MARCHING TOWARD A SLENDER SHIP WAVE RESISTANCE THEORY

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Slender ship theory (at least in the primitive 1960's version by Vossers (1962), Maruo (1962) and Tuck (1964)) shares with Michell's (1898) thin ship theory the 'defect' that it distributes its wave-making singularities (which are fully three-dimensional Havelock sources) on the centerplane of the ship. Indeed, it is 'worse' than Michell, in that it places those singularities on a line, usually (but not necessarily) located at the free surface, so that it lacks sensitivity to draft as well as beam. The wave resistance formula depends only on the section-area curve, and specifically measures interference effects between sections of the ship, as communicated along the length by the transverse waves. Unfortunately, the resulting wave resistance fails to agree at all well with model experiments, and even fails to tend to zero for high speed. This high-speed defect was addressed by one of us at the previous Workshop, and the present paper is a continuation of the research outlined therein.

A specifically high-speed slender ship theory has also been in existence for some time, e.g. as outlined by Ogilvie (1967,1972). In that class of theory, which is analogous to the strip theory of ship motions, one assumes that the problem is approximately two-dimensional in each cross section. Each such section thus appears like a two-dimensional wave maker, the sections growing fatter from the bow, then shrinking toward the stern. Note that this strip theory has the advantage that the wave-making capability of the ship is not collapsed onto the centerplane, but lies on the actual hull. However, it has the disadvantage that only two-dimensional wavemaking is possible. In the language of Kelvin's ship waves, only the diverging waves are predicted, not the (all-important) transverse waves. The other important feature of this strip theory is that it is a marching process, the coordinate measured along the hull's length playing only a time-like role. That is, there is no disturbance ahead of the ship, and a given section only affects sections aft of itself. This is good for numerical purposes, but again does an injury to the real problem.

Clearly these two theories are complementary; what one does well the other does badly. A successful slender ship theory should combine (good) elements of both. A clue to the solution is the fact that (because of the lack of draft-dependence and hence of an exponential decay factor) the slender ship theory fails to predict the correct rate of decrease in the diverging part of the free-wave spectrum as the wave propagation direction approaches 90° to the ship's track. This is the reason why the wave resistance fails to tend to zero at high speed. Roughly speaking, we need to use slender ship theory for the transverse waves, and strip theory for the diverging waves.

It is possible to make this idea more rigorous using a 'composite' formula

$$R = \frac{R_{(\text{slender})} \cdot R_{(\text{strip})}}{R_{(\text{common limit})}}$$

where the denominator is the (unphysical) non-zero limit of the slender ship resistance for large Froude number, which can be shown (at least for sufficiently-pointed ends) to be identical with the (equally unphysical) non-zero limit of the strip theory resistance for small Froude number. This type of composite theory is quite similar to the 'unified' theories for ship motions (Newman and Sclavounos 1980), which provide an analogous link between the low-frequency ('slender body') and high frequency ('strip theory') limits in that context.

At the previous Workshop, it was shown how this process could work for a model problem in which the Michell thin-ship assumption was made before any slender or strip assumptions were made. The results were therefore contained within Michell's theory, and could not improve upon it. In the oral presentation at that Workshop, the actual matching was carried out for a strut with a cusped section area curve. New computations have now been made giving a similar matching for a body of revolution with a parabolic waterline, having the same section area curve as the strut, but of more practical relevance, e.g. for canoe hulls of yachts. Note that the slender ship theory, depending only on section area, makes no distinction between these two hulls with identical section area curves, but the strip theory does, and indeed so does Michell's integral. However, the actual wave resistance is insignificantly (less than 10%) different; note that the computation of Michell or strip resistance for a body of revolution is several orders of magnitude more difficult than for a strut.

But these computations still beg the question, since they stay within a framework where the wavemaking occurs on the centerplane, i.e. an *a priori* **thin-ship** assumption has been made. The strip-slender matching procedure is in principle quite independent of any such assumption. For example, it could equally well apply to a case in which an *a priori* **flat-ship** assumption has been made. Indeed the 'low-aspect-ratio flat-ship theory' of Tuck (1973) is precisely the strip theory for that case, being valid only for moderately-high Froude numbers, and we have made some progress in matching that theory to slender-ship theory.

However, to do this job properly, we need to obtain solutions of the actual strip problem for ship sections that have arbitrary beam to draft ratios, as formulated by Ogilvie. Before solving, one must query whether the problem is linear or nonlinear. The answer would seem to be the latter; at least that is the conclusion from the Ogilvie derivation. The slenderness assumption has been already used up in making the problem two-dimensional, and is not a further justification for linearization. That is, we must have available, in principle, a computer program for solving a nonlinear, two-dimensional, wave-maker problem, for a wavemaker whose shape changes as a function of 'time' t (this being actually the length-wise coordinate of the body). There are such programs, developed for other purposes (Lin, Newman and Yue(1984) and Grosenbaugh and Yeung(1988) for example) but not yet in forms that are useful, in practice being used only for wavemakers of unchanging shape.

