NONLINEAR WAVE-BODY MOTION IN A CLOSED DOMAIN

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A finite-difference method based on boundary-fitted coordinates is combined with a Lagrangian description of the free surface to solve the time-dependent nonlinear free surface problem.

The forced nonlinear motion of a body in a tank is considered. The problem is formulated as an initial-value problem based on potential theory. The initial conditions are taken as that at time equals to zero, both the velocity potential and free surface elevation are zero. Then the body is set into a prescribed oscillatory motion of constant amplitude.

Automatic grid generation is introduced by solving a set of Poisson's equations, which transforms an irregular domain (x, y) into a regular (usually rectangular) domain (ξ,η) so that boundary conditions can be employed more conveniently. An additional advantage of the boundary-fitted grid is that it is able to adjust to large deformation of the boundary, e.g., the free surface.

The Laplace equation and other boundary conditions, except free surface conditions, are then tranformed into the (ξ , η)-plane. To discretize the equations, a nine-point central-difference scheme is used inside the domain and a five-point one-sided difference scheme of second order is used on the boundaries with Neumann Boundary conditions. Then the resulting linear system is solved by a Gauss elemination and iterative methods for banded matrix.

To cope with large deformation of the free surface, a Lagrangian description based on that of Longuet-Higgins and Cokelet (1976) is used. In our combined Lagrangian-boundary-fitted coordinate method, the free surface conditions are advanced by a second-order predictor-corrector scheme. The velocity potential and (x,y) values of free surface particles are obtained in both predictor and corrector stages, and hence enter the solution of Laplace equation as a Dirichlet boundary condition.

Linear and nonlinear numerical results of free surface elevations and forces on a heaving two dimensional cylinder in a closed tank are obtained and are compared. The linear results are obtained by simply linearizing all boundary conditions. Numerical results indicate that when the amplitude of body motion is large, the linear and nonlinear free surface elevations are quite different from each other, especially at relatively large time when waves have had a substantial opportunity to interact among themselves. The nonlinear free surface becomes quite irregular at relatively large time and waves with shallower trough and steeper crest are observed. The linear free surface retains a smooth and regular behavior. It is also observed that the dynamic nonlinear force is more negative than the linear force for large amplitude of body motion.

As a stability and convergence study, the energy in the fluid and the work done by the body to the fluid are calculated for both linear and nonlinear cases. Even in the absence of numerical smoothing of the free-surface profile, the numerical calculations are very stable and have been carried to very large time. The energy error becomes smaller as the mesh is refined or the time interval is reduced. Excellent convergence characteristics of the numerical scheme is observed. The normalized energy error does not exceed 3% for all time.

In order to have a better understanding of the singular behavior at the intersection point of body and free surface, an analysis based on a local solution near the intersection point is performed. It is found that there exists a critical local intersection angle of the free surface and the body which gives rise to either a singular or nonsingular local flow. (a) If the local intersection angle is $\pi/2$, i.e., the side wall of a body is vertical near the free surface, a logarithmic singularity in velocity is found in the swaymotion problem, which is consistent with Lin (1984)'s result. However in the heave-motion problem, if the local intersection angle is $\pi/2$, no singularity is found in the velocity. (b) If the local intersection angle is large than $\pi/2$, e.g., the body is wedgelike, an integrable singularity is found. (c) If the local intersection angle is less than $\pi/2$, no singularity in the velocity can be found. This analysis suggests that the singularity for a wedge is more severe than that of a vertical wall.

References

1. Longuet-Higgins, M.S. and Cokelet, E.D., 'The Deformation of Steep Surface Waves on Water ---I. A Numerical Method of Computation', Proc. R. Soc. London, Series A. 350,1-26 (1976)

