Force calculations using the linearized radiation potentials at steady forward speed

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A three-dimensional panel method is used to calculate the transient, linearized radiation potentials for a ship traveling with steady forward speed U. [2] [3] The fluid motion is assumed to be potential, free of separation and lifting effects. Under these assumptions, the total fluid velocity induced by the combination of steady translation and small unsteady motions may be expressed as the gradient of the potential,

$$\Phi_0(\vec{x}) + \sum_{k=1}^{6} \phi_k(\vec{x}, t) \qquad (1)$$

where the coordinate system $\vec{x}$ is traveling with the mean position of the body. The first term, Φ_0, represents the limiting form of the body's steady disturbance, while the remaining six terms are the radiation potentials. There is a radiation potential corresponding to an impulsive motion of the ship in each of its six rigid body modes of motion. [1] When this decomposition is substituted into the exact initial/boundary value problem, the free-surface and body boundary conditions may be linearized. The result of this linearization, however, depends upon what part of the steady potential is considered to be $\mathcal{O}(1)$. The simplest assumption to be made about the steady potential is that it is a small perturbation to the free-stream potential,

$$\Phi_0 = -Ux + \psi(\vec{x})$$

where both ψ and ϕ_k are $<< 1$. This assumption leads to the familiar Neumann-Kelvin linearization of the problem and the following boundary conditions,

$$\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)^2 \phi_k(\vec{x}, t) + g\frac{\partial \phi_k(\vec{x}, t)}{\partial z} = 0 \qquad \text{on } z = 0 \qquad (2)$$

$$\frac{\partial \phi_k}{\partial n} = n_k \dot{\alpha}_k(t) + m_k \alpha_k(t) \qquad \text{on } S_0 \qquad (3)$$

where, n_k is the generalized unit normal to the body, α_k and $\dot{\alpha}_k$ are the body's unsteady modal displacements and velocities respectively, and $m_k = (0, 0, 0, 0, U n_3, -U n_2)$. There exists a Green function which solves this linearization of the initial/boundary value problem (with the exception of the body boundary condition) and it may be written,

$$G(\vec{x}; \vec{\xi}, t - \tau) = G^{(0)} + G^{(f)} = \left(\frac{1}{r} - \frac{1}{r'}\right) + 2\int_0^\infty dk\, [1 - \cos(\sqrt{gk}\,(t - \tau))] \exp(kZ)\, J_0(kR) \qquad (4)$$

This Green function may be used to derive an integral equation for the radiation potentials and it is upon the solution to this equation that the results presented here will be based.

$$\begin{aligned} 2\pi\phi + \iint_{S_0} dS\, \phi\, G_n^{(0)} + \int_0^t d\tau \iint_{S_0} dS\, \phi\, G_{tn}^{(f)} \qquad (5) \\ -\frac{U}{g}\int_0^t d\tau \int_{\Gamma_0} n_1\, dl \left[G_t^{(f)}(2\phi_\tau - U\phi_\xi) + U\phi\, G_{t\xi}^{(f)}\right] \\ = \iint_{S_0} dS\, G^{(0)}\phi_n + \int_0^t d\tau \iint_{S_0} dS\, \phi_n\, G_t^{(f)} \end{aligned}$$

(Subscripts here denote partial differentiation, and the spatial integrals are to be carried out over the mean position of the body surface, S_0, and the mean waterline, Γ_0.) Once the radiation potentials have been calculated, the force on the body in mode j due to an impulse in mode k (the impulse response function) may be calculated by integrating the consequent pressure over the body:

$$F_{jk} = -\rho \int\int_{S_0} dS \, p_k \, n_j \tag{6}$$

where the correct linearizion of the Bernoulli equation is in this case,

$$p_k = -\rho \left(\frac{\partial \phi_k}{\partial t} - U \frac{\partial \phi_k}{\partial x} \right) \tag{7}$$

In practice, the force is not usually calculated directly from the combination of equations (6) and (7), but instead an extension of Stokes' theorem (Tuck's theorem) is used to write the force as,

$$F_{jk} = -\rho \int\int_{S_0} dS \left(\frac{\partial \phi_k}{\partial t} n_j - \phi_k \, m_j \right) \tag{8}$$

