Wave Resistance of a Submerged Body Moving with an Oscillating Velocity

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There are numerous papers treating stationary waves due to the forward motion of a body (see, e.g., [1,2] and bibliography cited there). Non-stationary ship waves are investigated to a lesser degree (see [3,4]). The situation is opposite from the mathematical point of view. Some solvability and uniqueness theorems are proved for different formulations of the linear initial value problem describing forward motion of a submerged body (see [5,6]). In the same time only for 2D Neumann - Kelvin problem the well-posed formulations exist [7].

In this work the effect of high-frequency oscillations of forward velocity on the wave-making resistance is cosidered. Let a body is submerged into an incompressible inviscid heavy fluid of infinite (for simplicity) depth. Let its dimensionless velocity has the form $U(t/\varepsilon)$, where $U(\tau)$ is a positive differentiable function having the unit period and t is the dimensionless time. Only dimensionless quantities will be used in the paper. It means that t is obtained by dividing the dimensional time by, e.g., $(l/g)^{1/2}$, where g is the acceleration of gravity and l is the characteristic length. In the same manner U is obtained by dividing of dimensional velocity by $(lg)^{1/2}$. If $\varepsilon \ll 1$, then the velocity oscillates at the frequency, which is high in comparison with $(g/l)^{1/2}$. In this case the singular perturbations method is applicable.

Our aim is to derive the asymptotic formula for the wave-making resistance $R(t,\tau)$ of the body, $\tau = t/\varepsilon$. Then we shall compare the mean value $\langle R\rangle = \int_0^1 R\, d\tau$ with the wave-making resistance $R_0(t)$ of the same body moving with the mean velocity $\langle U\rangle$. Numerical computations for the 2D problem show that there exist cylinders such that $|\langle R\rangle| < |R_0|$ (up to a term $O(\varepsilon)$).

Formulation and solution

Let a solid body occupies the domain $D \subset R^3_- = \{ (x,y,z) : y < 0, (x,z) \in R^2 \}$ and D is bounded by the closed surface $S \subset R^3_-$. Let the plane $\{ y=0, (x,z) \in R^2 \}$ is the free surface of fluid at rest. So the body is submerged and it moves in the direction of the x-axis with the velocity $U(t/\varepsilon)$. We choose vertical size of the body as the characteristic length l.

We seek a pair (Φ, η) satisfying

$$\left.\begin{array}{ll} \nabla^2 \Phi = 0 \quad \text{in } W = R^3_- \setminus \overline{D} & (1)\\ \Phi_t - U\Phi_x + \eta = 0, \quad y = 0 & (2)\\ \eta_t - U\eta_x - \Phi_y = 0, \quad y = 0 & (3)\\ \partial\Phi / \partial n = U \cos(n,x) \quad \text{on } S & (4) \end{array}\right\} \; t \geq 0$$

$$\left.\begin{array}{ll} \Phi = f_0, \quad y = 0 & (5)\\ \eta = f_1, \quad y = 0 & (6) \end{array}\right\} \; t = 0$$

Thus $\Phi(x,y,z,t,\varepsilon)$ can be regarded as a velocity potential of induced waves in the coordinate system moving with the body, $\eta(x,z,t,\varepsilon)$ is the corresponding elevation of free surface. The unit normal $\vec{n}$ is directed into W.

For the pair (Φ, η) satisfying (1) - (6) with $U(t/\varepsilon)$ described above the following asymptotic fomulae are true as $\varepsilon \longrightarrow 0$:

$$\Phi = [\, U(t/\varepsilon) - \langle U \rangle \,]\, v_0(x,y,z) + \Phi_0(x,y,z,t) + \varepsilon [\, \beta(t/\varepsilon)\, v_1(x,y,z,t) + \Phi_1(x,y,z,t) \,] + O(\varepsilon^2) \tag{7}$$

$$\eta = \eta_0(x,z,t) + \varepsilon \{ \beta(t/\varepsilon) [(\partial \eta_0/\partial x)(x,z,t) + (\partial v_0/\partial y)(x,0,z)] + \eta_1(x,z,t) \} + O(\varepsilon^2) \tag{8}$$

Here

$$\beta(\tau) = \int_0^1 (\partial G/\partial \tau)(\tau, \sigma)\, U(\sigma)\, d\sigma , \quad 0 \leqslant \tau \leqslant 1$$

$$G(\tau, \sigma) = H(\tau - \sigma) - (\tau - \sigma) - 1/2$$

where H is Heaviside-function. To the half-axis $\tau > 1$ the function $\beta(\tau)$ is extended periodically. The function $G(\tau,\sigma)$ is generalized Green function of periodic boundary value problem for the operator $d/d\tau$.