In the meantime, we have made an attempt at a linear version of the problem. That is we solve a two-dimensional potential-flow problem, with a linearized free-surface condition, but with the actual (section-wise) hull as the wavemaker. This is inconsistent, but is intended to provide insight into the way in which the real strip problem would be solved, and to give some finite-beam effects. Note that it is the correct strip theory limit of the Neumann-Kelvin problem, so that if one accepts that problem as a sufficiently accurate model of the three-dimensional wave resistance problem, then the present approach is a candidate to replace it numerically, with a potential accuracy comparable with that to which the matched strip-slender wave resistance theory approximates Michell's integral. That is, if one really believes in the relevance of the Neumann-Kelvin problem (which does not require any slenderness assumption about the hull, in general), then the present approach should give results of insignificantly reduced accuracy whenever the ship is actually slender. This is a nontrivial observation in view of the now apparent (Beck and Doctors 1987) numerical intractability of the Neumann-Kelvin problem in its general form.

However, even this linearized task is not easy, since the time-like lengthwise coordinate t enters the problem in a general way. An integral equation which is of Fredholm type in the transverse spatial dimensions (x laterally, y upward) and of Volterra type in the time-like longitudinal dimension t may be derived via Green's theorem, making use of the two-dimensional, transient, free-surface Green function. Yeung and Kim (1981) present a version of this equation in their marching approach to the forward-speed radiation problem.

$$
\begin{aligned}
-\pi\phi(\vec{x},t) + \int_{S(t)} d\vec{\xi}\ & [\phi(\vec{\xi},t)G_n(\vec{x};\vec{\xi},0) - \phi_n G] \\
& - \int_{0+}^{t} d\tau \int_{S(\tau)} d\vec{\xi}\ [\phi(\vec{\xi},\tau)G_{\tau n}(\vec{x};\vec{\xi},t-\tau) - \phi_n G_\tau] \\
& + \int_{0+}^{t} d\tau\ \frac{db^-}{d\tau}[\phi(b^-,0,\tau)G_{\tau\tau}(\vec{x};b^-,0,t-\tau) - \phi_\tau G_\tau] \\
& - \int_{0+}^{t} d\tau\ \frac{db^+}{d\tau}[\phi(b^+,0,\tau)G_{\tau\tau}(\vec{x};b^+,0,t-\tau) - \phi_\tau G_\tau] = 0
\end{aligned}
\tag{1}
$$

where $b^-(t)$ and $b^+(t)$ are the intersection points of the body section and the free surface at time t. Note that the body surface in equation (1) is free to move rigidly or distort without restriction. This equation may be regarded as an extension of the familiar two-dimensional transient radiation equation. The additional complications in (1) are the integrations following the body-free-surface intersection points. These terms introduce the unknown, $\phi(\vec{x},t)$, into the discrete form of the time integration, whereas the convolution over the body surface does not, due to initial conditions for $G(\vec{x};\vec{\xi},t)$. Also, these terms are analogous to the waterline integral in the Neumann-Kelvin problem could contribute similar computational difficulties.

Taking a simpler approach, we have sought to generate the potential by distributing vertical dipoles over the surface of the body, with a strength $P(\vec{x},t)$ to be determined. This gives the expression for the potential:

$$\phi(\vec{x},t) = \int_0^t d\tau \int_{S(\tau)} d\vec{\xi}\ P(\vec{\xi},\tau)K_1(\vec{x};\vec{\xi},t-\tau) \tag{2}$$

where the kernel is:

$$K_1(\vec{x};\vec{\xi},t) = -\frac{1}{2}\frac{y-\eta}{r_1^2} + \frac{1}{2}\frac{y+\eta}{r_2^2} + \int_0^\infty dk\ \ \cos k(x-\xi)e^{k(y+\eta)}\cos\sqrt{gk}t \tag{3}$$

in which $\left.\begin{matrix} r_2^2 \\ r_1^2 \end{matrix}\right\} = (x-\xi)^2 + (y \pm \eta)^2$ and g is the acceleration due to gravity.

Direct substitution of equation (2) into the problem statement verifies that the potential $\phi(\vec{x},t)$ is a solution. This potential is sought indirectly, by solving an integral equation which is the harmonic conjugate of equation (2), and has the (known) stream function on the left-hand side, but with a kernel $K_2(\vec{x};\vec{\xi},t)$ that is the harmonic conjugate of $K_1(\vec{x};\vec{\xi},t)$, *i.e.*replaces the cosine (of the lateral coordinate) with a sine. Then once $P(\vec{x},t)$ is known, equation (2) gives the potential by quadrature. We have also investigated a formulation which is essentially the derivative of equation (2) with respect to t. The two approaches are equivalent while the body is dilating, but are distinctly different during the contraction. We do not fully understand the physical implications of these differences.

At this time, we are not able to report computational results. We have developed input and output routines to handle arbitrary body geometries and efficient economized polynomial routines for the evaluation of the kernels $K_1(\vec{x};\vec{\xi},t)$ and $K_2(\vec{x};\vec{\xi},t)$; however, we have not obtained convergent results for the resistance coefficient of simple bodies of mathematical description.

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DISCUSSION

Miloh: 1) The results you presented are very interesting and somewhat surprising that one can recover Michell's theory by simply multiplying the results of slender body and strip theory. I think that a way to justify it would be by using a singular matching procedure between inner and outer solutions.
2) My second remark concerns your remark concerning Ursell's technique of getting a small parameter equation of his Green's function integral. I wonder if your trick of replacing the lower bounds by a and integrating by parts will work for a general kernel and will enable you to get asymptotic expansion beyond the first (most singular) term?

Tuck: 1) Yes, I would like to do that, but the present more-limited objective is being pursued first.
2) It is not easy to get more terms my way, and Ursell's method is preferable if a complete series is wanted.