This form is more convenient since it does not involve any spatial derivatives of the potential. This manipulation, however, is hard to justify since it relies on the $\mathcal{O}(1)$ part of the steady potential satisfying the steady body boundary condition (which the free-stream potential clearly does not do). Even so, it is interesting to compare the results calculated directly through (6) with those calculated using (8). The most striking difference between the two is that calculations made using equation (8) preserve the symmetry between the zero frequency limits of the cross-coupling damping coefficients while calculations made using equation (6) do not. (This will be exhibited at the workshop.)

Other assumptions about the steady flow can be made as well, for example the steady potential might be considered as a small perturbation to the double-body potential (that due to the flow around the body plus its reflection about the $z = 0$ plane in an infinite fluid.) Such a linearization leads to a different pair of free-surface and body boundary conditions. [5] It is hard to imagine a Green function which solves the free-surface condition when linearized about the double-body flow, since it would neccessarily be a function of the body's geometry, but this particular linearization of the ship motions problem has been solved successfully using a Rankine panel method. [4] The body boundary condition obtained from this linearization is identical to (3) except that the 'm-terms' are now given by,

$$\begin{aligned} (m_1, m_2, m_3) &= -(\vec{n} \cdot \vec{\nabla}) \vec{\nabla} \Phi_{db}(\vec{x}) \\ (m_4, m_5, m_6) &= -(\vec{n} \cdot \vec{\nabla})(\vec{r} \times \vec{\nabla} \Phi_{db}(\vec{x})) \end{aligned} \tag{9}$$

where Φ_{db} is the double-body potential. Using the double-body 'm-terms' while satisfying the Kelvin linearized free-surface condition is hard to justify, (although a bulbous bow might be such a situation) however, in the spirit of a numerical experiment, results will be presented which have been calculated using equation (5), and the 'double-body' m-terms. A fairly elaborate procedure has been developed to implement the solution with double-body m-terms. The extra effort is motivated by the desire to avoid the difficulties associated with calculating second gradients of the potential, when this potential has been calculated from a form of equation (5) which been discretized using constant strength, planar panels. The procedure is as follows: An application of Stokes' theorem is used to replace the m-terms in equation (5) with gradients of the double-body potential,

$$\int\int_{S_0} d\vec{\xi} \, m_k(\vec{\xi}) G(\vec{x}; \vec{\xi}, t) = -\int\int_{S_0} d\vec{\xi} \, n_k(\vec{\xi}) \, \vec{\nabla} \Phi_{db}(\vec{\xi}) \cdot \vec{\nabla}_{\xi} G(\vec{x}; \vec{\xi}, t) \tag{10}$$

Equation (5) is then solved to get the radiation potentials. The properly linearized Bernoulli equation in this case is,

$$p_k = -\rho\left(\frac{\partial \phi_k}{\partial t} + \vec{\nabla}\Phi_{db} \cdot \vec{\nabla}\phi_k\right) \tag{11}$$

which means that gradients of the potential are required, along with the potential itself, in order to calculate the impulse response functions. In order to avoid taking these gradients by some sort of finite difference scheme, the values of the radiation potentials are now used to solve a first kind integral equation for the source strengths,

$$\begin{aligned}\phi(\vec{x},t) = & \iint_{S_0} dS\; G^{(0)}\sigma(t) + \int_0^t d\tau \iint_{S_0} dS\; G_t^{(f)}(t-\tau)\,\sigma(\tau) \\ & -\frac{U_0^2}{g}\int_0^t d\tau \int_{\Gamma_0} n_1^2 dl\; G_t^{(f)}(t-\tau))\,\sigma(\tau)\end{aligned} \tag{12}$$

With the source strengths known, the gradients of the potentials may be calculated by taking the gradient of equation (12). Finally, with both the potentials and their gradients known, equation (11) may be used in equation (6) to calculate the impulse response functions. Results from these calculations will also be presented and compared to the consistent Neumann-Kelvin results, as well as to results obtained using a Rankine panel method.

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References

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