Minimization of wave forces on an array of floating bodies - The inverse hydrodynamic interaction theory -

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1 Introduction

Given the location and the diffraction/radiation characteristics of each member of a floating-body assembly, the hydrodynamic interactions among the members can be accounted for exactly within the linear potential theory[1].

In this paper we now consider an inverse hydrodynamic interaction problem. That is, given the diffraction/radiation characteristics of each member of a floating-body assembly, what the locations of the members should be in order that, say, the wave force on the assembly is minimized.

The most naive way to accomplish this purpose is to calculate the wave force while changing the locations of the bodies systematically and identify the minimum point in the contour curves(surfaces) of the corresponding force. This is, however, inefficient and practically impossible when the number of bodies is not small.

On the other hand, in the field of structural design of large structures such as ships the nonlinear programming techniques have been in common use to solve such inverse problems in which certain effectiveness functions related to the structures are optimized.

Combining these conventional nonlinear programming techniques and the hydrodynamic interaction theory of Kagemoto and Yue[1], we show that the inverse hydrodynamic interaction problems can be solved efficiently.

2 Theory

We consider an array of N floating bodies in plane progressing waves under the usual assumption of the linearized potential theory.

According to the theory of Kagemoto and Yue[1] the velocity potenital Φ that represents the wave field around each member (say body-j) of the floating-body array is expressed as

$$\phi_j^s = \{\mathbf{B_j}(\tilde{a}_j + \sum_{i=1(i\neq j)}^{N} \mathbf{T_{ij}}^T \tilde{A}_i)\}^T \tilde{\psi}_j^s \quad (1)$$

where $\Phi = \Re(\phi_j^s e^{-i\omega t})$ and $\tilde{\psi}_j^s$ is the vector of cylindrical partial waves of $H_n^{(1)}$ and K_n. $\tilde{a}_j$ and $\tilde{A}_i$ are the vectors of amplitudes of each partial wave modes of the plane incident wave and the addtional incident waves due to the interactions among the bodies respectively. $\mathbf{B_j}$ is the "diffraction transfer matrix" for body-j. The superscript T indicates that the transverse should be taken.

Once the velocity potential that represents the flow field around each member is determined, the hydrodynamic force F on the whole assembly acting in β direction is calculated as

$$F = \sum_{j=1}^{N} \int\int i\omega\rho\phi_j^s n_\beta dS_j = \sum_{j=1}^{N} i\omega\rho(\tilde{a}_j^T + \sum_{i=1(i\neq j)}^{N} \tilde{A}_i^T \mathbf{T_{ij}})\mathbf{B_j}^T \int\int \tilde{\psi}_j^s n_\beta dS_j \qquad (2)$$

Now we consider the following inverse problem. That is, what should the locations of the bodies be if you want the hydrodynamic force F to be minimal ?

Let $x_k(k = 1,2,\cdots,K)$ be the variables that determine the locations of the bodies. x_k can be the coordinate of each body or the distance & the relative angle between two bodies of the assembly.

Then the necessary condition for F to be minimal is

$$\frac{\partial}{\partial x_k}F = 0 \qquad (k = 1,2,\cdots,K) \qquad (3)$$

From Eq.(2)

$$\frac{\partial}{\partial x_k}F = \sum_{j=1}^{N} i\omega\rho[\frac{\partial}{\partial x_k}\tilde{a}_j^T + \sum_{i=1(i\neq j)}^{N} \{(\frac{\partial}{\partial x_k}\tilde{A}_i^T)\mathbf{T_{ij}} + \tilde{A}_i^T(\frac{\partial}{\partial x_k}\mathbf{T_{ij}})\}\mathbf{B_j}^T] \int\int \tilde{\psi}_j^s n_\beta dS_j \qquad (4)$$

Here the derivative of a vector or a matrix with respect to x_k means that every component of the corresponding vector or matrix is differentiated with x_k.

Since the components of $\tilde{a}_j$ and $\mathbf{T_{ij}}$ are given explicitly in $x_k(k = 1,2,\cdots,K)$, the differentiation of $\tilde{a}_j, \mathbf{T_{ij}}$ can be carried out analytically. On the other hand, $\tilde{A}_i$ and $\frac{\partial}{\partial x_k}\tilde{A}_i$ are determined numerically by the theory of Kagemoto and Yue[1].

After obtaining these derivatives we can solve Eq.(3) by the conventional nonlinear programming techniques. Since the derivatives with respect to $x_k(k = 1,2,\cdots,K)$ can be calculated from Eq.(4), the descent method of Fletcher and Powell[2] can be used.

