve numerically. Naturally, we ask whether one can derive a KP type equation including higher dispersion in y-direction that is equivalent to Boussinesq equations. Yes, one can. It is written down here:

$$2\varepsilon U\varphi_{x\tau} + (1-U^2)\varphi_{xx} + (1+\underline{\varepsilon U\varphi_x})\varphi_{yy} + \frac{\varepsilon U^2}{3}\varphi_{xxxx} + \underline{\frac{\varepsilon U^2}{3}\varphi_{xxyy}} + \frac{3\varepsilon U}{2}(\varphi_x^2)_x + \underline{2\varepsilon U\varphi_{xy}\varphi_y} = 0. \quad (12)$$

where $y = \varepsilon^{\frac{1}{2}} y^*/h^*$ and the underlined terms are additional to KP equation (2). It has the same precision and same applicable speed range as Boussinesq equations in the slowly time-varying problem, while it keeps KP equation's advantage of easy numerical solution.

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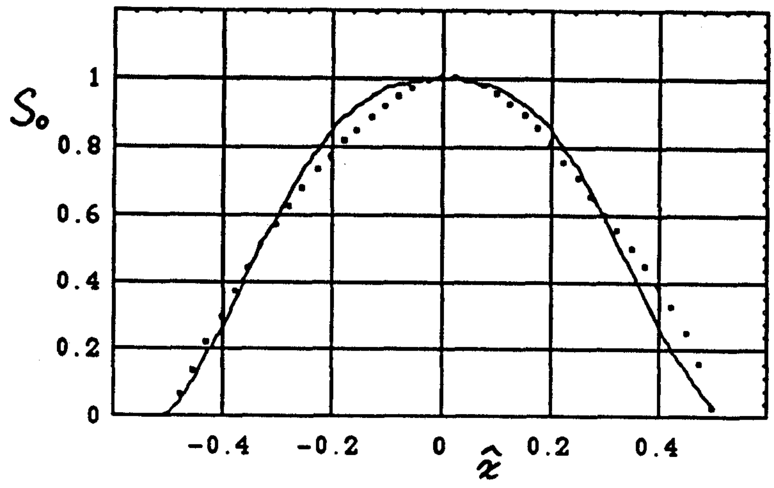

Figure 1: The sectional area curves where solid lines are drawn for original TSS model and dots for modified model optimized at $U = 0.9$ and $h^*/l^* = 0.125$.

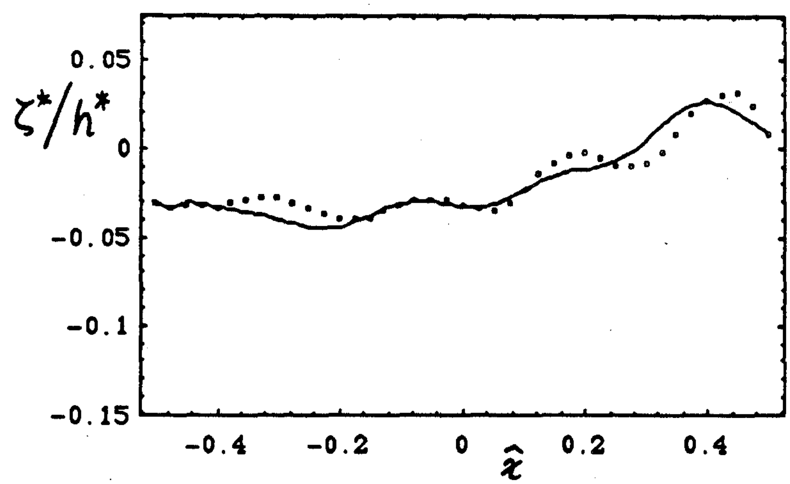

Figure 2: Calculated wave profiles along the ship hulls at $\tau = 2.5$, $U = 0.9$ and $h^*/l^* = 0.125$ where solid lines are drawn for original TSS model and dots for modified model.

$U = \frac{U^*}{\sqrt{g^* h^*}}$	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20
Exp. [5]	0.00484	0.00806	0.02692	0.03583	0.03609	0.03005	0.02628	0.02602	0.02613
Orig. R_{w0}	0.00375	0.00611	0.02240	0.03283	0.03475	0.02894	0.02620	0.02723	0.02701
Mod. R_{w1}	0.00433	0.00632	0.02059	0.03149	0.03353	0.02562	0.02312	0.02413	0.02456
$\frac{R_{w0}-R_{w1}}{R_{w0}}$	-0.155	-0.0344	0.0808	0.0408	0.0351	0.1147	0.1176	0.1138	0.0907

Table 1: Calculated time-averaged wave resistances for the original TSS model and modified model along with the experimental results of Graff et al. (1964) for the original TSS model at $h^*/l^* = 0.125$.