SECOND-ORDER WAVE FORCES ON FLOATING BODIES

by
Chang-Ho Lee
Department of Ocean Engineering
MIT, USA
(Abstract for the 4th Int. Workshop on Water Waves and Floating Bodies
Øystese, Norway, 7-10 May 1989)

The complexity of the second-order problem compared with the linear one resides in the infinte extent of the non-homogeneity of the free-surface condition. The prediction for the solution of the second-order problem (second-order velocity potential) or second-order forces exerted on a body requires an integration which involves the nonhomogeneous free-surface forcing term over the entire free-surface extrior to the body waterplane. A few effective algorithms for this integration are reported. But those methods not only require the discretization of the free-surface with panels but also may have difficulties as the water depth increases.

An alternative approach which does not involve the integral over the free-surface is proposed by Sclavounos(1988) for the infinite water depth problem. The non-homogeneity of the free-surface condition is extended throughout the entire free-surface by decomposing the second-order potential appropriately. Specifically the second-order potential is decomposed into two parts, the one is named the particular solution(φ_P) which is subject to the non-homogeneous free-surface condition throughout the entire free-surface, and the other is named the homogeneous solution(φ_H) which satisfies the homogeneous free-surface condition on the free-surface exterior to the body and an appropriate body boundary condition which insures that $\varphi_P + \varphi_H$ satisfies the homogeneous body boundary condition. This decomposition facilitates the analytic integration over the free-surface and produces an explicit expression for φ_P. The complete second-order potential is obtained by adding the second-order incident wave potential and second-order radiation potential to $\varphi_P + \varphi_H$. However we will restrict our discussion only on the latter component.

The present study derives new expressions for the second-order forces in the form of integrals involving "modified Kochin functions". By examining these modified Kochin functions, it is shown that both submerged and surface piercing bodies share the same expression for the second-order forces despite the singular behaviour of second-order potential at the body waterline.

SECOND-ORDER POTENTIAL

For simplicity, we consider only two elementary components of the particular solution("second-order Green functions"). Each Green function satisfies following nonhomogeneous free-surface conditions,

$$g\frac{\partial \mathcal{D}_A^+}{\partial z} - (\omega_1+\omega_2)^2 \mathcal{D}_A^+ = -i(\omega_1+\omega_2)(\nabla\varphi_1^I \cdot \sigma_2 \nabla G_2) \tag{1}$$

$$g\frac{\partial \mathcal{R}_A^+}{\partial z} - (\omega_1+\omega_2)^2 \mathcal{R}_A^+ = -i(\omega_1+\omega_2)(\sigma_1 \nabla G_1 \cdot \sigma_2 \nabla G_2) \tag{2}$$

where $\mathcal{D}_A^+$ is the solution of the diffraction problem defined as the interaction of the incident wave φ_I with the linear wave source potential of strength σ_2, and $\mathcal{R}_A^+$ is the solution of the radiation problem defined as the interaction of the two linear wave source potentials of the strenths σ_1 and σ_2. The superscript + represents the sum frequency problem.

These diffraction and radiation Green functions accept the solutions

$$\mathcal{D}_A^+ = \frac{\omega_1(\omega_1+\omega_2)}{2\pi g} A_1 e^{i\delta_1} \sigma_2(\vec{\xi}) \int_0^{2\pi} d\theta \int_0^{\infty} dk \frac{k}{k-N_\delta} \frac{\nu_1+\nu_2-k\cos(\theta-\beta_1)}{l-\nu_2}$$
$$e^{kz-ikx\cos\theta-iky\sin\theta} \cdot e^{l\zeta+i(k\cos\theta-\nu_1\cos\beta_1)\xi+i(k\sin\theta-\nu_1\sin\beta_1)\eta} \quad (3)$$

$$\mathcal{R}_A^+ = \frac{-i(\omega_1+\omega_2)}{2\pi^2 g} \sigma_1(\vec{\xi}_1)\sigma_2(\vec{\xi}_2) \int_0^{2\pi} d\theta \int_0^{\infty} \frac{k}{k-N_\delta} \int_0^{2\pi} d\theta_1 \int_0^{\infty} dk_1 \frac{k_1}{k_1-\nu_1^\epsilon} \frac{\nu_1\nu_2+k_1^2-kk_1\cos(\theta-\theta_1)}{l_2-\nu_2^\epsilon}$$
$$e^{kz-ikx\cos\theta-iky\sin\theta} \cdot e^{k_1\zeta_1+ik_1\xi_1\cos\theta_1+ik_1\eta_1\sin\theta_1}$$
$$e^{l_2\zeta_2+i(k\cos\theta-k_1\cos\theta_1)\xi_2+i(k\sin\theta-k_1\sin\theta_1)\eta_2} \quad (4)$$

