DETAILS OF A PANEL-METHOD SOLUTION TO THE FIRST-ORDER TRANSIENT RADIATION PROBLEM

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In the past decade, several investigators have reported solutions to first-order transient problems, for instance: Yeung (1982), Newman (1985), Beck and Liapis (1987), and Korsmeyer and Sclavounos (1988). Primarily, their results have consisted of integrated quantities, such as the impulse response function or the force on a freely floating body. In these cases, knowledge of the potential on the body alone is required, and integration obscures any local errors which may be present. If we wish to make other use of these first-order solutions, such as for the quadratic quantities in the right-hand sides of second-order problems, detailed knowledge of the flow field may be necessary.

Since boundary-integral equation formulations and panel methods are the dominant choice for solution of these problems, we investigate the ability of the discretized Green formulation to evaluate the potential, and its spatial and temporal derivatives, in the semi-infinite fluid field surrounding a general 3-D body. As a model problem, this body is a right-circular cylinder in the heave mode; but to retain generality, the panel method is not specialized to this case. A Fredholm-Volterra integral equation may be derived for this problem, which when discretized in a panel method, and normalized by setting the acceleration due to gravity, fluid density and a representative body length equal to one, appears as (Korsmeyer, 1988):

$$\begin{aligned}2\pi\phi^{(1)}_{i,M} + \sum_{j=1}^{N}\phi^{(1)}_{j,M}\iint_{S_j} d\vec{\xi}\, G^{(0)}_{n_\xi}(\vec{x};\vec{\xi}) = &-\sum_{m=0}^{M-1}{}'\Delta t\sum_{j=1}^{N}\phi^{(1)}_{j,m}\iint_{S_j} d\vec{\xi}\, G^{(F)}_{n_\xi t}(\vec{x};\vec{\xi},t_{M-m}) \\ &+\sum_{m=0}^{M}{}'\Delta t\sum_{j=1}^{N}\mathcal{B}^{(1)}_j(t_m)\iint_{S_j} d\vec{\xi}\, G^{(F)}_{t}(\vec{x},\vec{\xi},t_{M-m}) \\ &+\sum_{j=1}^{N}\mathcal{B}^{(1)}_j(t_M)\iint_{S_j} d\vec{\xi}\, G^{(0)}(\vec{x};\vec{\xi}) \\ & i=1,2\ldots N \qquad M=0,1\ldots M_T\ ,\end{aligned} \tag{1}$$

where $\mathcal{B}^{(1)}(t)$ is the right-hand side of the first-order body boundary condition, S_j is the surface of the j^{th} panel, the prime on the summation in time indicates a weight of one-half is applied when $m = 0$, and the Green functions $G^{(0)}$ and $G^{(F)}$ are defined in Newman (1985). As is well understood in potential theory, equation (1) may first be solved for the potential on the body by letting $\vec{x}$ and $\vec{\xi}$ both be on the body surface, and subsequently may be used to compute the potential anywhere in the fluid domain by letting $\vec{x}$ be a field point in that domain, with the factor 2π changed to 4π. In addition, (1) may be used to compute temporal and spatial derivatives of the potential at field points, by first taking a partial derivative with respect to the field point variables.

We would like to evaluate the potential and its spatial and temporal derivatives in the fluid field by another method, in order to evaluate the accuracy of the panel method by comparison. Usually a closed-form solution is sought for this purpose, but in the case of the transient radiation problem there is none available. However we can make beneficial use of a formulation which is often employed for matching nonlinear inner solutions to first-order outer solutions. This matching takes place on a vertical, right-circular cylinder which encloses the nonlinear portion of the solution and is of semi-infinite vertical extent. Exterior to the cylinder is a first-order potential which may be expressed in terms of the radial fluid velocity through the cylinder and a Green function. For the present application, the interior potential *is* the first-order solution and the surface over which the radial fluid velocity must be evaluated is an infinitely deep continuation of the body itself. We can express the first-order, axisymmetric potential exterior to the infinite cylinder (of radius equal to 1) by:

