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THE INVERSE SHIP-WAVE PROBLEM

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The free-surface elevation in the far field, behind a ship which moves with constant velocity U, can be expressed in the form

$$\varsigma(x,y) = \mathrm{Re} \int_{-\pi/2}^{\pi/2} H(k_0 \sec\theta)\, e^{i\mathbf{k}\cdot\mathbf{x}} \sec^3\theta \, d\theta \qquad (1)$$

Here θ is the angle of each wave component with respect to the longitudinal x-axis, $k_0 = g/U^2$, the vector wavenumber $\mathbf{k}$ is horizontal with components $(k_x = k_0 \sec\theta)$ and $(k_y = k_0 \sec^2\theta \sin\theta)$, and H is the free-wave spectrum (or Kochin function). If the ship is represented by a longitudinal source distribution $\sigma(x)$, the free-wave spectrum is defined by the Fourier transform

$$H(k) = \int_{-\infty}^{\infty} \sigma(x)\, e^{ikx} dx, \quad (k \geq k_0) \qquad (2)$$

The corresponding wave resistance

$$R = \int_{-\pi/2}^{\pi/2} |H(k_0 \sec\theta)|^2 \sec^3\theta \, d\theta \qquad (3)$$

can be derived from energy radiation in the far-field. (For simplification multiplicative factors are omitted in (1-3), involving U, g, and the fluid density; these constants can be absorbed in the definitions of the parameters ς, H, and R.)

With appropriate modifications of the variable of integration, (1) can be expressed as a Fourier integral with respect to the longitudinal (k_x) or transverse (k_y) component of the wavenumber. Thus from Fourier inversion it is possible to evaluate H from either a longitudinal or transverse 'wave-cut' in terms of the wave amplitude. (The longitudinal wave slope also is required in the latter case.) The wave resistance (3) can be evaluated on this basis, as discussed by Eggers, Sharma & Ward (1967).

Is it possible, in an analogous manner, to specify the wave amplitude (1) and calculate the effective source strength σ? Since H can be evaluated from ς by the wave-cut procedure, this question can be re-stated in a simpler form: is it possible to solve (2) as an integral equation for the unknown σ? If $H(k)$ were known for *all* real values of k Fourier inversion would be straightforward, and since σ is real it would be sufficient to specify $H(k)$ for $k \geq 0$. But there is a gap between zero and k_0 which corresponds, in some sense, to the local (evanescent) wave field. Does this mean that σ is nonunique?

Before this question can be answered some restrictions must be imposed. With appropriate nondimensionalization of the coordinates, it will be assumed here that $\sigma = 0$ for $|x| > 1$ and that σ

is a sufficiently well-behaved function to be approximated by polynomials in $(-1,1)$. With these restrictions (2) is an entire function (analytic in the complex k-plane for all $|k|<\infty$), with specified values on the segment $k \geq k_0$. Thus, from a theorem in complex-variable theory (*cf.* Churchill, *et al*, 1974, §106), $H(k)$ is unique for all finite values of k, and σ is defined uniquely by the inverse Fourier transform. From a different perspective, Krein (see Kostyukov, 1959, §40) has shown that the wave resistance (3) is nonzero, except for pathological cases which violate our restrictions. Thus H cannot vanish for all wavenumbers. Since the difference between two source strengths σ with the same free-wave spectrum (2) would give $H \equiv 0$, it follows that σ is unique for any specified H.

The 'inverse ship-wave problem,' where we seek to identify a ship from its wave system, is a motive for solving the integral equation (2). In addition, it has been an unproven assumption in ship-wave theory that any practical hull form can be represented by a longitudinal source distribution. This assumption is justified in the context of slender-body theory, where the source strength is proportional to the first derivative $S'(x)$ of the ship's sectional-area curve. The existence of such a simple singularity distribution is less obvious if a ship of finite transverse dimensions is considered. It is plausible to assume that (1-2) apply in this more general context, at least for the far-field waves, with σ the ship's 'effective' source strength. This issue can be resolved if a solution of (2) is shown to exist for any physically relevant free-wave spectrum.

In order to calculate σ from H, we assume an expansion in Legendre polynomials,

$$\sigma(x)=\sum_{n=0}^{\infty} c_n P_n(x) \tag{4}$$

The corresponding slender-body approximation of the ship's sectional area is given by

$$S(x)=\int_{-1}^{x}\sigma(x)\,dx=c_0(1+x)+\sum_{n=1}^{\infty}\frac{c_n}{2n+1}\Big[P_{n+1}(x)-P_{n-1}(x)\Big] \tag{5}$$

If the conditions $S(\pm 1)=0$ are imposed, $c_0=0$. Substituting (4) in (2) gives the corresponding expansion for the free-wave spectrum in the form

$$H(k)=2\sum_{n=0}^{\infty} i^n c_n\, j_n(k) \tag{6}$$

where j_n is the spherical Bessel function. Our objective is to calculate the coefficients c_n, for given functions H.