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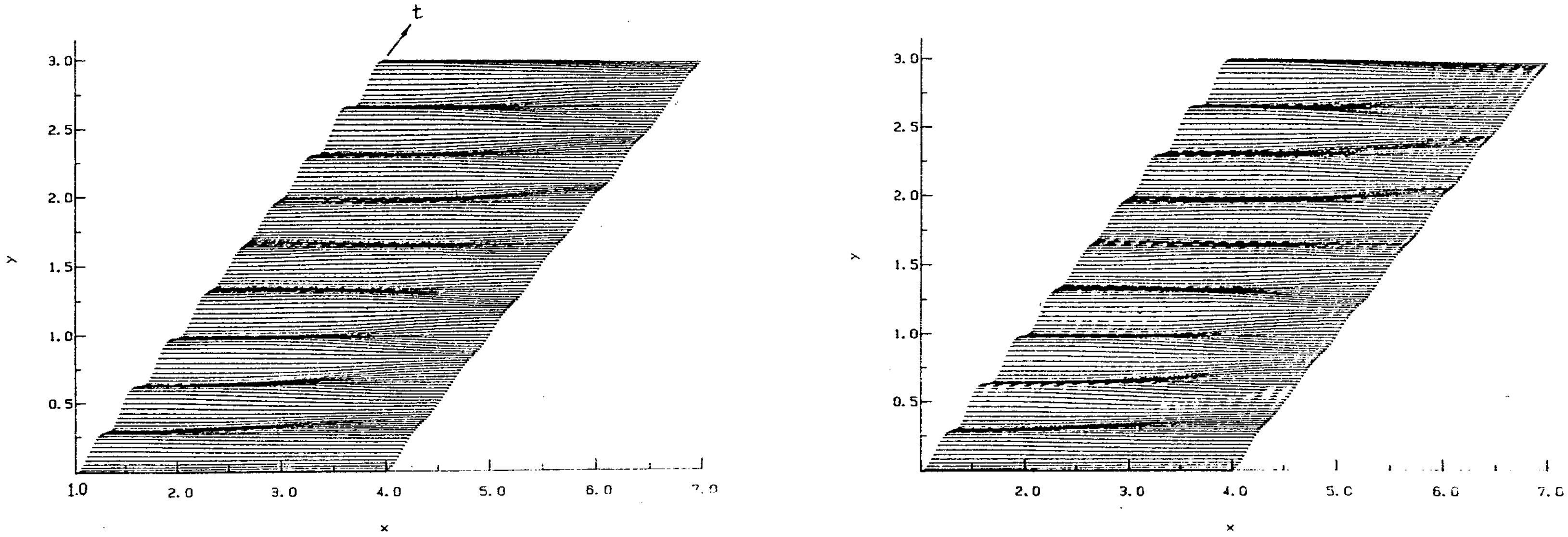

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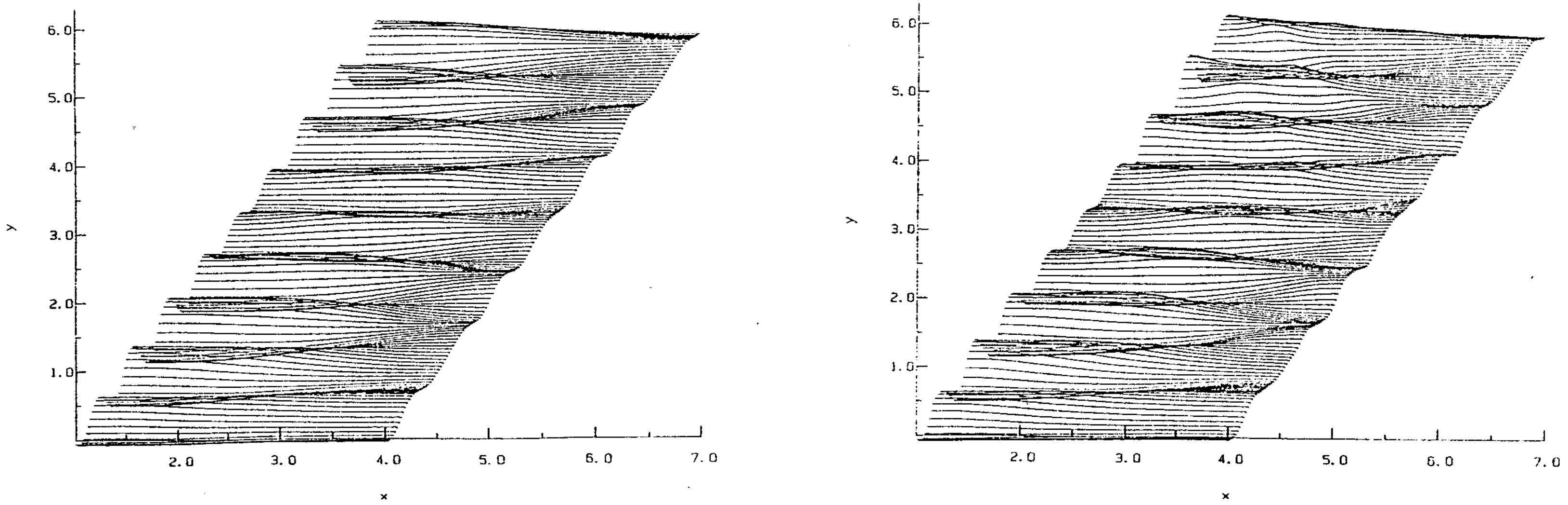

amplitude = .50

Fig. 1 Free surface elevations excited by forced body oscillations at $\omega\sqrt{\frac{a}{g}} = 1.253$,