$$\begin{aligned}\phi^{(1)}(r,z,t) = &\int_{-\infty}^{0} dz' \phi_r^{(1)}(1,z',t) D^{(0)}(r,z;1,z') \\ &+ \int_0^t d\tau \int_{-\infty}^{0} dz' \phi_r^{(1)}(1,z',t) D^{(F)}(r,z;1,z',t-\tau),\end{aligned} \tag{2}$$

where z' is on the infinite cylinder, (r,z) is a field point, and the Green function, $D = D^{(0)} + D^{(F)}$, is an axisymmetric function, with Rankine portion:

$$D^{(0)} = \frac{1}{\pi}\int_0^\infty dk\, \frac{1}{k}\mathcal{L}(k,r)\left[e^{-k|z-z'|} - e^{-k|z+z'|}\right]; \tag{3}$$

and free-surface wave portion:

$$D^{(F)} = \frac{2}{\pi}\int_0^\infty dk\, \frac{1}{\sqrt{k}}\sin(\sqrt{k}t)\mathcal{L}(k,r)e^{-k|z+z'|}; \tag{4}$$

in which $\mathcal{L}(k,r)$ is a rational function of ordinary Bessel functions:

$$\mathcal{L}(k,r) = \frac{J_0(kr)Y_1(k) - J_1(k)Y_0(kr)}{J_1^2(k) + Y_1^2(k)}. \tag{5}$$

In the application of (1) (and its partial derivatives) to find quantities in the fluid field, particularly on the free surface, several numerical difficulties arise. We find that very close to the free-surface-body intersection, the potential or its derivatives do not converge, and in fact 'blow-up', in spite of the fact that there is no singularity at this intersection for this problem. Also, near the body there are large-amplitude short waves that may be resolved, but only by great computational effort. These two problems make the evaluation of the free-surface integral in the second-order problem difficult by a panel method, and we suspect that they are a result of the evaluation of field quantities close to the edge of a constant strength panel.

Note that in (2), $\phi_r^{(1)}$ vanishes on the portion of the infinite cylinder which is the body itself, so that for field points on the free surface, $|z \pm z'|$ is never smaller than the extent of the body draft. The effect is that even though the panel method may be employed to find $\phi_r^{(1)}(1, z, t)$ on the infinite cylinder, the constant-strength panels near the free surface, in a Green formulation like (1), play no direct role in the evaluation of quantities on the free surface in (2). A further benefit of this formulation is that the behavior of the first-order potential and its derivatives is more apparent than it is in (1). In equation (2) we may observe that the non-zero limit of $|z \pm z'|$ ensures that very short waves are indeed damped exponentially with wave number, so that at large time, very close to the body, we can expect no large-amplitude, rapid oscillation of the free surface.

The implementation of (2) requires that (1) be used to find $\phi^{(1)}(\vec{x}, t)$ on the body, and $\phi_r^{(1)}(1, z, t)$ on a portion of the infinite cylinder. $\phi_r^{(1)}(1, z, t)$ is required from the corner of the body to a truncation point at sufficient depth to allow accurate evaluation of free-surface quantities. Computational effort may be reduced by matching the computed velocity at that point, to that due to a dipole located at the origin. Accurate evaluation of $\phi^{(1)}(r, z, t)$ close to the submerged body corner cannot be expected, because like the free-surface-body intersection, this requires field points close to the edge of a panel. However, empirical evidence suggests that depth improves this situation, and more importantly depth allows an infinite-fluid analytical approximation to the locally two-dimensional, corner flow.

Comparison of the results from the two methods shows that the apparently singular behavior of the potential and its derivatives at the free-surface-body intersection is an artifact of the panel method solution to the Green formulation. The results also show that the large-amplitude short waves in the solution of (1) are due to the discrete form of the Green formulation. As discussed above, (2) does not predict this large-amplitude behavior. Rather, we see that the initial impulsive disturbance of the free surface, which contains all wave lengths, disperses with increasing time and wave amplitude is attenuated exponentially with wave number. Away from the free-surface-body intersection the results of both methods are in excellent agreement.

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