At first glance the idea of solving (6) numerically for its coefficients does not seem promising. For a Froude number equal to 0.3, for example, $k_0 = 5.55...$ and the range of practically useful data might be restricted to, say, $k_0 \leq k \leq 2k_0$. Since the spherical Bessel functions $j_n(k)$ are remarkably similar for moderate values of n and k (*cf.* Abramowitz & Stegun, 1964, Figure 10.1), numerical instabilities can be anticipated.

Despite this pessimistic outlook it is possible to calculate the coefficients in (6) using a least-squares algorithm. Two simple cases are used here for illustration, corresponding to the sectional-area curves $S_1(x)=\cos(\pi x/2)$ and $S_2(x)=\cos^2(\pi x/2)$. With $\sigma_m(x)=S'_m(x)$ substituted in (2) the corresponding free-wave spectra H_m are given by

$$H_m=\frac{\pi i}{2}\Big[j_0(k+\frac{\pi}{2}m)-j_0(k-\frac{\pi}{2}m)\Big],\quad (m=1,2) \tag{7}$$

where $j_0(x) = (\sin x)/x$. These spectra are plotted in Figure 1.

Since the coefficients c_n are real, only the odd terms in (6) are required to fit (7). We truncate (6) after the term $n = 2N - 1$, assign the values (7) at M evenly spaced points in a finite interval (k_1, k_2) with $M \geq N$, and derive a linear system of N equations for the odd coefficients c_n by minimizing the sum of squares of the residual errors. As anticipated, the linear system is poorly conditioned if N is substantially larger than the interval $k_2 - k_1$, but converged results with 3 or 4 decimals accuracy for the sectional-area curves $S_{1,2}$ are obtained with $N = 3$ or 4. These results were obtained using LINPACK subroutines in double precision (15-16 decimals).

The sectional-area curves plotted in Figure 2 have been computed in this manner, taking an arbitrary interval in the wavenumber space of Figure 1, typically with $k_2 - k_1 = 5$, $N = 5$, and $M = 20$. No changes were discerned in the fourth decimal place when these parameters were varied moderately. (The results in Figure 2 are unchanged when $N = 10$, but the LINPACK estimate of the condition number is then of order $10^{15} - 10^{20}$! The growth in error of c_n is obvious if N is too large or the interval $k_2 - k_1$ is too small; an adaptive algorithm might rely on truncation when $|c_n|$ is a minimum, in an analogous manner to an asymptotic sequence.)

No computational difficulties are envisaged in applying this method to actual free-wave spectra for practical ships, aside from the obvious difficulties of accurate wave measurement. One limitation of the method, somewhat masked by our nondimensionalization, is the need to specify in advance the ship's velocity or to determine this parameter by further analysis of the wave pattern. Another point to note is that the ship's length is not determined.

Finally we emphasize that the effective source strength $\sigma(x)$ is uniquely determined from the far-field wave system. To the extent that slender-body approximations are valid the sectional area $S(x)$ is likewise unique, and it is fundamentally impossible to identify other features of the hull shape from the far-field waves.

