# Active devices for the reduction of wave intensity

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## Introduction

In this paper we look at the possibility of using active breakwaters - structures which move in response to the incident waves - to reflect waves away from installations or to limit wave motion inside harbours. In particular we shall explore to what extent it is possible to utilise the inherent resonance of the moving system to create downstream waves which cancel the transmitted waves. Thus we shall be working in two dimensions.

## Formulation and solution

We consider a single body spanning a narrow wave tank and constrained to oscillate at the same frequency as the incident waves in a single mode of motion, either heave, sway, or roll. Its motion is opposed by an external force proportional to its (small) displacement. For example this could be regarded as the hydrostatic restoring force for a surface-piercing body, or a mooring force.

Cartesian coordinates are chosen with the origin in the undisturbed free surface, with x to the right and y vertically upwards. The depth of the water is h. On linear water-wave theory, there exists a velocity potential $\Phi(x,y,t) = \text{Re}\ \phi(x,y)e^{i\omega t}$ where the time independent potential $\phi(x,y)$ satisfies

$$\nabla^2\phi = 0 \quad \text{in the fluid}$$

$$\kappa\phi - \frac{\partial\phi}{\partial y} = 0 \quad \text{on} \quad y = 0, \qquad \kappa = \omega^2/g$$

$$\frac{\partial\phi}{\partial y} = 0 \quad \text{on} \quad y = -h$$

$$\frac{\partial\phi}{\partial n} = \underline{U}.\underline{n} \quad \text{on the body.}$$

As $x \to +\infty$ we assume

$$\phi \sim \frac{gA}{\omega}\frac{\cosh k(y+h)}{\cosh kh}\left(e^{ikx} + R_1e^{-ikx}\right)$$

whilst as $x \to -\infty$

$$\phi \sim \frac{gA}{\omega}\frac{\cosh k(y+h)}{\cosh kh}\ T_1e^{ikx}. \tag{1}$$

Here k is the real positive root of

$$\kappa = k \tanh kh.$$

It is clear from (1) and the equation

$$\eta(x) = -i\omega g^{-1}\phi(x,0)$$

for the time-independent surface elevation, that $\phi$ describes a wave of amplitude A incident upon the body and giving rise to reflected and transmitted waves of amplitude $A|R_1|$ and $A|T_1|$ respectively.

If we restrict considerations to bodies which are symmetric in the sense that they produce waves of identical amplitudes at either infinity, then it can be shown that

$$T_1 = T(C \mp \chi)/(C - i)$$
$$R_1 = R(C \pm \chi^{-1})/C - i)$$

where $R, T$ are the reflection and transmission coefficients assuming the body to be held fixed in the incident waves, and $R/T = i\chi$, $\chi$ real.

Here $$C = \{(M + I)\omega^2 - \lambda\}/B\omega$$

where $M,B$ are the added mass (inertia) and damping of the body. The upper sign refers to heave, the lower to sway or roll motions. Again $\lambda$ is defined by

$$X_{ext} = i\lambda\omega^{-1}U$$

where $X_{ext}$, $U$ are the time-independent amplitudes of external force on and velocity of the body. Thus $\lambda$ is a 'spring' constant perhaps arising from the hydrostatic restoring force or a mooring restraint. We shall assume $\lambda$ is real, so that $C$ is real also.

It follows that $T_1 = 0$ provided $C = \pm\chi$ or

$$\lambda = (M + I)\omega^2 \mp B\omega\chi \qquad (2)$$

Equation (2) provides a resonance condition under which a wave will be totally reflected by the oscillating body.

Suppose now that the body is a totally submerged buoyant cylinder having its axis parallel to the wave crests and held down by two inextensible cables one at each end. Then

$$\lambda = M'(1 - s)g/\ell$$

where $M'$ is the mass of water displaced by the cylinder, and $s$ its specific gravity and $\ell$ the length of each cable. Thus here $\lambda$ is provided by the restoring force when the cylinder makes small horizontal displacements.

It follows that

$$s = \frac{1 - (\mu + \nu\chi)K\ell}{1 + K\ell} \quad , \quad K = \omega^2/g$$

as the requirement on $s$ for complete reflection of a wave of frequency $\omega$. Here $\mu = M/M'$, $\nu = B/M'$ are non-dimensional added mass and radiation damping coefficients.

Finally, if $\ell$, $s$, $M'$ and hence $\lambda$ are chosen so as to cancel a wave a frequency $\omega_o$, we obtain

$$T_1 = \frac{T\{(\mu + s) + \nu\chi - \{(\mu_o + s) + \nu_o\chi_o\}(\omega_o/\omega)^2\}}{(\mu + s) - i\nu - \{(\mu_o + s) + \nu_o\chi_o\}(\omega_o/\omega)^2} \qquad (3)$$

where $\mu_o$, $\nu_o$ are the values of $\mu$, $\nu$ at $\omega = \omega_o$ Clearly $T_1$ vanishes when $\omega = \omega_o$.