The functions v_m $(m = 0,1)$ must be determined from:

$$\nabla^2 v_m = 0 \text{ in } W, \; v_m = \delta_{1m} (\partial \Phi_0/\partial x) \text{ for } y = 0$$
$$\partial v_m/\partial n = \delta_{om} \cos(n,x) \quad \text{on } S \tag{9}$$

The pairs (Φ_m, η_m) $(m = 0,1)$ are the solutions of the initial-boundary value problems

$$\left.\begin{aligned} &\nabla^2 \Phi_m = 0 \text{ in } W && (10)\\ &\frac{\partial \Phi_m}{\partial t} - \langle U \rangle \frac{\partial \Phi_m}{\partial x} + \eta_m = 0, \; y = 0 && (11)\\ &\frac{\partial \eta_m}{\partial t} - \langle U \rangle \frac{\partial \eta_m}{\partial x} - \frac{\partial \Phi_m}{\partial y} = 0, \; y = 0 && (12)\\ &\partial \Phi_m/\partial n = \delta_{om} \langle U \rangle \cos(n,x) \quad \text{on } S && (13)\end{aligned}\right\} \; t \geqslant 0$$

$$\left.\begin{aligned} &\Phi_m = [-\beta(0)]^m \partial^m f_0/\partial x^m, \; y = 0 && (14)\\ &\eta_m = [-\beta(0)]^m \left[\frac{\partial^m f_1}{\partial x^m} + \delta_{1m} \frac{\partial v_0}{\partial y} \right], \; y = 0 && (15)\end{aligned}\right\} \; t = 0$$

Here δ_{nm} is the Kronecker delta.

Thus Φ_0 can be regarded as the velocity potential of waves due to the body at the forward speed $\langle U \rangle$, η_0 is the corresponding elevation of free surface. The formula (8) demonstrates that the free surface elevations η and η_0 coinsides up to a term $O(\varepsilon)$. But the same is not valid for potentials Φ and Φ_0.

For m = 0 the problems (9) and (10)-(15) can be solved independently of each other. Then we have to find v_1 from (9), and at last we obtain the solution Φ_1 of the problem (10) - (15). In analagous manner one can derive an arbitrary number of terms to prolong the expansions (7) and (8).

Wave resistance and other characteristics

Applying usual formula we get from (7) that for the force $\vec{F}(t,\tau)$ acting on the body the following asymptotics is true

$$\vec{F} = \frac{U'(\tau)}{\varepsilon} \int_S v_o \vec{n}\, dS + \int_S \left(\frac{\partial \Phi_o}{\partial t} - \langle U \rangle \frac{\partial \Phi_o}{\partial x} \right) \vec{n}\, dS$$

$$- [\, U(\tau) - \langle U \rangle \,]^2 \int_S \frac{\partial v_o}{\partial x} \vec{n}\, dS$$

$$+ [\, U(\tau) - \langle U \rangle \,] \int_S [\, v_1 - \langle U \rangle \frac{\partial v_o}{\partial x} - \frac{\partial \Phi_o}{\partial x} \,] \vec{n}\, dS + O(\varepsilon)$$

Then, averaging in variable τ we find that

$$\langle \vec{F} \rangle = \vec{F}_o(t) - (\langle U^2 \rangle - \langle U \rangle^2) \int_S \frac{\partial v_o}{\partial x} \vec{n}\, dS + O(\varepsilon) \qquad (16)$$

where

$$\vec{F}_o(t) = \int_S \left(\frac{\partial \Phi_o}{\partial t} - \langle U \rangle \frac{\partial \Phi_o}{\partial x} \right) \vec{n}\, dS$$

is the force acting on the body moving with the mean velocity $\langle U \rangle$. The second term in (16) is proportional to the dispersion of velocity $\langle U^2 \rangle - \langle U \rangle^2 \geqslant 0$.

Let consider the horizontal component $R(t,\tau)$ of force (wave-making resistance). Using the boundary value problem (9) we get from (16) that

$$\langle R \rangle = R_o(t) - (\langle U^2 \rangle - \langle U \rangle^2)\, I_o + O(\varepsilon) \qquad (17)$$

$$I_o = \int_S \frac{\partial v_o}{\partial x} \cos(n,x)\, dS = \frac{1}{2} \int_S |\nabla v_o|^2 \cos(n,x)\, dS$$

$$R_o(t) = \int_S \left(\frac{\partial \Phi_o}{\partial t} - \langle U \rangle \frac{\partial \Phi_o}{\partial x} \right) \cos(n,x)\, dS$$

The last is the wave resistance of the body at the constant speed $\langle U \rangle$. The asymptotic formula for R then may be written in the following form

$$R(t,\tau) = \langle R \rangle - \frac{U'(\tau)}{\varepsilon} \int_W |\nabla v_o|^2\, dxdydz$$

$$+ [\, \langle U \rangle - U(\tau) \,] \int_S \left(\frac{\partial v_o}{\partial x} \frac{\partial \Phi_o}{\partial n} + v_o \frac{\partial^2 \Phi_o}{\partial x \partial n} \right) dS + O(\varepsilon)$$

The supplied power can be obtained multiplying (-R) by U. Then the average supplied power is given by the following expression

$$-\langle U \rangle R_o + (\langle U^2 \rangle - \langle U \rangle^2) \{ \langle U \rangle I_o + \int_S \left(\frac{\partial v_o}{\partial x} \frac{\partial \Phi_o}{\partial n} + v_o \frac{\partial^2 \Phi_o}{\partial x \partial n} \right) dS \} + O(\varepsilon)$$

It differs both from the power required for the motion at the mean speed $\langle U \rangle$ and from the power required for overcoming of the mean wave resistance $\langle R \rangle$.