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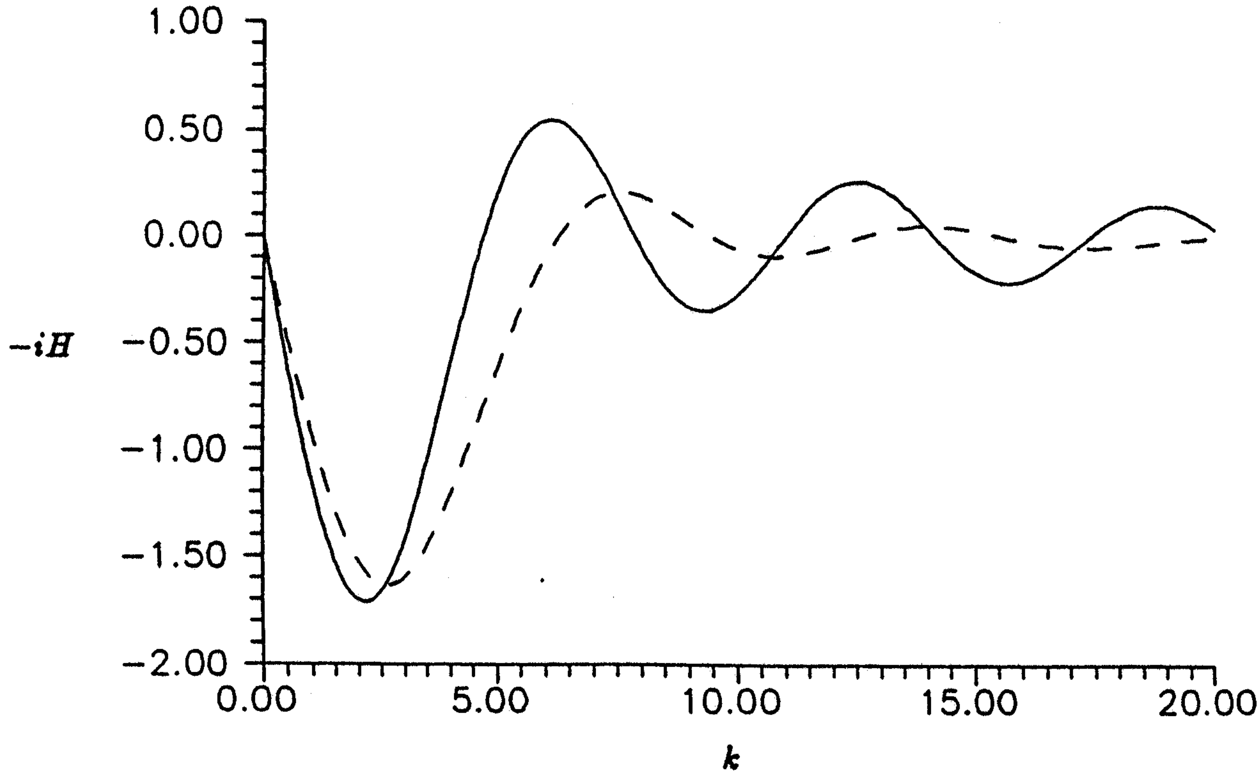

Figure 1 – Plot of the free-wave spectra $H_{1,2}(k)$ defined by (7) (solid curve is for m=1; dashed curve is for m=2).

