RADIATION PROBLEM OF A TWO-DIMENSIONAL SURFACE-PIERCING BODY WITH FORWARD SPEED

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ABSTRACT

When we analyse the unsteady flow field around a ship advancing and oscillating on the free surface in the framework of linear theory, we usually adopt the classical linearized free-surface condition which takes into account only the contribution of uniform flow:

$$K\phi + i2\tau\frac{\partial\phi}{\partial x} - \frac{1}{K_0}\frac{\partial^2\phi}{\partial x^2} + \frac{\partial\phi}{\partial y} = 0 \qquad \text{on } y = 0 \tag{1}$$

where $$K=\omega^2/g, \quad \tau=U\omega/g, \quad K_0=g/U^2 \tag{2}$$

In eq.(2), ω denotes the circular frequency of oscillation, U the forward speed of a ship, and g the acceleration of gravity.

Numerical solutions of radiation problem for a 2-D surface piercing body satisfying eq.(1) does not satisfy the principle of energy conservation; the damping coefficient by the pressure integration over the wetted portion of the body is not equal to the one obtained from the amplitude of outgoing waves at infinity, and hydrodynamic forces computed sometimes fluctuate anomalously against the frequency [1][2]. This fault in those solutions is certainly attributed to the singularities of the fluid velocity at intersections of body and free surfaces; the so-called line integral term. (The term "line integral" is not appropriate for the 2-D problem, but for convenience we use here this term.) Ursell [3] discussed this line integral term in the 2-D "Neumann-Kelvin" problem and proved that a unique solution can be obtained provided the boundedness of the fluid velocity is assumed at the intersection points. This solution is referred to as the "least singular" solution. He also showed that the least singular solution can be constructed either by the multipole expansion method or by the integral equation method.

With the same line as Ursell's for the steady translation problem, we gave numerically the "least singular" solution of the unsteady problem with forward speed by the multipole expansion method. The derivation of velocity potential and the numerical scheme based on this multipole expansion method are briefly described and then the results of numerical computations are discussed in the first half of

the present study. As seen in Figure 1, there seems to be no anomalous fluctuation of hydrodynamic forces against the frequency, but the satisfaction of the energy conservation principle is not improved.

Zhao & Faltinsen [4] may be the first to have obtained a numerical solution including the effect of steady perturbation flow on the free surface condition. They neglected the terms of order U^2 on the assumption of small forward speed; this makes the wave system at infinity reduce to only two components with longer different wavelength (other two wave components with shorter wavelength, which exist if the terms of $O(U^2)$ are not neglected, disappear) and thus makes the problem easy to be solved.

In the present study, we made no assumptions on the order of U and/or ω except that the unsteady potential is of small order of quantity. Retaining consistently the contribution of steady perturbation potential ϕ_S which satisfies the rigid-wall free surface condition, we can obtain the modified free surface condition correct to $O(U^2)$ in the following form [4]

$$K\phi - i2\tau\frac{\partial\phi_S}{\partial x}\frac{\partial\phi}{\partial x} - \frac{1}{K_0}\left(\frac{\partial\phi_S}{\partial x}\right)^2\frac{\partial^2\phi}{\partial x^2} + \frac{\partial\phi}{\partial y}$$

$$-i\tau\frac{\partial^2\phi_S}{\partial x^2}\phi + \frac{1}{2K_0}\left\{1 - \left(\frac{\partial\phi_S}{\partial x}\right)^2\right\}\left(\frac{\partial^2\phi}{\partial x^2} - K\frac{\partial\phi}{\partial y}\right) - \frac{3}{K_0}\frac{\partial\phi_S}{\partial x}\frac{\partial^2\phi_S}{\partial x^2}\frac{\partial\phi}{\partial x} = 0 \qquad (3)$$

$$\text{on } y = 0$$

Since ϕ_S becomes $-x$ at infinity, eq.(3) reduces to eq.(1) at a large distance from the body. In the second half of the present study, we show through numerical calculations that if the free surface condition (3) is used in place of the classical free-surface condition (1), a solution for the radiation problem satisfies the principle of energy conservation within the allowable numerical error.

Following Zhao & Faltinsen, we utilized the integral equation method with the fundamental logarithmic singularity as the Green function. But unlike their numerical scheme, a circular boundary surrounding the body is chosen as the radiation boundary, on which the multipole expansions derived in this study are introduced to impose the radiation condition. The numerical scheme employed is essentially analogous to the "hybrid" method contrived by Nestegard & Sclavounos [5] for the zero-speed problem. In this study, we refer to our numerical calculation method as the "hybrid" method.

An example of the numerical computation, the surge damping coefficient, is shown in Figure 1 for a half-immersed circular cylinder with Froude number 0.128. The result at zero forward speed is also given in this figure for comparison. It can be seen from this figure that the effect of forward speed on the damping coefficient is small. However it should be kept in mind that the effect of forward speed arises also from the body boundary condition owing to the interaction between steady and unsteady flow fields, and this effect tends to suppress the forward speed effect arising from the free surface condition.