The quantities $\mu$ and $\nu$ were calculated from the radiation problem using the method of multipoles first used by Ursell for the infinite depth case. The reflection and transmission coefficients for the scattering problem, R and T, were calculated by means of the Newman relations.

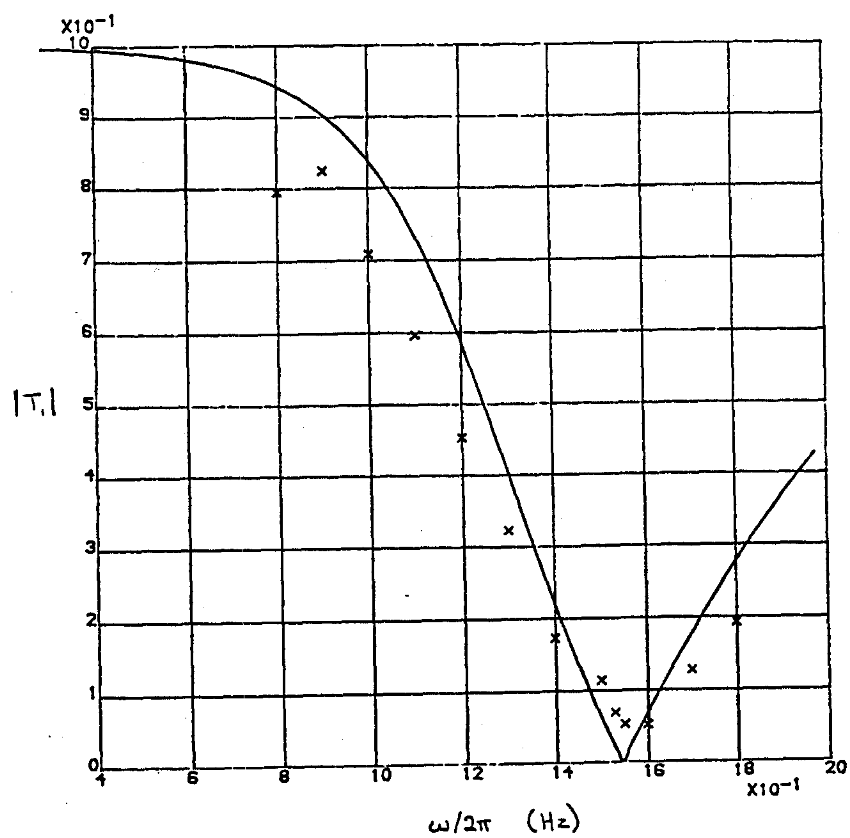
## 3. Results

The figure shows the variation of $|T_1|$ with $\omega/2\pi$ for a submerged horizontal cylinder in a wave tank. The cylinder of diameter 102mm and specific gravity 0.06 was submerged to a clearance of 51mm in water of total depth 204mm. The solid line shows the theoretical prediction, the crosses are the experimental results. There is clearly good agreement.

The experiments were performed with a constant voltage applied to the wavemaker. As a result the incident wave amplitude in the experiments varied between 2mm and 3.5mm. No account was taken of attenuation along the tank which may account for the fact that the experimental readings are slightly lower than the theoretical prediction.

## 4. Conclusion

Further results confirm the ability of the cylinder to reflect an appreciable amount of the wave energy over a broad band-width of frequencies as shown in the figure. The results in the figure imply that a cylinder of radius 4m submerged to a clearance of 4m will reflect over 85% of the incident wave energy for waves with a period of between 4.5 and 7 seconds.



**Tuck**: If this is an *active* absorber, in what way is it caused to move, and how is the phase of the forced motion determined relative to the phase of the wave?

**Linton & Evans**: The device is active in the sense that it moves, rather than implying some external influence on its motion. The motion of the device is purely that induced by the wave motion when it is subject to a restoring force. The word active is included to distinguish between these devices and fixed devices like trenches for which the reflection characteristics are much poorer.

**Martin**: Can you make any progress with cylinders whose cross-sections are not symmetric about a vertical line?

**Linton & Evans**: There is no reason why progress should not be made in the asymmetric case, though the method would be more complicated and it is not clear whether total reflection could be achieved without some energy input.

**Wu**: This is very interesting work, showing how effectively the transmission of incident waves can be reduced. Can this approach be extended to have both wave transmission and reflection be minimized for the purpose of wave-energy extraction?

**Linton & Evans**: This has already been considered by Evans (*J. Fluid Mech.* (1976) vol. 77 p. 1-25) and also by Evans *et al.*(*Applied Ocean Research* vol. 1, 1979) where it was shown that a submerged circular cylinder held down by two cables from each end, making angles of 45 degrees to the horizontal, could, in theory, absorb 100% of the incident wave energy. The idea was subsequently developed as part of the U.K. wave-energy program. See for example Clare *et al.*(Institution of Civil Engineers, 1982).