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Strongly Nonlinear Waves in a Tapered Channel

Introduction

The energy conversion unit of a TAPCHAN wave power plant consists of a tapered channel which converts the wave energy into potential energy. Water is stored at a certain hight above sea level in a storage magazin. The waves entering the channel are sqeezed side-ways; they thereby increase in hight as they run down the channel until they spill over the horizontal rim into the magazin. The depth of the channel is constant.

Up to now the design of the channel has been based on extensive empirical investigations on scale models performed about 10 years ago.

We have now had the first TAPCHAN device operating in full scale for more than 4 years. In this period large amounts of data and experience have been collected in addition to the older results from scale models.

We have now started a mathematical/numerical study of the strongly nonlinear waves running down the channel. We intend to compare the results with the older empirical data.

The final goal of this endavour is to develop a mathematical/numerical model which can be used in future full scale design work of such channels.

Mathematics

The analysis is based on LUKE´s variational criterion. The width of the channel is supposed to be small and slowly varying. The velocity potential ϕ will then not vary in the transverse direction so that we can write:

$$\phi = \phi(x,y,t)$$

x = coordinate along the channel
y = depth coordinate pointing upwards. y=0 is at the still water level
t = time
b(x) = half width of channel
h = constant dept of channel
$\eta(x,t)$ = elevation above still water level
$H = \eta + h$ = total depth

We make an ansatz for the y dependence:

$$\phi = A(x,t) + B(x,t)(y+h)^2 + \ldots\ldots$$

The possibility of breaking is not included.
We have indicated a continuation to higher orders of y-dependence. To prevent excessively long formulae in this abstract, the continuation is dropped from here on.

This ansatz is inserted into LUKE´s variational criterion. The variational problem is thus:

$$\int\int b(x)\int_{-h}^{\eta}\left\{\frac{1}{2}(A_x + B_x\,(y+h)^2\,)^2+\frac{1}{2}\,(2B(y+h))^2 + A_t + B_t\,(y+h)^2+gy\right\}dydxdt = \text{stationary}$$

The unknown functions in this problem are A(x,t), B(x,t), H(x,t).

The Euler equations for these unknowns are (within second order accuracy in y+h)

$$A_t + B_t H^2 = -g\eta - \frac{1}{2}(A_x + B_x H^2)^2 - \frac{1}{2}(2BH)^2$$

$$H_t = 2BH - H_x (A_x + B_x H^2)$$

$$2bB = -(bA_x)_x$$

The subscripts x and t denote partial derivatives. The first two equations are the dynamic and kinematic surface conditions. The third is an approximation to Laplace´s equation. These Euler equations are solved by a numerical scheme.

Numerics and presentation

To solve the system of nonlinear partial differential equations, we have deviced a numerical scheme based on the Lax-Wendroff two step method. This scheme is numerically stable and has a very weak "numerical dissipation". We treat the problem as an inital value problem such that initially there are no waves in the channel. At t = 0 we let a specified wave train enter the channel and see what happens to it as it runs down the channel and is changed in shape, amplitude and spectrum.

Most of the actual presentation at the workshop will be to show such wave developments as video recordings and/or slides from the computer output. If they are finished by March 1990, we will also show comparions between the computer results and the older empirical data.

DISCUSSION

Peregrine: This appears to be an excellent way to obtain high-order unsteady solutions. How many terms do you retain?

Mehlum: At present, we retain four terms. In principle, there are no limitations.

Peregrine: Some more details of your overtopping algorithm would be appreciated.

Yeung: The removal of mass during the calculations when the wave elevation exceeds the spill height would seem to violate mass conservation, which is implied in Luke's variational principle. Doesn't that lead to an inconsistency?

Mehlum: This is a tricky and highly relevant point. A partial answer, of course, is that the analysis is valid when mass *is* conserved. This means that the analysis can be used by researchers not interested in spilling.

The way we think about the spilling part of the analysis is as follows. Initially we have an initial-value problem which we can follow in time steps until the wave elevation exceeds the rim at one or more points. Then we stop the time stepping and remove the excess water from the calculation. This means that elevations and hydrostatic variables are modified, but velocities and dynamic variables in the remaining volume are not. Then the time stepping is restarted from a new initial state with variables modified as described. ('Initial state' does not necessarily mean that the water is at rest.) We thus run the calculation as a wave-spill-wave-spill-$\cdots$ algorithm.

McGregor: Concerning the effects of changing the depth of the tapered channel, would you expect the energy extraction to be greater or lesser if this freedom in channel design were introduced?

Raven: Judging from your slides, I do not think that the waves in the channel have a very 'potential' appearance. Strong breaking and dissipation appear to be present. Is your method only suited to predicting the gross features of the flow?

Mehlum: We want the waves to be 'potential'. The goal of the work is to find channel geometries which *prevent* breaking and dissipation.

Miloh: Luke's variational principle was originally derived for a bounded domain where the normal derivative of ϕ is prescribed on the boundary. It states that, in this case, the pressure is precisely the Hamiltonian. Now in your problem you do not satisfy exactly the no-flow condition on the walls. In addition, you have spray (rotational) and viscous dissipation in the narrow part of the wave guide. Under these conditions, Luke's principle may not be applied as stated, and one has to 'augment' it. Please specify what kind of modifications (or assumptions) you imposed on this variational principle when applied to your problem.

Mehlum: See my previous replies.

Thomas: I would like to comment on the tricky point of the side-wall boundary condition, as raised by Touvia Miloh. Any formulation, variational or otherwise, should ensure that the normal component of velocity on the vertical side-wall is zero. In your notation, the correct variational principle is an extension of that given by Luke,

$$\delta \iint L(x,t)\, dx\, dt = 0,$$

where

$$L(x,t) = -\rho \int_{-b(x)}^{b(x)} \int_{-h(x)}^{\eta(x,z,t)} \left[\phi_t + \tfrac{1}{2}(\operatorname{grad}\phi)^2 + gy\right] dy\, dz.$$

The further variations then produce the equations associated with slowly-varying flows in the manner which you outline.

If you compare the Lagrangian principle given above with the one in your abstract, you will see that the z-integration has essentially been evaluated. This is only permissible if there is no transverse variation in the flow, i.e. the waves are locally straight-crested. Thus, if the waves are regarded as being locally plane, your variational approach is valid and all boundary conditions are satisfied.

Miloh: A comment: Luke's variational principle may also be considered as a special case of Bateman's principle, which was derived many years ago, before Luke; I think that Bateman does not receive his due credit.

Mehlum: I agree.

Thomas: You assume that the channel width $b(x)$ is both small and slowly-varying. There is no need for $b(x)$ to be small. If $b(x)$ was constant, the waves in the channel would not exhibit any variation across the channel width. The slowly-varying constraint is enough to ensure that the waves are locally plane and this is all that is required (it is also consistent with my previous discussion).

Mehlum: The assumption that $b(x)$ is small is necessary when one wants to couple the channel to the external (linearized, three-dimensional) world.

Tulin: What is the relation between your equations of the Korteweg-de Vries type for a slowly varying channel?

Mehlum: I do not know!

Tuck: Are you neglecting wave reflection?

Mehlum: $\partial\phi/\partial x$ is set equal to zero at the end of the channel, i.e. reflections are included.

McGregor: A wave-energy station is being constructed on Islay in the Hebrides. Is your analysis relevant to its potential performance?

Mehlum: Probably.