In practice, inverse problems usually entail certain addtional restraints on $x_k' s$ (otherwise the problems often end up with some trivial solutions). The additional restraints usually encountered are categorized into the following two equations.

$$h_\ell(x_1, x_2, \cdots, x_K) = 0 \quad (\ell = 1,2,\cdots,L) \qquad (5)$$

$$c_m(x_1, x_2, \cdots, x_K) \geq 0 \quad (m = 1,2,\cdots,M) \qquad (6)$$

In order to incorporate these restraints into our inverse hydrodynamic interaction theory, we follow the method of Carroll[3] and define the following function f and search for $\tilde{x}_k(= (x_1, x_2, \cdots, x_K)^T)$ that makes the function f, instead of F, to a minimum.

$$f \equiv F + r \sum_{m=1}^{M} \frac{1}{c_m(\tilde{x}_k)} + r^{-1/2} \sum_{\ell=1}^{L} h_\ell^2(\tilde{x}_k) \qquad (7)$$

where r is a certain very small positive real number.

In the procedure for the search of $\tilde{x}_k$, we start with certain initial values of $\tilde{x}_k$ such that $c_m(\tilde{x}_k) \geq 0$ $(m = 1, 2, \cdots, M)$, then $c_m(\tilde{x}_k)$ can not become negative because as $c_m(\tilde{x}_k)$ approaches zero the second term of Eq.(7) blows up and thus assures that Eq.(6) is fulfilled. Similarly, since r is a very small positive number, even a tiny deviation of $h_\ell(\tilde{x}_k)$ from zero makes the third term of Eq.(7) blow up and therefore Eq.(5) is assured to be satisfied. In this way the minimization of f ensures that $c_m(\tilde{x}_k) \geq 0, h_\ell(\tilde{x}_k) = 0$. Moreover, the minimum value of f is almost the same as that of F as far as $h_\ell(\tilde{x}_k)$ is zero and r is very small.

The detailed procedures of how to search $\tilde{x}_k$ that minimize f are well established and can be found in many literatures.

3 An example problem

We applied the theory of Section 2 to an array composed of two rows of four-cylinder arrays shown in Fig.1(diameter:0.5m, draft:0.5m, water depth:1.0m) and identified the configuration of the array for which the horizontal drift force $\bar{F}_x$ is minimal in head seas (at waveperiod T=1.15 sec.). (The theory described in Section 2 can be readily extended to the minimization of drift forces.) The length L and the width B of the array are assumed to be fixed as 3.0m and 1.5m respectively. The waveperiod(T=1.15 sec.) was chosen because the drift force is the largest for an equally-spaced array($\ell_1 = \ell_2 = \ell_3 = 1.0m$) at this waveperiod.

As shown in Fig.2 there exist three minima and which one is identified depends on the initial values of $\tilde{x}_k$, which in this case are ℓ_1, ℓ_2, ℓ_3. The points identified by the present inverse theory are indicated by the symbol $'+'$ in Fig.2. Among the three minima the drift force is the smallest at No.3 point and the corresponding configuration of the array is depicted in Fig.3. The frequency response characteristics of $\bar{F}_x$ acting on the corresponding array is shown in Fig.4 with the comparison to $\bar{F}_x$ of the equally-spaced array.

[References]

[1]H. Kagemoto and D.K.P. Yue: J. Fluid Mechanics, 166, 189-209, 1986.
[2]Fletcher, R. and Powell, M.J.D.: The Computer Journal, Vol.6, 163-168, 1963.
[3]Carroll, C.W.: J. Operations Research Society of America, Vol.9, March-April, 169-185, 1960.