The numerical accuracy of the solutions by the present calculation scheme is not always improved as the number of segments on the free surface increases beyond certain limit. Presumably there might be some inaccuracy in the numerical differentiation of the potential on the free surface. We are now investigating further to overcome this problem and studying the mathematical reason why the principle of energy conservation is satisfied when the free surface condition (3) is used in place of the classical free surface condition (1).

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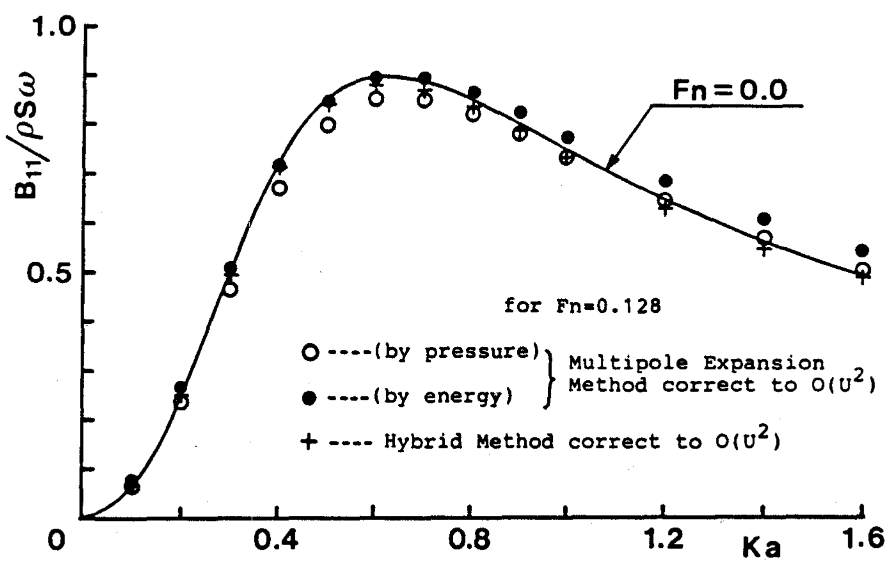

Figure 1 Surge damping coefficient of a half-immersed circular cylinder (Fn=0.0, 0.128)

Grue: When the discussed solution does not fulfill conservation equations (energy or mass) it tells us that something is wrong with the solution, not that the energy relation is wrong. The energy relation is not affected by the free-surface boundary condition close to the body.

Kashiwagi & Ohkusu: The expression for the energy relation is not affected, but the difference in the free-surface condition near the body is reflected in the Kochin function. As you may know, there is not such a shortcoming in the submerged-body problem regardless of whether the steady disturbance is included. In addition, judging from that, the energy relation seems to be improved by using the modified free-surface condition. It is likely that a problem exists near (or possibly at) the intersection of the body and the free-surface, especially when the classical free surface condition is used.

Faltinsen: When you solve the problem with the classical free-surface condition you show results with three different methods. Could you comment on why you get differences between the methods in some cases. Is the same boundedness condition used? Do the differences become more pronounced for variables which are more sensitive to Froude number than the added-mass and damping?

Kashiwagi & Ohkusu: The results from the integral-equation method are coincident with those from the multipole-expansion method except for anomalous fluctuations. Although almost the same results are obtained by the hybrid method, there exist slight differences as you point out. The former two methods will produce the same least singular solution, but we do not use any special treatment for the singularity at intersection points in the hybrid methods, which might be a reason. However as long as we use the integral equation based on Green's theorem, the solution will be a least singular solution. In the presentation, we compared the values of the damping coefficient, but of course we have to compare the diffraction force and the wave drift force, or the pressure distribution.

Palm: When you use the "classical" boundary conditions you find that the energy relation is not satisfied. However, why is the energy relation not satisfied when you use the modified (correct) boundary conditions?

Kashiwagi & Ohkusu: We presume that the energy relation must be satisfied when the free-surface condition is modified. Judging from the numerical results indicating that the satisfaction of energy relation is improved as compared to the case of the classical free-surface condition, I think that the reason why the energy relation is not satisfied as well as we might expect is due to numerical inaccuracy.

Newman: I do not understand why the flux across the plane y=0 should be zero, so long as it is an oscillatory quantity with zero time-average.

Kashiwagi & Ohkusu: We explain this as an extension of the steady problem. In the steady Neumann-Kelvin problem, the flux of water becomes

$$Q = -\frac{1}{k_0}\left[\frac{\partial \chi}{\partial x}\right]_{-1}^{1}$$

where χ denotes the steady perturbation potential. $\partial\chi/\partial x$ at intersection points is not zero. This fact means that the stream line on the free surface is not continuous to the body surface. This flow seems strange from the physical point of view. The same kind of argument is possible in the unsteady problem. We understand your point is quite important. In order to convince you, it seems to me that we have to consider the energy flux across the free surface.