*Abstract submitted for the 4th Workshop on Water Waves and Floating Bodies*
*Øystese, Norway, 7 - 10 May 1989*

# Waves Traveling over a Submerged Cylinder: Nonlinear vs. Second-order Theory

**Raymond Cointe**
Institut Français du Pétrole

## Introduction.

At the last Workshop in Woods Hole, Grue and Granlund (1988) presented experiments related to incoming deepwater Stokes waves passing over a restrained submerged circular cylinder. For a small cylinder submergence, a strong local nonlinearity is introduced at the free surface above the cylinder and free higher order harmonic waves are generated.

We remind of the observations of Grue and Granlund concerning the wavefield far away from the cylinder:

• upstream of the cylinder: an incoming Stokes wavetrain. No reflected waves, even to higher order;

• downstream of the cylinder: shorter free second harmonic waves of considerable amplitude are riding on the transmitted Stokes wave.

If these trends are well predicted by second-order theory (Vada, 1987), the quantitative agreement with experiments is rather disappointing. The amplitude of the second harmonic free wave, $a_2$, only increases as the square of the amplitude of the incoming wave, $a$, for very small values of $a$. A "saturation" rapidly appears; thereafter $a_2$ remains almost constant — see figure 2.

These findings suggest that the range of validity of second-order theory is quite narrow in this case. Observing that this theory predicts amplitudes of the second-order free wave as large as that of the incoming wave, this should not appear as totally unexpected. In order to see if nonlinear free-surface effects — and not viscous effect — are, indeed, responsible for this deficiency, a fully nonlinear potential flow computation has been attempted.

## Nonlinear simulation.

This simulation was made using Sindbad, a code already described partly at the last Workshop (Cointe, 1988) and elsewhere. This code has for purpose to solve numerically the fully nonlinear potential flow problem in the time-domain. It uses a mixed Eulerian-Lagrangian scheme: particles at the free surface are followed in their motion; the kinematical constraint $\Delta\phi = 0$ is taken into account by a boundary integral method. In order to avoid difficulties related to the prescription of incoming and outcoming wave trains, a two-dimensional wavetank is simulated. A transient simulation is made: the flow is initially at rest; waves are generated by a piston-type wavemaker and reflection is limited by the use of an absorbing zone.

In order to compare our results with those of Grue and Granlund, we write (once a steady state is reached):

- *for the incident wave:*

$$\eta_i = a \ \cos(\kappa x - \omega t + \theta) + a^\ell \ \cos 2(\kappa x - \omega t + \theta) + \cdots ,$$

where $a^\ell$ is the amplitude of the second-order locked wave;

- *for the diffracted wave:*

$$\eta_d = a_1 \ \cos(\kappa x - \omega t + \theta_1) + a_2^\ell \ \cos 2(\kappa x - \omega t + \theta_1) + a_2 \ \cos(4\kappa x - 2\omega t + \theta_2) + \cdots ,$$

where $a_2^\ell$ and $a_2$ are the amplitudes of the second-order locked and free wave, respectively.

In order to exhibit the second-order free wave, it is necessary to take a rather fine grid. The wavenumber of this free wave is, indeed, four times larger than the wavenumber of the incoming wave. The units are defined so that the depth of the tank and the acceleration of gravity are equal to 1. The wavenumber of the incoming wave is chosen equal to $\kappa = 3.42$ (so that we are in deep water). With these choices, the results shown by Grue and Granlund correspond to a cylinder radius $r = 0.117$ and a depth of immersion of the center of the cylinder $y_c = -0.1755$.

The simulation was perfomed in a tank of length 8, with a damping zone of length 3. The cylinder center was located at a distance $x_c = 2$ from the wavemaker. 340 nodes were distributed on the free surface and 60 time steps used per period of the incoming wave.

**Results.**

On figure 1, free surface profiles after 7 periods are shown for several amplitudes of the incident wave. The apparition of a perturbation of wavenumber $4\kappa$ is obvious. However, its amplitude does not increase as the square of the incident wave amplitude.

A Fourier analysis of the diffracted wave confirms this trend. We show on figure 2 the comparison between Grue and Granlund's experiments, Vada's second-order theory and the present calculation. The agreement between the numerical simulation and the experiments is very good, indicating that the "saturation" is, indeed, a nonlinear free-surface phenomenon not accounted for by second-order theory.

