MOTIONS OF SURFACE EFFECT SHIPS

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Introduction

The steady and unsteady flow around a Surface Effect Ship (SES) is analyzed. The motivation is to predict the motions of a SES with forward speed in sinusoidal waves. A SES is a twin hull with an air cushion confined by the sidewalls and bow and stern seals. When the SES moves in waves, the relative motions between the vessel and the waves cause an oscillating pressure inside the cushion which again exerts a dynamic force on the vessel. Hence, the cushion pressure is coupled to the rigid body motions and must be treated as an additional unknown dynamic variable. The coupling is provided by contributions from the air-cushion pressure in the equations for linear and angular momentum as well as from a continuity equation for the air inside the cushion.

Mathematical Formulation

The SES is moving in the negative x-direction with a constant velocity U. Viewed from an inertial frame (x,y,z) following the ship, there is an incident stream of velocity U in the positive x-direction. The fluid is assumed to be ideal and the fluid motion irrotational. The cushion pressure p is oscillating around a mean value p_0.

The velocity potential is decomposed as

$$\Phi = Ux + \phi_s + [A(\phi_I + \phi_D) + \sum_{j=1}^{6} \eta_j \phi_j + (p - p_0)/p_0 \, \phi_p] \, e^{i\omega_e t} \tag{1}$$

where ϕ_s is the steady disturbance potential, ϕ_I, ϕ_D and ϕ_j are the incident wave-, diffraction- and radiation potentials. η_j , $j = 1,6$ are the rigid body motions of the SES. ϕ_p is an additional feature of the SES, it is the potential caused by a travelling oscillating pressure patch. ω_e is the frequency of encounter.

Restricting the sidewalls to be slender the longitudinal component of the normal vector to the ship hull $\mathbf{n}$ will be small (except near the bow and stern). Therefore, $\mathbf{n}$ can be written $\mathbf{n} = (\epsilon N_x, N_y, N_z)$ where $N_x, N_y, N_z = O(1)$ and ϵ is the slenderness parameter. The slenderness also implies that the variation of flow quantities in the transverse direction is much greater than in the longitudinal direction. We assume that

$$\frac{\partial}{\partial y} , \frac{\partial}{\partial z} = O(\epsilon^{-1/2}) \; , \; \frac{\partial}{\partial x} = O(1) \tag{2}$$

The boundary value problem for ϕ_s takes the following (linear) form to leading order in the slenderness parameter,

$$\frac{\partial^2 \phi_s}{\partial y^2} + \frac{\partial^2 \phi_s}{\partial z^2} = 0 \text{ ; in fluid domain} \tag{3}$$

$$U^2 \frac{\partial^2 \phi_s}{\partial x^2} + g \frac{\partial \phi_s}{\partial z} = -\frac{U}{\rho} \frac{\partial p_0}{\partial x} \text{ ; } z = 0 \text{ inside cushion} \tag{4}$$

$$\frac{\partial \phi_s}{\partial N} = -U n_x \text{ ; on hull surface} \tag{5}$$

Outside the air cushion the free surface condition (4) is homogeneous. This boundary value problem ($p_0 = 0$) was considered by Ogilvie (1972). The free surface condition is the only equation where the variable x appears explicitly. For each station $x =$ constant, the two-dimensional Laplace equation (3) is solved subject to the conditions (4) and (5) on the free surface and the hull surface respectively. Upstream flow effects are passed downstream by the first term in the free surface condition. The boundary value problem (3) - (4) is parabolic in nature and hence 'initial' conditions must be supplied at $x = 0$. For high speed one can assume that the potential and the surface elevation both vansish at x=0.

For a station at x_n, let S_H, S_{F_1} and S_{F_2} denote the intersection curve between $x = x_n$, the hull surface, the free surface outside and inside the cushion respectively.

Applying Green's third identity to ϕ_s and a Green function G, the following integral equation is obtained

$$-\alpha\phi_s(\mathbf{r}_0) + \int_{S_H} \phi_s(\mathbf{r}) \frac{\partial}{\partial n} G(\mathbf{r}_0;\mathbf{r})\,ds + \int_{S_{F_1}} \phi_s(\mathbf{r}) \frac{\partial}{\partial z} G(\mathbf{r}_0;\mathbf{r})\,ds$$
$$+ \frac{U^2}{g} \int_{S_{F_1}} G(\mathbf{r}_0;\mathbf{r}) \frac{\partial^2 \phi_s}{\partial x^2}\,ds = -U \int_{S_H} G(\mathbf{r}_0;\mathbf{r}) n_x\,ds - \frac{U}{\rho g} \int_{S_{F_2}} G(\mathbf{r}_0;\mathbf{r}) \frac{\partial p_0}{\partial x}\,ds \tag{6}$$

where α is the angle between the two line segments adjacent to the source point (as seen from the fluid domain). The Green function is chosen to be the logarithmic source function

$$G(\mathbf{r}_0;\mathbf{r}) = \ln|\mathbf{r} - \mathbf{r}_0| \tag{7}$$

The unsteady potentials ϕ_D, ϕ_j and ϕ_p satisfy the boundary conditions

$$\frac{\partial \phi}{\partial n} = B \text{ on hull} \tag{8}$$

$$[(i\omega + U\frac{\partial}{\partial x})^2 + g\frac{\partial}{\partial z}]\phi = F \text{ on } z = 0 \tag{9}$$

where $B = 0$ for ϕ_p, $-\partial\phi_I/\partial n$ for ϕ_D and $i\omega_e n_j$ for ϕ_j. $F = 0$ for ϕ_D and ϕ_j. $F = -(i\omega_e + U\partial/\partial x)p_0/\rho$ for ϕ_p inside the air cushion.

