SECOND-ORDER DEFORMATION OF THE FREE-SURFACE ELEVATION AROUND A VERTICAL CYLINDER

B. MOLIN
Institut Français du Pétrole
BP 311
92 506 Rueil-Malmaison Cedex
France

L. BOUDET
Laboratoire d'Hydrodynamique Navale
ENSM
1 rue de la Noe
44 072 Nantes Cedex

Deck height determination is a crucial part of the design of semi-submersible or tension leg platforms. Rules incorporate safety factors under various forms but do not specifically demand that modifications of the wave system, due to diffraction effects, be taken into account. However interaction effects between the vertical columns may result into large increases of the vertical motion of the free surface, and lead to critical situations.

Figure 1 shows calculated contour plots of the RAO of the free surface elevation around a 4 column structure. The columns diameter is about one third of their axis to axis distance, and the wave-length is twice that value. This is a situation where a resonating sloshing effect takes place, and the RAO reaches values as high as 1.85 in front of the platform, which acts as a reflecting wall.

These plots were obtained according to the (linear) theory developed by Mc Iver and Evans /1/. Obviously the assumption of linearity can hold only for moderate steepness of the incoming waves.

This has motivated the present study, which attempts to extend Mc Iver and Evans' theory to second order, and to derive the resulting free surface motion. To check the validity of the proposed method the case of one isolated vertical cylinder (extending to the bottom) was considered first.

The method proposed here follows from considerations that were presented at the First Workshop /2/, and has been outlined in /3/. That is the second-order diffraction potential is assumed to consist, some distance away from the cylinder, of two components, one locked to the first-order wave system, and one free obeying the Sommerfeld radiation condition. The locked component may be determined to its leading order by assuming the diffracted waves to behave locally as plane progressive waves (which is consistent with the approximations underlying Mc Iver and Evans' theory).

A fictitious vertical cylinder is thus introduced and the following BVP is written for the second order diffraction potential:

$$\Delta \varphi_D^{(2)} = 0 \qquad \text{in the inner fluid domain}$$

$$\frac{\partial \varphi_D^{(2)}}{\partial n} = -\frac{\partial \varphi_I^{(2)}}{\partial n} \qquad \text{on the cylinder}$$

$$g\frac{\partial \varphi_D^{(2)}}{\partial z} - 4\omega^2 \varphi_D^{(2)} = \alpha_L^{(2)} \qquad \text{on the mean free surface}$$

$$\frac{\partial \varphi_D^{(2)}}{\partial n} = 0 \qquad \text{on the bottom}$$

$$\frac{\partial \varphi_D^{(2)}}{\partial R} + ik_2 \varphi_D^{(2)} = \frac{\partial \varphi_{DL}^{(2)}}{\partial R} + ik_2 \varphi_{DL}^{(2)} \qquad \text{on the surrounding cylinder}$$

where k2 is the wave number associated with the double frequency and

$\varphi_{DL}^{(2)}$ is the locked component of $\varphi_D^{(2)}$. Its expression is (/3/):

$$\varphi_{DL}^{(2)} = i a a_D \frac{g^2 k^2}{\omega} \frac{(2\cos\theta + 3\,\mathrm{th}^2 kh - 1)\,\mathrm{ch}\,k\sqrt{2-2\cos\theta}\,(z+h)\; e^{-ikR(1-\cos\theta)}}{g k\sqrt{2-2\cos\theta}\,\mathrm{sh}\,k\sqrt{2-2\cos\theta}\,h - 4\omega^2\,\mathrm{ch}\,k\sqrt{2-2\cos\theta}\,h} + \frac{3i}{8} a_D^2 \omega \frac{\mathrm{ch}\,2k(z+h)}{\mathrm{sh}^4 kh} e^{-2ikR}$$

where aD = a H(θ) / $\sqrt{kR}$ is the local complex amplitude of the diffracted waves.

The condition specified on the fictitious cylinder is thus parent of the radiation condition used by Yeung in his Ph. D. thesis.

This interior problem is solved classically by distributing Rankine singularities over the boundaries and using the two symmetries of the geometry. Because it is necessary to locate the fictitious cylinder at more than one wave-length away from the body (/3/) the obtained matrix is quite large but its construction is straight forward. The right hand side of the free surface equation is easily obtained from the analytical expression of the first-order potentials.

Preliminary results have shown that the second-order diffraction loads are correctly predicted, in accordance with the results published in /4/ or /5/. Experiments are under way at ENSM to validate the calculation of the second-order free surface deformation.