where

ω_i : frequency of linear wave source potential

σ_i : strength of linear wave source potential

$\vec{x}$: location of field point

$\vec{\xi}_i$: location of linear wave source potential

A_i, β_i, δ_i : amplitude, heading angle, and phase of incident wave

$$l = \sqrt{k^2+\nu_1^2-2k\nu_1\cos(\theta-\beta_1)}$$
$$l_2 = \sqrt{k^2+k_1^2-2kk_1\cos(\theta-\theta_1)}$$

The contributions from the poles in equations (3) and (4) are accounted for by introducing complex wave numbers defined by

$$N_\delta = \frac{(\omega_1+\omega_2-isgn(\omega_1+\omega_2)\delta)^2}{g}$$
$$\nu_i^\epsilon = \frac{(\omega_i-isgn(\omega_i)\epsilon)^2}{g}$$

where δ and ϵ are small positive parameters.

The other elementary components of the particular solution have similar expressions. The total particular solution can be obtaind by the integration of the sum of elementary solutions over the body surface.

The particular solution of the diffraction problem is

$$\varphi_P(\vec{x}) = \iint_{S_b} d\vec{\xi}\,\mathcal{D}(\vec{x};\ \vec{\xi}) \quad (5)$$

and that of the radiation problem is

$$\varphi_P(\vec{x}) = \iint_{S_b} d\vec{\xi}_2 \iint_{S_b} d\vec{\xi}_1\,\mathcal{R}(\vec{x};\vec{\xi}_1,\vec{\xi}_2) \quad (6)$$

It should be noticed that the velocity potential φ_P is harmonic in the entire fluid domain including the domain interior to the body boundary, but is singular near the intersection of the body with the free-surface.

SECOND-ORDER FORCES AND MOMENTS

The second-order forces (moments are understood hereafter) on the body are obtained by the integration of second-order hydrodynamic pressure force over the mean position of the body boundary

$$X_i = -i(\omega_1 + \omega_2)\rho \iint_{S_b} (\varphi_P + \varphi_H) n_i ds, \qquad i = 1, \ldots 6 \tag{7}$$

where $\vec{n} = (n_1, n_2, n_3), (n_4, n_5, n_6) = (x, y, z) \times \vec{n}$ and ρ is the fluid density. φ_H is a linear velocity potential subject to

$$g\frac{\partial \varphi_H}{\partial z} - (\omega_1 + \omega_2)^2 \varphi_H = 0, \qquad on \quad z = 0 \tag{8}$$

$$\frac{\partial \varphi_H}{\partial n} = -\frac{\partial \varphi_P}{\partial n}, \qquad on \quad S_b \tag{9}$$

An alternative form of the exciting-force expression can be obtained by the introduction of an auxilliary linear velocity potential ψ_i subject to the homogeneous free-surface condition (8) and

$$\frac{\partial \psi_i}{\partial n} = n_i, \qquad on \quad S_b \tag{10}$$

Applying Green's identity between φ_H and ψ_i and making use of the boundary condition (9), the alternative force expression takes form

$$X_i = -i(\omega_1 + \omega_2)\rho \iint_{S_b} (n_i \varphi_P - \psi_i \frac{\partial \varphi_P}{\partial n}) ds \tag{11}$$

For submerged bodies we can express X_i as the integration of product of Kochin functions using the definitions (3) and (4) of the second-order Green functions. The forces for the diffraction problem is

$$X_{iA}^{+} = \frac{-i\rho\omega_1(\omega_1 + \omega_2)^2}{2\pi g} A_1 e^{i\delta_1} \int_0^{2\pi} d\theta \int_0^{\infty} dk \frac{k}{k - N_6} \frac{\nu_1 + \nu_2 - k\cos(\theta - \beta_1)}{l - \nu_2} \cdot H_i(k, \theta) \cdot S_2(l, \alpha) \tag{12}$$

and the forces for the radiation problem is

$$X_{iA}^{+} = \frac{-\rho(\omega_1 + \omega_2)^2}{2\pi^2 g} \int_0^{2\pi} d\theta \int_0^{\infty} \frac{k}{k - N_6} \int_0^{2\pi} d\theta_1 \int_0^{\infty} dk_1 \frac{k_1}{k_1 - \nu_1^{\epsilon}} \frac{\nu_1\nu_2 + k_1^2 - kk_1\cos(\theta - \theta_1)}{l_2 - \nu_2^{\epsilon}} \cdot H_i(k, \theta) \cdot S_1(k_1, \theta_1) \cdot S_2(l_2, \alpha_2) \tag{13}$$