Grue and Granlund observed breaking for $a\kappa \simeq 0.085$, while we were able to perform the numerical simulation up to $a\kappa = 0.12$. It is rather interesting to note that this does not seem to affect the amplitude of the second-order free wave.

A little more surprising is the reason for which the numerical computation fails for $a\kappa = 0.12$ after 3.2 periods. The distribution of the nodes along the free surface just before this failure is shown on figure 3. The computation does not blow up because of the overturning of the crest, as would have been expected, but because of a concentration of particles just aft the cylinder. Physically, it seems that particles flow very rapidly over the cylinder and are then decelerated. A numerical consequence is that the grid is more sparse over the cylinder. This might explain why overturning does not appear. Calculations using a regridding procedure similar to that used by Dommermuth (1987) are going to be done in order to study this phenomenon in more details.

Finally, it should be noticed that the loads acting on the cylinder are calculated during the simulation, even though they are not of primary interest for this study. All the calculations performed seem to indicate the existence of a negative horizontal drift, in agreement with the results of Dommermuth (1987).

**Conclusions.**

The fully nonlinear simulation of the potential flow over a submerged cylinder shows good agreement with experiments in a case where second-order theory has a very narrow range of validity. This suggests that second-order theory is not adequate to study this problem.

The case studied here is, however, critical. It was apparently chosen by Grue and Granlund to give a strong nonlinear response (according to Vada's second-order theory). The present comparison between nonlinear and second-order theory is, therefore, somehow unfair and no general conclusion concerning the validity of second-order theory is meant.

**Acknowledgements.**

This work is a result of research sponsored in part by the "Ministère de la Défense", DRET, under convention number 88/073. This support is gratefully acknowledged.

**References.**

Cointe, R., 1988, "Remarks on the numerical treatment of the intersection point between a rigid body and a free surface," *Proceedings,* Third International Workshop on Water Waves and Floating Bodies (Woods Hole).

Dommermuth, D.G., 1987, "Numerical methods for solving nonlinear water-wave problems in the time domain," Ph.D. Dissertation, MIT.

Grue, J., and Granlund, K., 1988, "Impact of nonlinearity upon waves traveling over a submerged cylinder," *Proceedings,* Third International Workshop on Water Waves and Floating Bodies (Woods Hole).

Vada, T., 1987, "A numerical solution of the second-order wave-diffraction problem for a submerged cylinder of arbitrary shape," *J. Fluid Mech.,* Vol. 174, pp. 23-37.