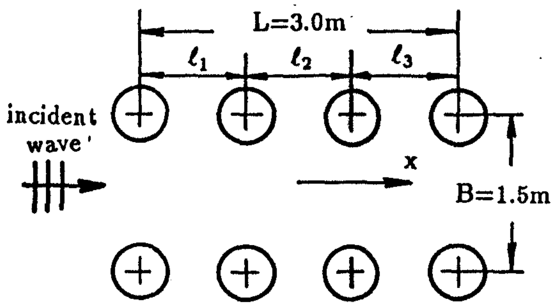

Fig.1 An assembly of two rows of four-cylinder arrays

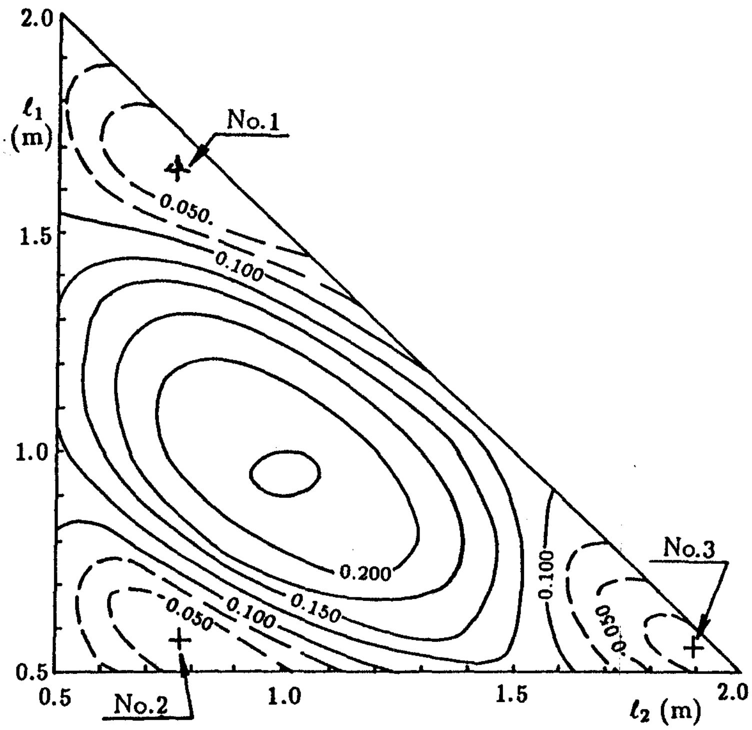

Fig.2 The contour curves of $\bar{F}_x$

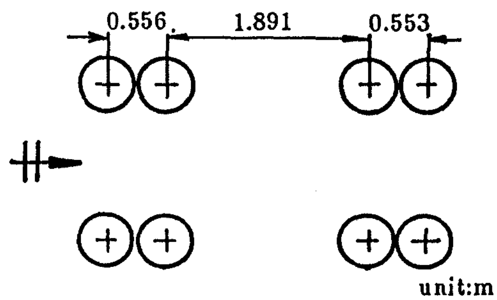

Fig.3 The configuration of the array for minimum $\bar{F}_x$

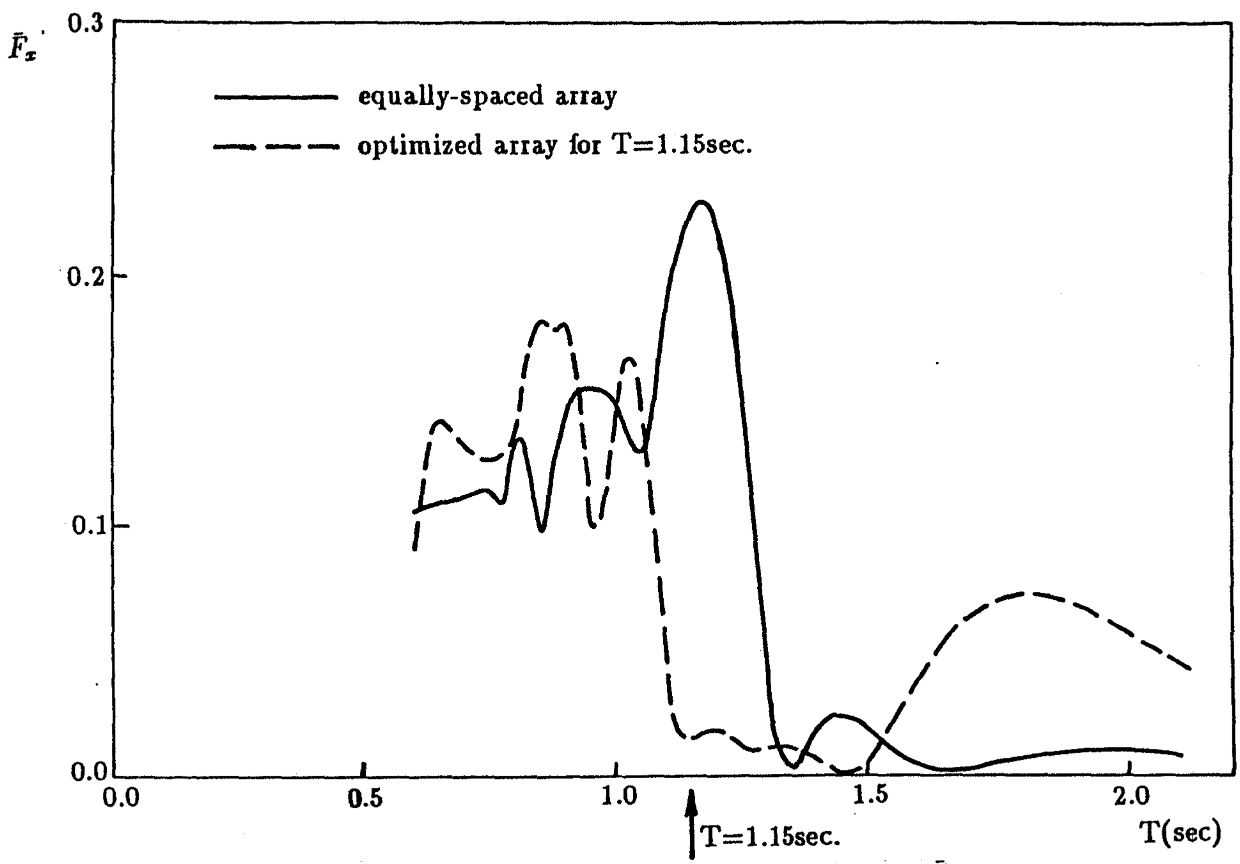

Fig.4 The frequency response characteristics of $\bar{F}_x$