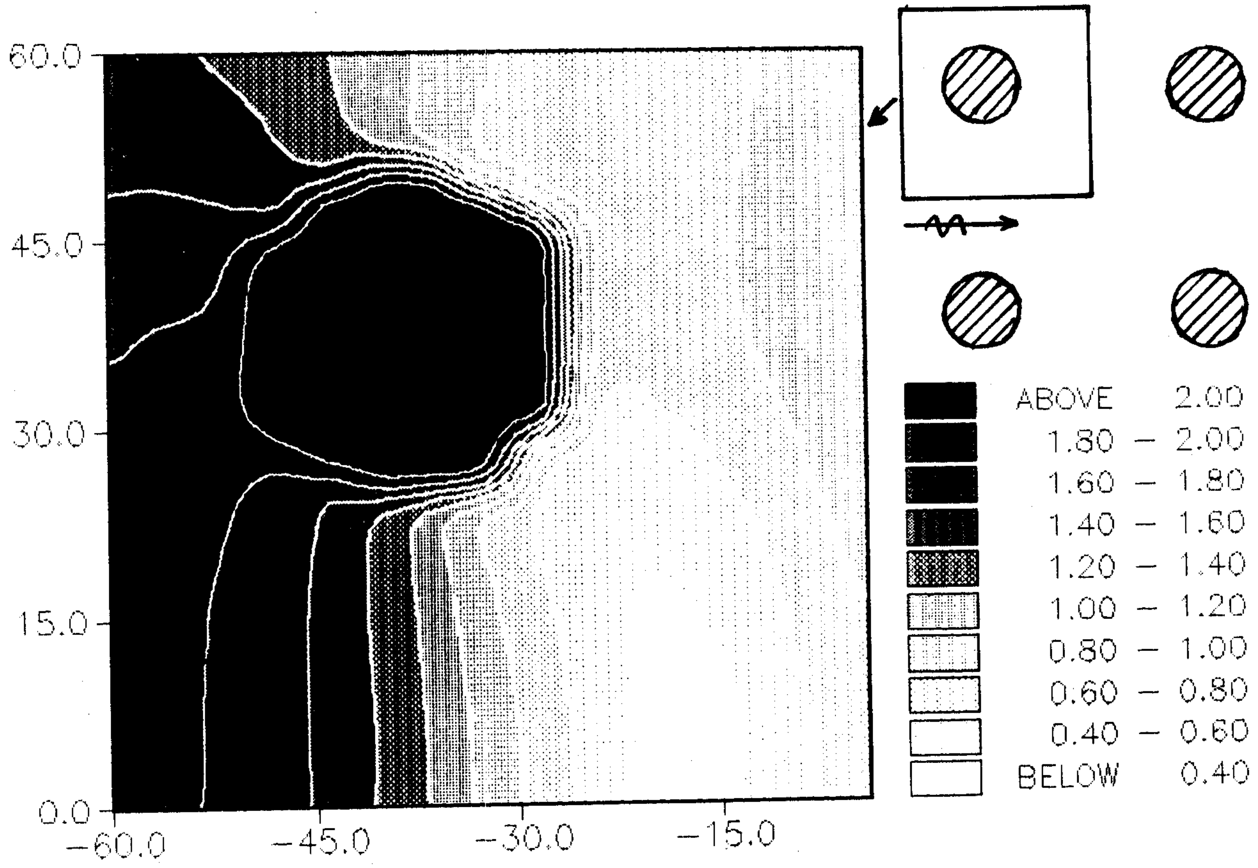
/1/ P. MC IVER and D. EVANS: Approximation of wave forces on cylinder arrays", Applied Ocean Research, Vol. 6, n°2, 1984.

/2/ B. MOLIN: "Second-order double frequency loads and motions for 3D bodies", First International Workshop on Water Waves and Floating Bodies, MIT, 1986.

/3/ B. MOLIN: "Quelques réflexions sur la résolution du problème de diffraction au deuxième ordre", Premières Journées de l'Hydrodynamique, ENSM, 1987.

/4/ B. MOLIN and A. MARION: "Second-order loads and motions for floating bodies in regular waves", OMAE Conference, Tokyo, 1986.

/5/ R. EATOCK TAYLOR and S. M. HUNG: "Second order diffraction forces on a vertical cylinder in regular waves", Applied Ocean Research, Vol. 9, n°1, 1987.



Grue:

1. Can you be sure that you record second harmonic free (outgoing) waves in your measurements?

2. If so, do you have an explanation for the pronounced decay in the dimensionless second harmonic wave amplitude for increasing incoming wave amplitude? This trend coincides with the experiments when (2-D) Stokes waves propagate over a submerged cylinder and a second harmonic transmitted wave is generated. In this case there is a pronounced local nonlinearity at the cylinder, which second order theory cannot account for when the incoming wave amplitude exceeds a certain (small) value.

Molin and Boudet:

1. I do not think that one can pretend to identify, in the vicinity of a structure, be it mathematically or physically, different components in the second-order wave elevation. It is only some distance away that a decomposition in locked and free components makes sense.

2. The second question refers to some experimental results for the second-order wave elevation in the 4-column case. The measured values showed that, at the most critical wave period, the second-order component did not quite vary like the incident wave amplitude squared. I do not have any explanation for this phenomenon.

Faltinsen: The experimental and numerical results presented by Zhao and Faltinsen showed that there may be significant sloshing effects due to tank wall interference with the structure. I wonder to what extent you have examined this effect and if you think it has influence on your results.

Aanesland: What are the dimensions of the wave tank with respect to the dimensions of the model? Is it a towing tank or an ocean basin?

Molin and Boudet: The discussers refer to the model tests on a 4-column structure, some results of which were presented at the Workshop, but are not described in the written contribution. These tests were carried out in the Ocean Basin of ENSM, the dimensions of which are: 18 x 9 x 2 (meters), at a 1:75 scale. During the tests it was checked, visually, that no resonant excitation of the tank transverse modes took place. That is not to say that tank wall interference was non-existent, but it was believed to be of minor importance, considering the relatively large width of the basin. (The purpose of this series of tests was not to obtain accurate figures for the second-order elevation of the free surface, but to obtain a quick answer on whether there would be wave impacts with the deck in regular or irregular waves.