Discussions and numerical examples

Formula (17) shows that the sign of the difference $\langle R \rangle - R_o$ depends on the value of I_o. As wave resistance is directed opposite to the x-axis then we have the inequality $|\langle R \rangle| \leqslant |R_o|$ if $I_o \leqslant 0$. So, it is of interest to find bodies with $I_o < 0$.

It is easy to see that $I_o = 0$ if the body is symmetric about the middle-plane (without loss of generality we can choose $x = 0$ as the middle-plane). Indeed, in this case $\cos(n,x)$ is an odd function of x. Then the solution $v_o(x,y,z)$ of the boundary value problem (9) is an odd function of x. The same is true for the integrand in I_o.

For numerical example the 2D problem (9) with an isosceles triangle ABC (see fig.) as contour S is convenient. For any contour S we have

$$I_o = \int_S [\cos^3(n,x) + (\partial v_o/\partial s)\cos(s,x)\cos(n,x)]\, dS$$

In the case of triangle ABC we get $\int_{ABC} \cos^3(n,x)\, dS = -\sin^2\alpha$

$$\int_{ABC} (\partial v_o/\partial s)\cos(s,x)\cos(n,x)\, dS = \frac{1}{2}[2v_o(B) - v_o(A) - v_o(C)]\sin 2\alpha$$

So, $I_o(ABC) < 0$ if $2v_o(B) - v_o(A) - v_o(C) < \mathrm{tg}\,\alpha$. For triangles that correspond to the points above the curve (see fig.) the inequality $I_o(ABC) < 0$ holds. The opposite inequality takes place for triangles that correspond to the points below the curve. For a triangle AB'C which is symmetric about y-axis with any triangle ABC shown in fig. the inequlity $I_o(AB'C) > 0$ is valid.

Numerical results are also obtained for right triangles that have one of its legs on the y-axis.

References

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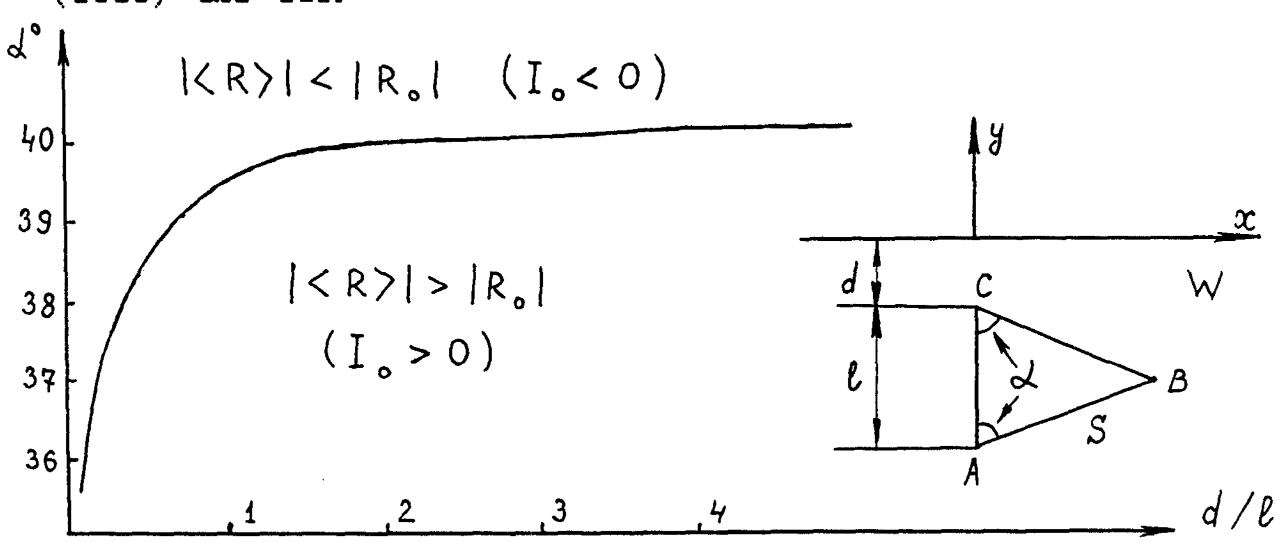

DISCUSSION

TUCK: Is there any circulation around your body? I ask because your body has sharp corners where in practice one would have to use a Kutta condition to fix the circulation?

KUZNETSOV: In the performed calculations the vertices had been replaced by small circular arcs. Hence, there was no need in the Kutta condition and circulation. Moreover, 2D problem was considered only to simplify numerical examples. My formulas are true for 3D problem.