Free Surface Flows for Semi-Displacement Craft

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Introduction

This abstract presents an application of a three-dimensional Rankine panel method for the solution of transom stern wave flows. Semi-displacement craft comprise the general class of vessels to be studied.

Semi-displacement craft, characterized by transom sterns and operating at moderate Froude numbers, demand attention to the stern flow. At moderate Froude numbers, the bow flow and spray are not crucial considerations. The behavior of the flow at transom sterns is the most imprtant complication to existing theory.

This paper is an initial numerical investigation intended to provide insight into the treatment of deep transom sterns within the framework of potential flow theory. Shallow transom sterns can be treated adequately by linear theory. At present only the steady problem is considered.

The discussion will begin with consideration of the non-linear conditions governing the boundary value problem. A Kutta condition must be applied to insure a unique solution. Two simplified models of this problem will then be presented. A study of the zero cavitation number flow will provide information about the affect of trailing dipole sheets on the upstream flow. A quasi-linear model for the wave flow will provide information about the position of the trailing wake sheet and the sensitivity of the Kutta condition to this position.

Formulation

Potential flow is assumed for this boundary value problem.

The boundaries for this problem are the body, where a no-flux condition is imposed, a wake region aft of the transom stern, and the free surface outside the wake region. The wake region consists of a free surface with continuity imposed on the velocity potential and its derivatives in order to insure tangency of flow and continuity of pressure. For the fully nonlinear problem, both streamwise and tranverse velocity components must be continuous.

The intersection of the wake region and the outer free surface which may take the form of a vortex segment is not discussed in this presentation.

Two idealizations of the nonlinear boundary value problem are discussed here. The first is a zero cavitation number flow for the double-body semi-displacment craft. This can be considered as the limiting case for infinite Froude number where gravity is insignificant. For this first model, the cavity position is assumed fixed and a dipole sheet with an imposed strength is applied to study the affect of a wake sheet on the body pressure.

The second model is a quasi-linear approach to the problem. The craft is studied at finite Froude numbers. Continuity of potential, streamwise velocity, and streamline slope will be enforced.

A standard Neuman-Kelvin linearization is employed for this second model using the decomposition of the total potential into a free-stream and perturbation potential, $\phi(\vec{x})$,

$$\Phi(\vec{x}) = -Ux + \phi(\vec{x}) \tag{1}$$

The boundary of interest is the free-surface trailing the transom stern. If the transom is assumed not to be wall-sided, a finite slope for the transom bottom, then a quasi-linear approach can be taken. The wave flow is assumed to be linear in the direction of the flow but not transversly across the transom. This leads to a modified free surface condition with a non-linear term retained from the kinematic boundary condition,

$$\frac{\partial \phi}{\partial z} = -\frac{U^2}{g}\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial \zeta}{\partial y}\frac{\partial \phi}{\partial y} \tag{2}$$

$$\text{with} \quad \zeta(x,y) = \frac{U}{g}\frac{\partial \phi}{\partial x} \quad \text{on } z = 0. \tag{3}$$

The term including the wave slope, ζ_y, is strictly nonlinear and could be considered in an iterative scheme. If the transom depth is zero or very small, this term can be neglected, but for many semi-displacement craft, the transom slope may be significant. In this study, the term is treated in an approximate sense. The transverse wave slope just aft of the transom stern is assumed to be equal to the transverse slope of the body at the stern. Using this assumption, the extra term becomes an additional forcing in the linearized problem.

Numerical Implementation

The governing equations are expressed in boundary-integral form through a derivation employing Green's second identity for the unknown potential, ϕ, on the free surface, S_F, body, S_B, and wake, S_W,

$$2\pi\phi(\vec{x}) - \iint_{S_F \cup S_W} \frac{\partial \phi(\vec{\xi})}{\partial z} G(\vec{\xi};\vec{x})d\xi + \iint_{S_F \cup S_W \cup S_B} \phi(\vec{\xi})\frac{\partial G(\vec{\xi};\vec{x})}{\partial n}d\xi$$
$$= \iint_{S_B} \frac{\partial \phi(\vec{\xi})}{\partial n} G(\vec{\xi};\vec{x})d\xi, \tag{4}$$

The Green function satisfying the Laplace equation and condition at infinity is the Rankine source potential,

$$G(\vec{\xi};\vec{x}) = \frac{1}{|\vec{x} - \vec{\xi}|}. \tag{5}$$

The numerical scheme used to solve this boundary-integral equation is the bi-quadratic panel method of Nakos and Sclavounos (1990). This method provides a convenient representation for the boundary conditions and has been shown to contain no numerical dispersion. The verification of this method for monohulls has been extensive and successful.