where

$$H_i(k, \theta) = \iint_{S_b} (n_i(\vec{\xi}) - \psi_i(\vec{\xi}) \frac{\partial}{\partial n}) e^{k\varsigma - ik\xi\cos\theta - ik\eta\sin\theta} \tag{14}$$

is the "classical Kochin function" and

$$S_i(l,\alpha) = \iint_{S_b} \sigma_i(\vec{\xi}) e^{l\zeta + il\xi\cos\alpha + il\eta\sin\alpha} \tag{15}$$

is named the "source Kochin function". In equations (12)-(15), l and l_2 haves the same definitions as in the equations (3)-(4), and α and α_2 are defined by

$$\alpha = \tan^{-1}\frac{k\sin\theta - \nu_1\sin\beta_1}{k\cos\theta - \nu_1\cos\beta_1}$$
$$\alpha_2 = \tan^{-1}\frac{k\sin\theta - k_1\sin\theta_1}{k\cos\theta - k_1\cos\theta_1}$$

The source Kochin function $S_i(l,\alpha)$ can be replaced by the Kochin function $H_i(l,\alpha)$ for submerged bodies by utilizing the jump condition on the boundary of distribution of sources. Expressions (12) and (13), after replacing S_i with H_i, are convenient to use for the derivation of explicit form of the second-order forces on deeply submerged elemetary singularities as well as distributions of them.

For surface piercing bodies, the singular behaviour of the second-order Green function near the intersection prevents the direct interchange between body integration and Fourier integration as is the case for the submerged bodies. To overcome this difficulty, for wall sided bodies, we divide the body integration into two parts by introducing an arbitrarily small quarter circle of radius δ excluding the waterline intersection. Denoting the circular strip by W^δ and the rest of the body surface by S_b^δ, the second-order forces can be written by

$$X_i = -i(\omega_1+\omega_2)\rho\lim_{\delta\to 0}\left(\iint_{S_b^\delta}(n_i\varphi_P - \psi_i\frac{\partial\varphi_P}{\partial n})ds - \iint_{W^\delta}(n_i\varphi_H - \psi_i\frac{\partial\varphi_H}{\partial n})ds\right) \tag{16}$$

The first integral of equation (16) can be expressed by equations (12) or (13) after interchanging the integrations. The limiting value of this integral can be shown to be finite by examining the behaviour of the Kochin functions for large values k and k_1. The Kochin functions H_i and S_i for the half infinite vertical circular cylinder decay like $O(k^{\frac{1}{2}})$ and $O(k^{\frac{3}{2}})$, respectively. The rates of decay of the Kochin functions are such that the expressions (12) and (13) are finite. This behaviour of Kochin functions holds for other body geometries for the large values of the wave numbers. The second integral vanishes according to the analysis given by Sclavounos(1988). Therefore, equations (12) and (13) express the forces on the floating bodies as well as submerged bodies.

REFERENCES

Sclavounos P.D.(1988) Radiation and diffraction of second-order surface waves by floating bodies. J. Fluid Mech. Vol. 196, pp. 65-91

DISCUSSION

Yue: In the more traditional way of treating the second-order diffraction/radiation problem (e.g. Kim & Yue, 1989, JFM vol. 2000), the requisite free-surface integral can likewise be reduced to a form involving generalized Kochin functions. This, incidently, results in a substantial savings in computational effort if the second-order potential itself is being solved for. Would you please comment on the (possible) relationships and perhaps contrast the key differences between that type of formulation and your present work?

Lee: The "source Kochin function" in the present work and the "generalized Kochin function" in Kim & Yue (1989) both arise due to the expression of the 1st order potential and its derivatives on the free surface in terms of a distribution of linear wave sources on the body surface. The appearence of Kochin functions in both formulations is also the consequence of the separation of the source point on the body and the field point on the free surface. This separation implies that the integration with respect to the field points can be simplified. A well known example is the mean horizontal drift force expression using the momentum conservation principle.

The key difference between the two approaches is that the present work is based on the analytic integration of the forcing term throughout entire free surface while in Kim & Yue separation of two variables is exploited only at far field where local wave effect is small. As a consequence the "source Kochin function" involves both propagating and local waves and the "generalized Kochin function" involves only propagating wave. In this regard, it may be more appropriate to put "generalized" in front of the former.