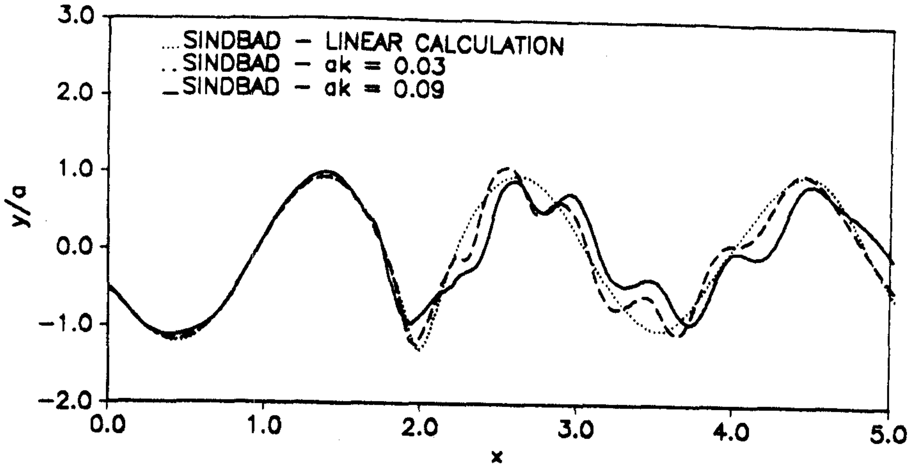


Figure 1 — Free surface profiles

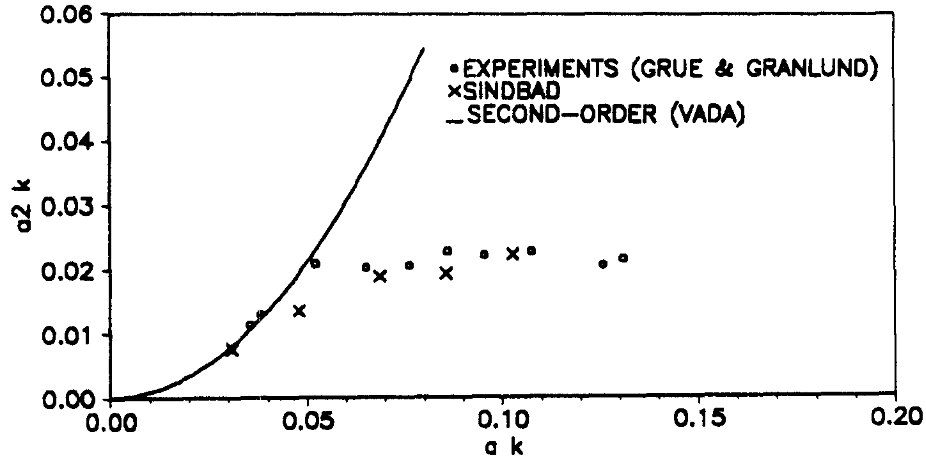


Figure 2 — Second-order free wave

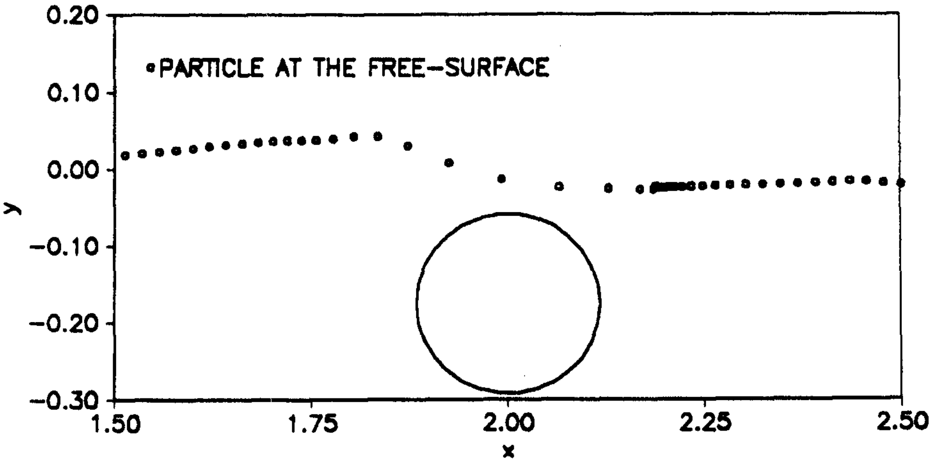


Figure 3 — Distribution of particles at free surface

**DISCUSSION**

Nakos: I am impressed by the efficiency of your "absorption" condition over the "numerical beach". Could you comment on the damping mechanism you use and its implementation? Do you think that it is going to be as efficient when a continuous spectrum of waves has to be absorbed?

Cointe: Our damping mechanism is similar to that used by Baker, Meiron & Orszag, 1981 (3rd Int.Conf.Num. Ship Hydro.). More details will be given in a paper that will be presented at the 5th Int.Conf.Num. Ship Hydro. next September. Obviously, difficulties are expected for a wide wave spectrum, but we have not performed any systematic test in this case.

Vinje: From our computations on submerged cylinders done some 8 years ago we found that the forces on the cylinder, caused by the breaking wave, were very close to those computed by linear theory, even with the cylinder rather close to the free surface.

Cointe: I think our results also show a similar trend. Our interest, however, has been more in evaluating the accuracy and limitations of our numerical scheme on a simple and well documented test case than in studying "extreme" wave forces.

Greenhow: When the first wave of a train breaks over the cylinder, i.e. a transient effect, does it make sense to compare *calculations* with second order steady theory? Where does negative pressure arise in the moving cylinder and does it cause any numerical problems?

Cointe: 1) Of course, a comparison with second-order steady theory can only be achieved once a steady (or quasi-steady) state is reached. Whenever a "numerical breaking" event occurs, the numerical computation stops and a steady state cannot be reached. in reality, however, breaking can be observed and a steady state reached. It would therefore be most useful to be able to "simulate" in some way breaking in the numerical computation.

2) The results concerning the moving cylinder are only preliminary and have not been carefully examined yet. I think that in our computation the pressure distribution is not very different from that observed in an unbounded fluid so that I expect negative pressures on the front face of the cylinder for a sufficiently large Froude number based on the submergence.

Grue: Does the formation of the very steep wave above the cylinder affect the forces in a particular way?

Cointe: Apparently, there is a small decrease of the inertia coefficient and "saturation" of the response at the double frequency.