Results

The Kutta condition was the focus for the numerical investigation.

The first model, zero cavitation flow, was studied in order to appreciate the affect of varying the wake strength on body pressure. Numerical investigations confirmed the experience gained from the study of lifting surfaces. For a highly three-dimensional flow, such as that encountered with transom sterns, a nonlinear pressure condition must be satisfied at the trailing edge. Attempting to satisfy zero linear pressure at the body trailing edge results in a boundary value problem with no solution. A Newton-Raphson iterative method was applied to find the wake strength for which the non-linear Kutta condition was satisfied in this cavity problem. This corresponds to a need to impose continuity of transverse as well as streamwise velocities.

The second study investigated the sensitivity of the body pressure to the free surface condition. A linear and a quasi-linear model were employed with tangency of flow imposed at the transom stern on the free-surface side. Although different wave patterns downstream of the body were observed, there was no appreciable change in the pressure on the body. The Kutta condition, was imposed at the transom stern in two variations. The first was continuity of potential and its first streamwise derivative, the linearized wave elevation. The second variation was continuity of wave elevation and slope. For both these variations, the Kutta condition was not adequately satisfied. Discontinuity in pressure existed.

References

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DISCUSSION

RAVEN: Your paper states that a vortex sheet is required to model the flow near the transom. I am not quite convinced of that. A Kutta condition states the pressure on both sides of a sharp edge must be equal; whatever the pressure. Here, you want to prescribe a particular pressure level, which is basically a Dirichlet condition. Also, the physical mechanism is different, since there is fluid only on one side of the dividing streamline. A more appropriate model could be the 'free streamline theory'. This might lead to different results. For example it allows the curvature of a streamline tends to infinity at the transom edge (G.H. Schmidt, JSR, circa 1980).
Irrespective of the use of a Kutta condition you may of course use a vortex sheet aft of the transom. This is just a singularity distribution on the boundary, used to satisfy a boundary condition, but does not imply a circulation or lift, so there is no need for a vortex sheet. Experience with other linearized models shows that it is possible to satisfy the conditions at the transom edge without using a vortex sheet. Nevertheless, it may be that your implementation has certain advantages. I will be pleased with your comments on this.
KRING: I apologize for the confusion in terminology. Through the application of Green's theorem, I distribute a "wake" sheet which is in fact, my free surface. The free surface condition imposes the position and strength of the trailing sheet, which as it consists of a dipole distribution, can be related to a vortex sheet.
I use "Kutta condition" as my terminology as I am enforcing tangency of flow and continuity of pressure. My case is analogous to a cavity flow rather than, simply, a lifting flow. The Kutta condition is a necessary jump condition along the streamline as its leaves the body boundary and becomes the free surface. The Kutta condition imposes a unique solution on the trailing free surface.

JENSEN: The Kutta condition on the transom could also be satisfied by source distribution outside the fluid domain. What is really the motivation for the vortex sheet? I think the major problem is that the nonlinear free-surface condition is required near the deeply submerged transom.
KRING: I am using a boundary integral approach to this problem, so through Green's theorem, the free surface can be considered as a distribution of source and dipole singularities. When I use the term, wake, in this context, I refer to the free surface.
My numerical study indicated that the Kutta condition was not strongly dependant upon the free surface condition. There was no difference in the body pressure between the linearized and quasi-linear models. The failure of this model lies in the fact that I can only impose a linearized version of the Kutta condition. In other words, I impose continuity of only streamwise velocities. It may turn out that a linear free surface condition in conjunction with a nonlinear treatment of the flow precisely at the trailing edge may be sufficient. This is beyond the ability of my numerical model at present, however.

YEUNG: In your quasi-linear condition, is it not true that the "higher-order" term simply reduces to zero when the side walls at the transom are parallel?

DISCUSSION

KRING: The transom slope, ζ, is defined as a function of y, the transverse coordinate. For vertical side walls, ζ_y becomes infinite and a new expression will be required for the free surface condition. In my study I have restricted ζ_y to be finite.
The case where the transom slope is zero is simply the zero-draft transom and the standard linear free surface condition is retrieved.