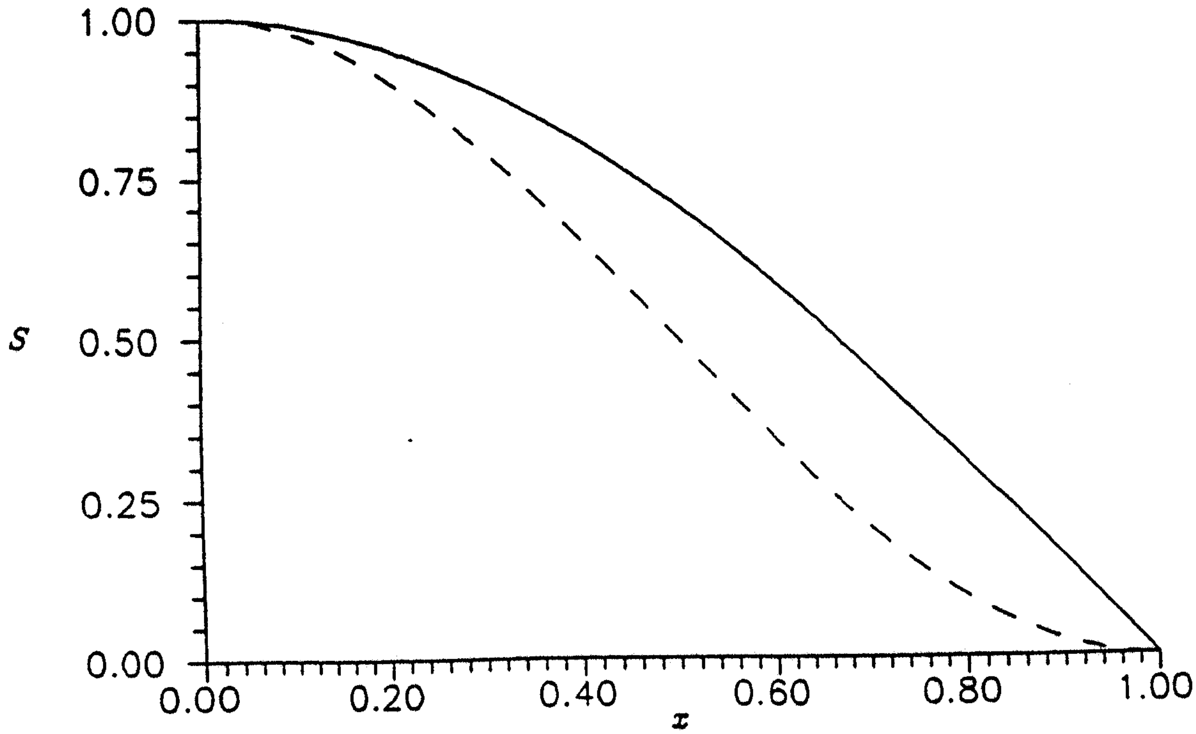

Figure 2 – Plot of the sectional-area curves $S_{1,2}(x)$ derived from the free-wave spectra shown in Figure 1.

Tuck: I am not sure why one can be so dogmatic as to assert that one can never obtain any more information about the ship's shape than the sectional area curve. This may be related to the question as to whether one can tell the difference between a ship, a submarine, a duck, a planing surface, a hovercraft, a hydrofoil, a deeply-submerged sphere, etc., in their respective far fields.

Newman: I would only state 'dogmatically' that there is one and only one equivalent source distribution for a given wave system. To relate this to the sectional area one has to make the additional assumption of slender-body theory. Different vessels could be responsible for the same wave system, and hence represented by the same $\sigma(x)$. If they are sufficiently slender they will all have the same sectional area distribution, but may have different vertical distributions of their displacement. Here the assumption of slenderness includes the requirement that transverse dimensions (beam and draft) are small compared to the characteristic wavelength. Going beyond the last restriction, as explained below, it appears that different physical source distributions (such as one on the free surface and another which is submerged) can be related to the same equivalent source distribution (on the free surface). Thus the vertical definition of the generating body must be inherently non-unique.

Martin: You are solving a Fredholm integral equation of the first kind with a very smooth kernel. This is a notoriously ill-posed problem, but you are getting good results! Have you tried putting some random errors into $H(k)$?

Newman: I have tested the algorithm with $O(\epsilon)$ random errors added to the free-wave spectrum $H(k)$. The effect of this on the final solution $\sigma(x)$ is of the same order as the random errors. This test was made for the two spectra plotted in Figure 1, with $\epsilon = 0.001,\ 0.01$. Another test was made with k replaced by ak in (7), corresponding to a source-strength which is non-zero in the interval $(-a, a)$. The algorithm was unchanged, i.e. it assumed a solution in the interval $(-1, 1)$. The results were reasonable when $a = 0.5$, with reduced accuracy and some indications of Gibbs' phenomena for $|x| > a$. When $a > 1$ the results were not reasonable.

Ursell: (1) If $\sigma(x) = 0$ for $|\, x \,| > 1$ then $H(k)$ is an entire function and is in principle known for all real k, thus $\sigma(x)$ can be found in principle but it may be found that $\sigma(x) \neq 0$ for all $|\, x \,| > 1$. Clearly the problem is very ill-conditioned. (2) I think that Kelvin sources can be combined to give a wavefree potential. (3) The expansion in terms of spherical Bessel functions is essentially a power series, fitted at the origin. We would like to fit it for $k > k_0$.

Newman: The simplest wavefree singularities to my knowledge are combinations of vertical dipoles and horizontal quadrupoles, corresponding to the free-surface differential operator acting on a source. In general these do not describe closed bodies of finite dimensions. The vertical dipole may be related in some sense to a lifting body, and thus connected with the submerged waveless body constructed by Tuck (J. Ship Res. 33, 2, 81-4, June 1989).

Tulin: How does the effective source strength take into account the vertical distribution of source strength and its effect on the wave resistance?

Yeung: I am curious to know whether your algorithm or conclusions would change in any way if you should let the depth of submergence of the line source $\sigma(x)$ be a variable, or a to-be-specified constant?

Newman: If the draft is sufficiently small to satisfy the slender-body assumption, the vertical distribution of volume (and hence of source strength in the Michell sense) has no effect on the wave resistance or generated waves. Some insight into the more general case can be inferred by considering a horizontal line of submerged sources at a constant depth $z = -h$. The free-wave spectrum is multiplied by the factor $\exp(-kh)$, resulting in a modified $H(k)$ and a different effective source-strength $\sigma(x)$ on the free surface. In general we can only say that distributing the actual sources below the free surface (as well as in the transverse direction) will modify the effective source-strength $\sigma(x)$ as defined by (2).