Following (Faltinsen, 1983) we write the unsteady potentials as

$$\phi(x,y,z) = e^{-i\omega_e x/U}\psi(x,y,z) \tag{10}$$

Assuming $U = O(1)$ and $\omega_e = O(\epsilon^{-1/2})$ the field equation for the unsteady potentials is to leading order

$$\frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} - \kappa^2\psi = 0 \tag{11}$$

where $\kappa = \dfrac{\omega_e}{U}$. The free surface condition becomes

$$U^2\frac{\partial^2\psi}{\partial x^2} + g\frac{\partial\psi}{\partial z} = -\frac{U}{\rho}\frac{\partial\Pi_0}{\partial x} \quad ; \; z = 0 \text{ inside cushion} \tag{12}$$

where $\Pi_0 = p_0 e^{i\omega_e x/U}$. The boundary value problems for the unsteady potentials are quite similar to the boundary value problem for ϕ_s. Hence the solutions are given by integral equations analogous to (6). The Green function is now chosen as

$$G(\mathbf{r}_0;\mathbf{r}) = K_0(\kappa|\mathbf{r} - \mathbf{r}_0|) \tag{13}$$

where K_0 is the modified Bessel function.

Numerical Solution

The integral equation (6) is solved at each station x_n by discretizing S_H and S_F with 3-node elements. On three consecutive stations, line elements can be arranged to compose 8-node surface elements, with an extra mid node. The second derivative of ϕ in the free surface condition is expressed in terms of ϕ-values at the corner points of this surface element.

The interpolation on the 8-node surface element is as follows

$$(x,y,\phi) = \sum_{k=1}^{8} N_k(\xi,\eta)(x_k,y_k,\phi_k) \tag{14}$$

where $N_k(\xi,\eta)$, $k=1,2,\ldots,8$ are interpolation functions, x_k,y_k are coordinates of the eight nodes on the element and ϕ_k are the corresponding potential values. The second order derivative of ϕ can be expressed in terms of nodal values,

$$\frac{\partial^2\phi}{\partial x^2} = \sum_{k=1}^{8}[N_{k\xi\xi}\xi_{xx} + 2N_{k\xi\eta}\xi_x\eta_x + N_{k\eta\eta}\eta_{xx} + N_{k\xi}\xi_{xx} + N_{k\eta}\eta_{xx}]\phi_k \tag{15}$$

At each station the one-dimensional integral equation is discretized using curved line elements (on the hull). Introducing (15) in the discretized equation the potential values on each station depends on potential values on two upstream stations. Considering all stations simultaneously the total equation system is banded and is solved efficiently by Gauss elimination.

Response in waves

The numerical solution of the integral equation for the unsteady potentials gives the generalized added mass and damping matrices The equation needed to close the problem is the continuity equation for the air inside the cushion. The changes of the air properties are assumed to be adiabatic. The forcing term in the continuity equation is provided by the change in air volume due to the relative motion between the vessel and the waves. The numerical results for the response are compared with results obtained by approximating the air cushion by a rectangular pressure patch with constant pressure and using three-dimensional theory neglecting the effects of the sidewalls.

References

1. Faltinsen, O.M., "Bow Flow and Added Resistance of Slender Ships at High Froude Number and Low Wave Lengths", Journ. Ship Research, Vol. 27, No.3, 1983.

2. Ogilvie, T.F., "The Wave generated by a Fine Ship Bow", Dept. of Naval Architecture and Marine Engineering, University of Michigan, Report No. 127, 1972.

DISCUSSION

Raven: Your formulation of the steady-potential problem is based on slender-body theory. But, due to your equation (4), ϕ_s includes the steady cushion potential. Slenderness is a valid assumption for the side hulls but it certainly is not valid for the cushion. Wouldn't it be much better to use a full 3-D method at just a small additional cost?

Nestegård & Vada: We agree that the slenderness assumption for the pressure potential is quite dubious. We have another code which includes the 3-D velocity potential due to an oscillating rectangular pressure. Comparison of the two codes should test the validity of the approximation. If it is reasonably good, the presented method has the advantage that a non-rectangular varying pressure can be implemented: it is known from experiments that the pressure inside the cushion is not constant.

We also agree that a full 3-D method would be better (but not necessarily much better), but think that the additional cost will not be small.

Yeung: The formulation based on the slender-ship approximation was discussed previously by Yeung & Kim [1],[2]. Pitch and heave characteristics were obtained for a monohull at moderate speed. In [2], the possibility and existence of a homogeneous solution in these types of approximation was discussed; this solution is associated with the presence of transverse waves.

Nestegård & Vada: Yeung & Kim used a Green's function that satisfies the free-surface condition. We have chosen a much simpler Green's function and integrate over the free surface. They presented results for rather low Froude numbers, whereas we are interested in higher Froude numbers.

References

[1] R.W. Yeung & S.H. Kim, 'Radiation forces on ships with forward speed', *Proc. 3rd Int. Conf. on Numerical Ship Hydrodynamics*, Paris (1981) 499–515.

[2] R.W. Yeung & S.H. Kim, 'A new development in the theory of oscillating and translating slender ships', *Proc. 15th Symp. on Naval Hydrodynamics*, Hamburg (1984) 195–212.